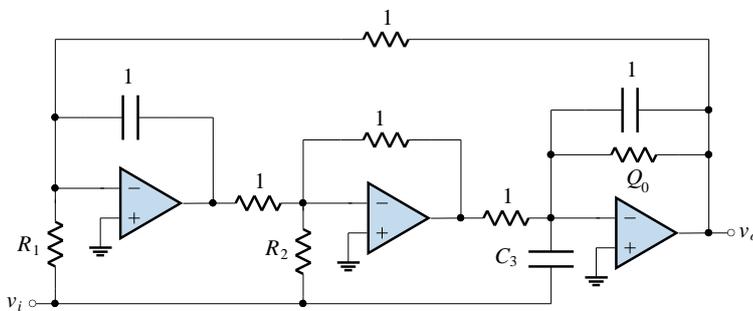
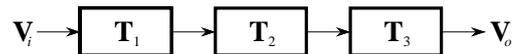
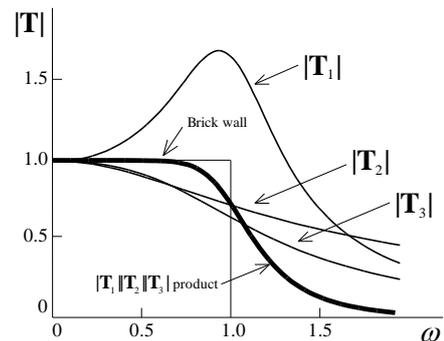


Analog Electronics

Topic Notes

2020



PMcL

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Answers

1 Simple Filters

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Introduction

Filters are essential to electrical engineering. They are used in *all* modern electronic systems. In communications, filters are essential for the generation and detection of analog and digital signals, whether via cable, optic fibre, air or satellite. In instrumentation, filters are essential in “cleaning up” noisy signals, or to recover some “special” part of a complicated signal. In control, feedback through a filter is used to achieve a desired response. In power, filters are used to inject high frequency signals on the power line for control purposes, or for removing harmonic components of a current. In machines, filters are used to suppress the generation of harmonics, or for controlling switching transients. The design of filters is therefore a useful skill to possess.

Filters can be of two types: analog and digital. In this subject, we will concentrate on analog filters. There are two reasons for this: analog filters are necessary components in “digital” systems, and analog filter theory serves as a precursor to digital filter design. The analog filters we will be looking at will also be of two types: passive and active. Active filters represent the most common, and use electronic components (such as op-amps) for their implementation. This is opposed to passive filters, which use the ordinary circuit elements: resistors, capacitors, inductors.

Filter has the commonly accepted meaning of something retained, something rejected. There are many examples of filters in everyday life:

- You’re being a filter right now. You have removed external distractions and are concentrating on reading these notes. Your brain is filtering out the unnecessary things going on around you (for a while, anyway).
- The media are information filters. They supposedly decide for us what is important information, and what is not. We rarely have time to investigate a topic for ourselves, and so we rely on them to pass us the essential points. The extreme of this is propaganda.
- More tangible filters are sunglasses, tinted windows, ear muffs, air filters, flour sifters, radios, TVs, etc.

For us, a filter is very simple: it is an electric circuit designed to implement a specific transfer function. Given a filter, obtaining the transfer function is just a matter of applying circuit theory. This is analysis. The choice of a transfer function and the choice of an implementation for a filter, however, are never unique. This is called design.

A filter is a circuit that implements a specific transfer function

To firstly analyse, and then design, active analog filters, a review of op-amps, transfer functions and frequency response is beneficial.

1.1 Op-Amp Circuits

A simplified model of an op-amp is:

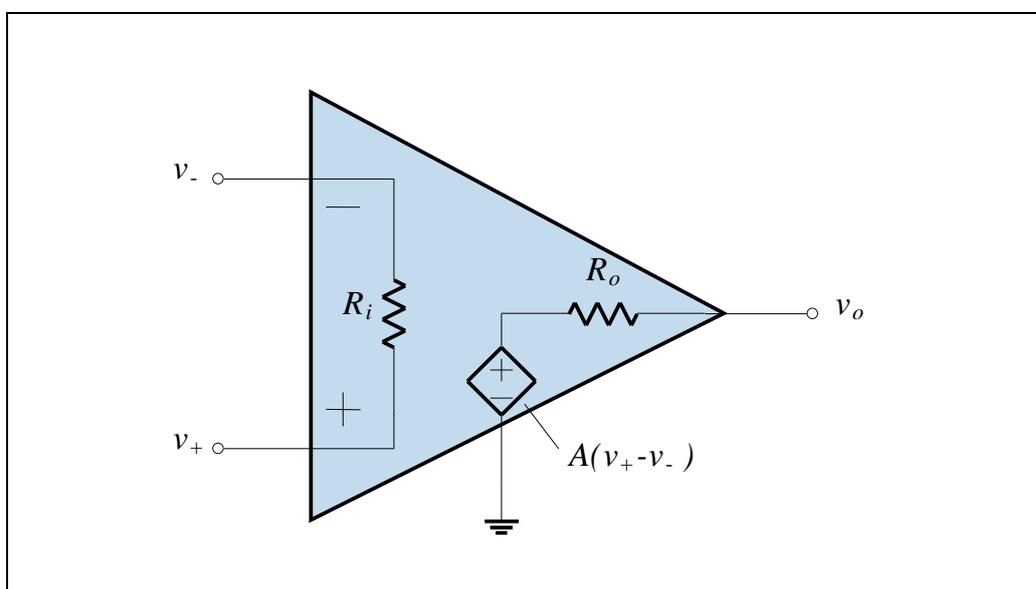


Figure 1.1

For most applications, we assume the op-amp is ideal.

1.4

An ideal op-amp has the following parameter values:

The ideal op-amp's parameters

$$\begin{aligned} A &= \infty \\ R_i &= \infty \\ R_o &= 0 \end{aligned} \tag{1.1}$$

If there is a *negative* feedback path (ie a connection between the output of the amplifier and the (-) input terminal), then the op-amp will have a finite output voltage. It follows that:

$$\begin{aligned} v_o &= A(v_+ - v_-) \\ v_+ - v_- &= \frac{v_o}{A} = \frac{v_o}{\infty} = 0 \\ v_+ &= v_- \end{aligned} \tag{1.2}$$

The *virtual* short circuit is the key to analysing op-amp circuits

The input to the op-amp looks like a short circuit for voltages, but due to the input resistance being infinite, it looks like an open circuit for currents. The input terminals can therefore be considered a *virtual short circuit*. We will use the virtual short circuit concept frequently.

1.1.1 Inverting Amplifier

An inverting amplifier is constructed as:

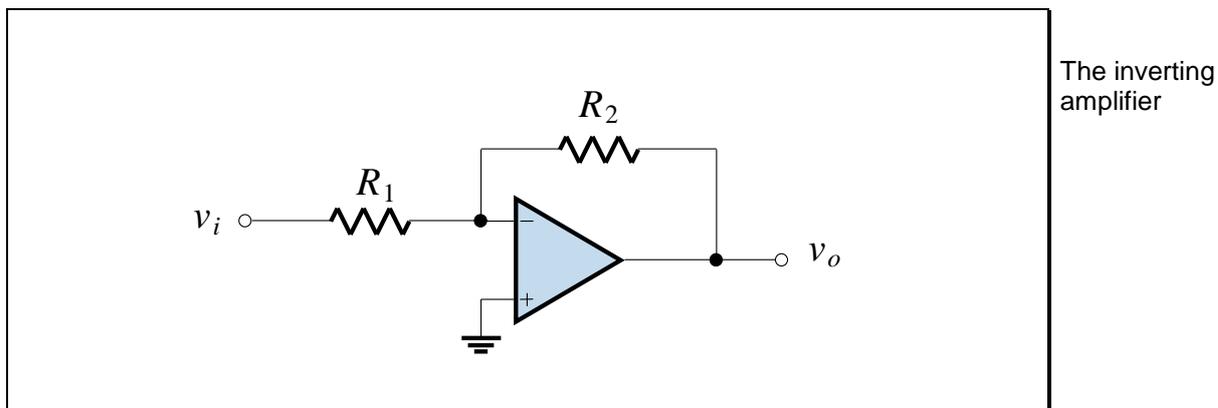


Figure 1.2

We analyse the circuit using a virtual short circuit:

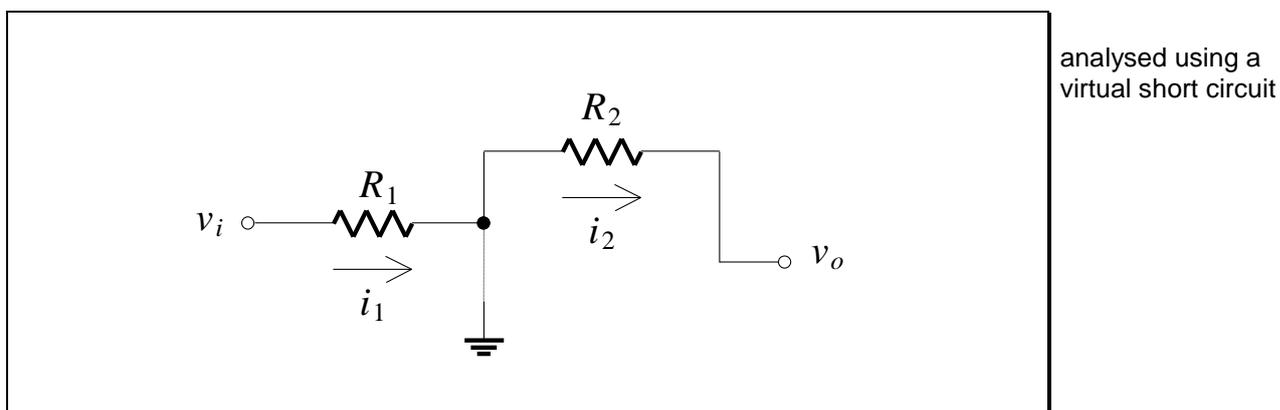


Figure 1.3

KCL at the middle node gives:

$$\frac{v_i}{R_1} = \frac{-v_o}{R_2}$$

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1} \quad (1.3)$$

1.6

1.1.2 Non-Inverting Amplifier

A non-inverting amplifier and its equivalent circuit for analysis are:

The non-inverting amplifier analysed using a virtual short circuit

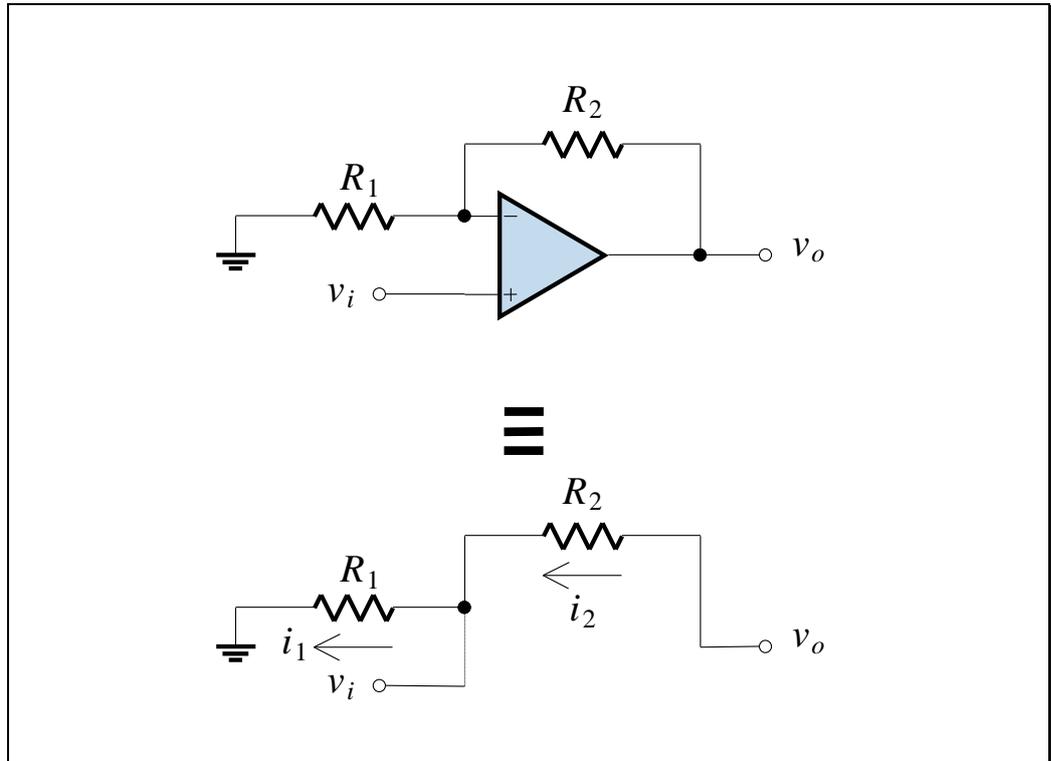


Figure 1.4

KCL at the middle node gives:

$$\frac{v_i}{R_1} = \frac{v_o - v_i}{R_2}$$
$$\frac{v_o}{v_i} = 1 + \frac{R_2}{R_1} \quad (1.4)$$

1.1.3 The Voltage Follower

A voltage follower is a special case of the non-inverting amplifier circuit. If we set $R_1 = \infty$, then any value of R_2 will give the gain as:

$$\frac{v_o}{v_i} = 1 \quad (1.5)$$

A simple value of R_2 is 0, a short circuit. The voltage follower circuit is therefore:

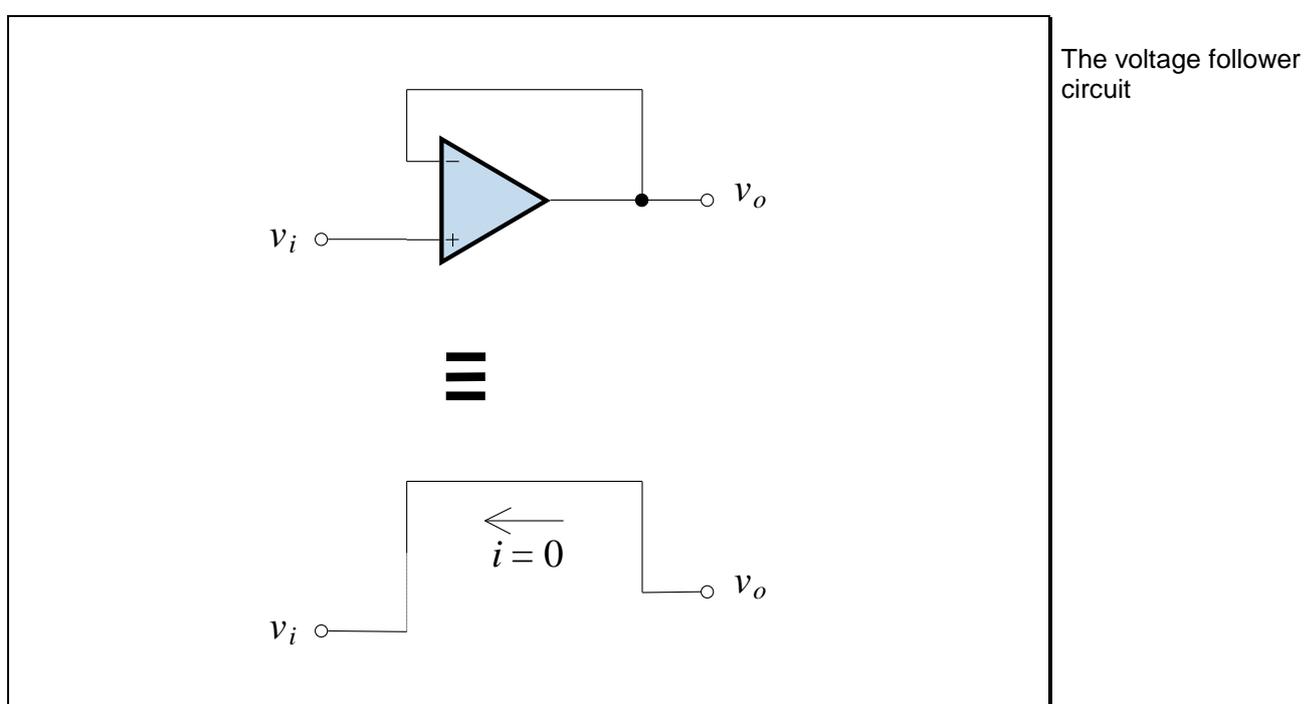


Figure 1.5

The circuit differs from a piece of wire in that the input resistance for v_i is infinite and the controlled-source nature of the circuit provides isolation. It is sometimes called a unity-gain *buffer*.

acts as a buffer between two parts of a circuit

The voltage follower is used to provide isolation between two parts of a circuit when it is required to join them without interaction.

1.8

For example, to couple a high resistance source to a low resistance load, without suffering a voltage drop, we insert a buffer between source and load:

A buffer is used to couple a high impedance to a low impedance

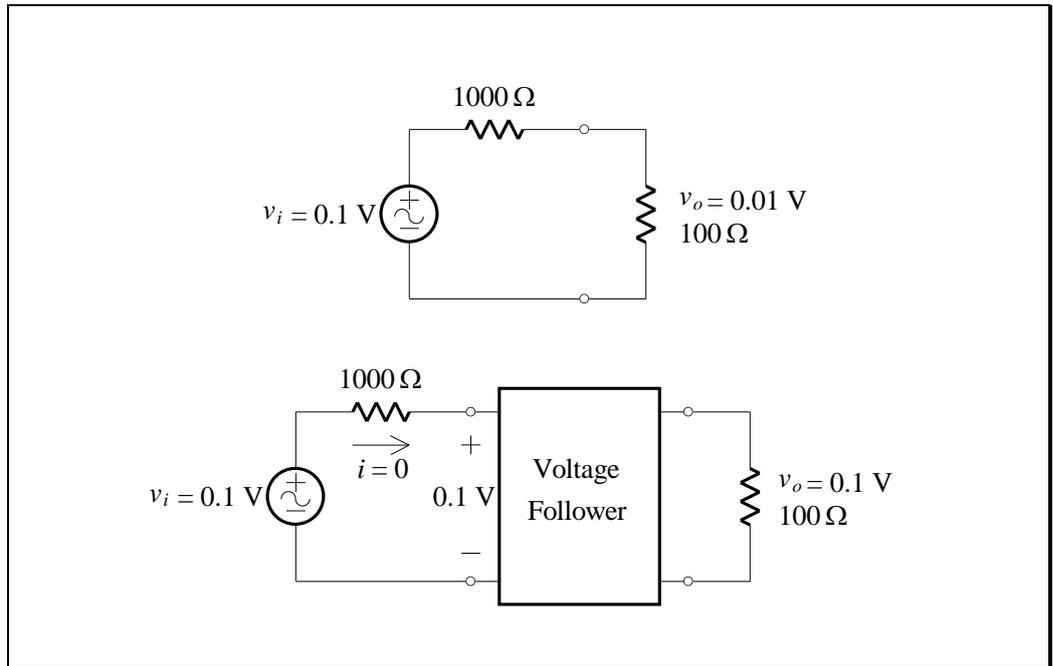


Figure 1.6

1.2 Bilinear Transfer Functions

Filter design and analysis is predominantly carried out in the frequency domain. The circuits we design and analyse will be assumed to be operating with sinusoidal sources and be in the steady state. This means we can use phasors and complex numbers to describe a circuit's response.

Define the voltage ratio transfer function as:

$$\mathbf{T}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i}$$

(1.6) Transfer function defined

For example, consider a simple RC circuit:

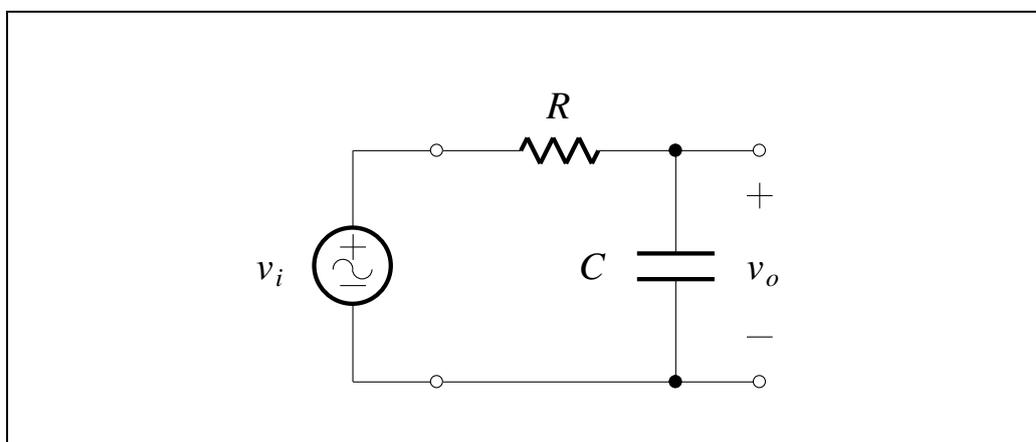


Figure 1.7

The transfer function is:

$$\mathbf{T}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1/RC}{s + 1/RC} \quad (1.7)$$

When a transfer function is the quotient of two linear terms like Eq. (1.7), it is said to be *bilinear*. Thus a bilinear transfer function is of the form:

$$\mathbf{T}(s) = \frac{a_1s + a_2}{b_1s + b_2}$$

(1.8) Bilinear transfer function defined

1.10

where the a and b constants are real numbers. If Eq. (1.8) is written in the form:

and rewritten in terms of poles and zeros

$$\mathbf{T}(s) = \frac{a_1 s + a_2/a_1}{b_1 s + b_2/b_1} = K \frac{s + z}{s + p} \quad (1.9)$$

then $-z$ is the *zero* of $\mathbf{T}(s)$ and $-p$ is the *pole* of $\mathbf{T}(s)$. In the s plane the pole and zero are located at:

$$s = -z \quad \text{and} \quad s = -p \quad (1.10)$$

Since z and p are real, the zero and pole of $\mathbf{T}(s)$ are located on the real axis of the s plane. For real circuits, p will always be on the negative real axis, while z may be on either the positive or negative part of the real axis.

For $\mathbf{T}(s)$ as in Eq. (1.7), the pole-zero plot in the s plane is:

The pole-zero plot of a bilinear transfer function

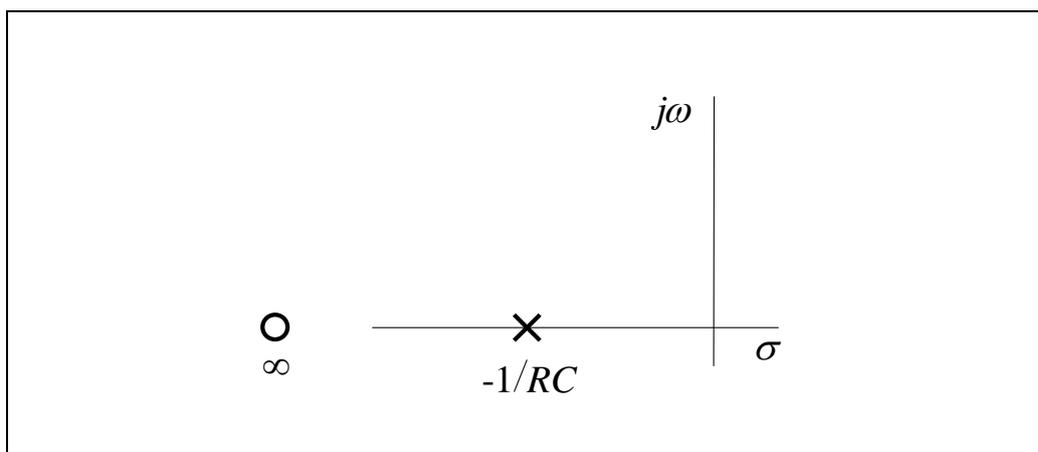


Figure 1.8

1.3 Frequency Response Representation

The sinusoidal steady state corresponds to $s = j\omega$. Therefore, Eq. (1.7) is, for the sinusoidal steady state, the frequency response:

$$\mathbf{T}(j\omega) = \frac{\omega_0}{j\omega + \omega_0}, \quad \omega_0 = \frac{1}{RC} \quad (1.11)$$

The complex function $\mathbf{T}(j\omega)$ can also be written using a complex exponential in terms of magnitude and phase:

$$\mathbf{T}(j\omega) = |\mathbf{T}(j\omega)|e^{j\theta(\omega)} \quad (1.12)$$

which is normally written in polar coordinates:

$$\mathbf{T}(j\omega) = |\mathbf{T}(j\omega)|\angle\theta(\omega) \quad (1.13)$$

The transfer function in terms of magnitude and phase

We also normally plot the magnitude and phase of $\mathbf{T}(j\omega)$ as a function of ω or f . We use both linear and logarithmic scales.

If the logarithm (base 10) of the *magnitude* is multiplied by 20, then we have the *gain* of the transfer function in decibels (dB):

$$A(\omega) = 20\log_{10}|\mathbf{T}(j\omega)| \text{ dB} \quad (1.14)$$

The magnitude of the transfer function in dB

A *negative* gain in decibels is referred to as *attenuation*. For example, -3 dB gain is the same as 3 dB attenuation.

The phase function is usually plotted in degrees.

1.12

EXAMPLE 1.1 Frequency Response of an RC Lowpass Circuit

For the RC circuit, let $\omega_0 = 1/RC$ so that the frequency response can be written as:

$$\mathbf{T}(j\omega) = \frac{1}{1 + j\omega/\omega_0}$$

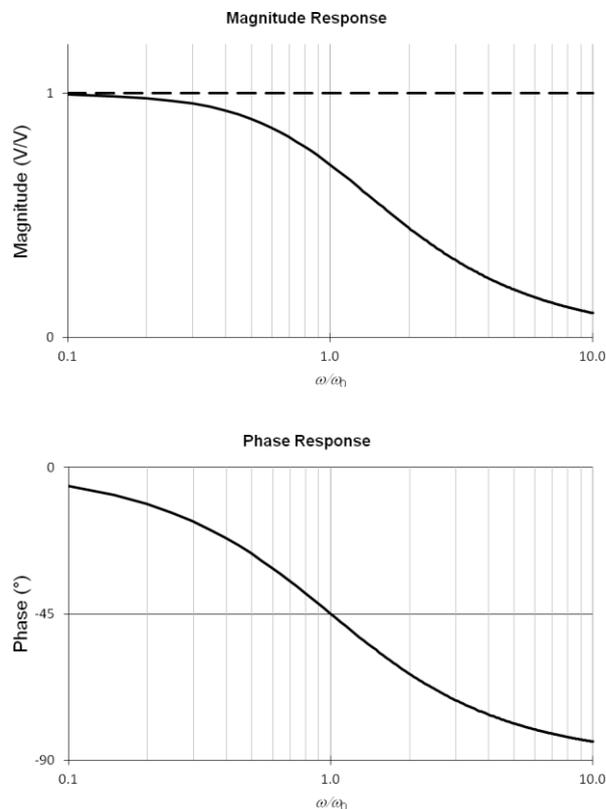
The magnitude function is found directly as:

$$|\mathbf{T}(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}$$

The phase is:

$$\theta(\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

These are graphed below, using a normalised log scale for ω :



1.4 Magnitude Responses

A magnitude response is the magnitude of the transfer function for a sinusoidal steady-state input, plotted against the frequency of the input. Magnitude responses can be classified according to their particular properties. To look at these properties, we will use linear magnitude versus linear frequency plots. For the RC circuit of Figure 1.7, the magnitude function has three frequencies of special interest corresponding to these values of $|\mathbf{T}(j\omega)|$:

The magnitude response is the magnitude of the transfer function in the sinusoidal steady state

$$\begin{aligned} |\mathbf{T}(j0)| &= 1 \\ |\mathbf{T}(j\omega_0)| &= \frac{1}{\sqrt{2}} \approx 0.707 \\ |\mathbf{T}(j\infty)| &\rightarrow 0 \end{aligned} \tag{1.15}$$

The frequency ω_0 is known as the *half-power frequency*. The plot below shows the complete magnitude response of $\mathbf{T}(j\omega)$ as a function of ω , and the circuit that produces it:

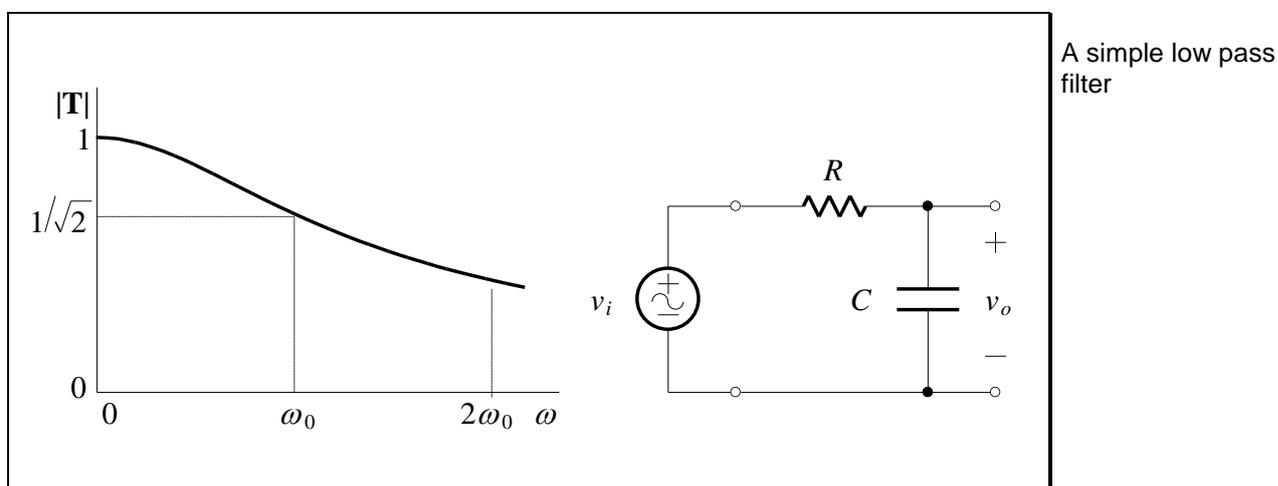


Figure 1.9

1.14

An idealisation of the response in Figure 1.9, known as a *brick wall*, and the circuit that produces it are shown below:

An ideal lowpass filter

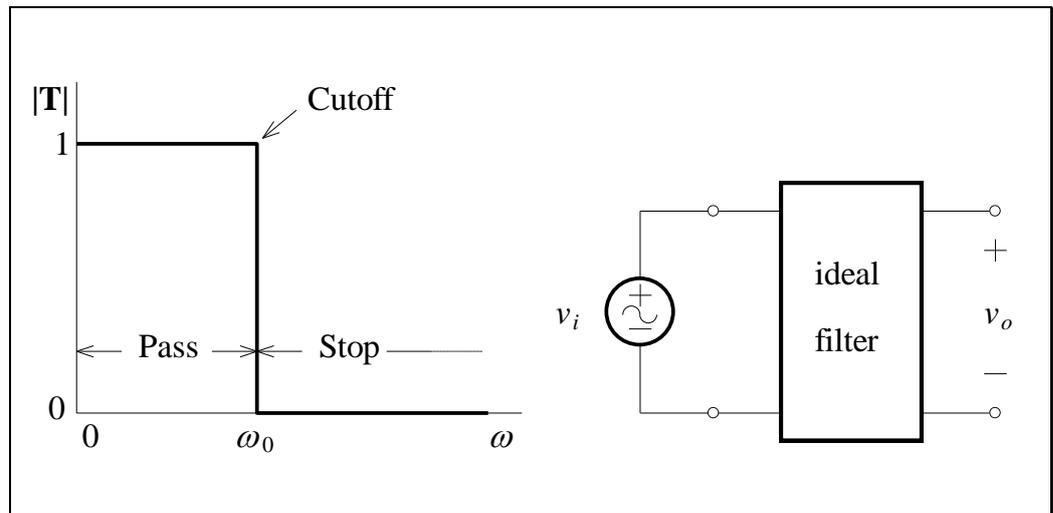


Figure 1.10

For the ideal filter, the output voltage remains fixed in amplitude until a critical frequency is reached, called the *cutoff frequency*, ω_0 . At that frequency, and for all higher frequencies, the output is zero. The range of frequencies with output is called the *passband*; the range with no output is called the *stopband*. The obvious classification of the filter is a *lowpass* filter.

Pass and stop bands defined

Even though the response shown in the plot of Figure 1.9 differs from the ideal, it is still known as a lowpass filter, and, by convention, the half-power frequency is taken as the cutoff frequency.

If the positions of the resistor and capacitor in the circuit of Figure 1.9 are interchanged, then the resulting circuit is:

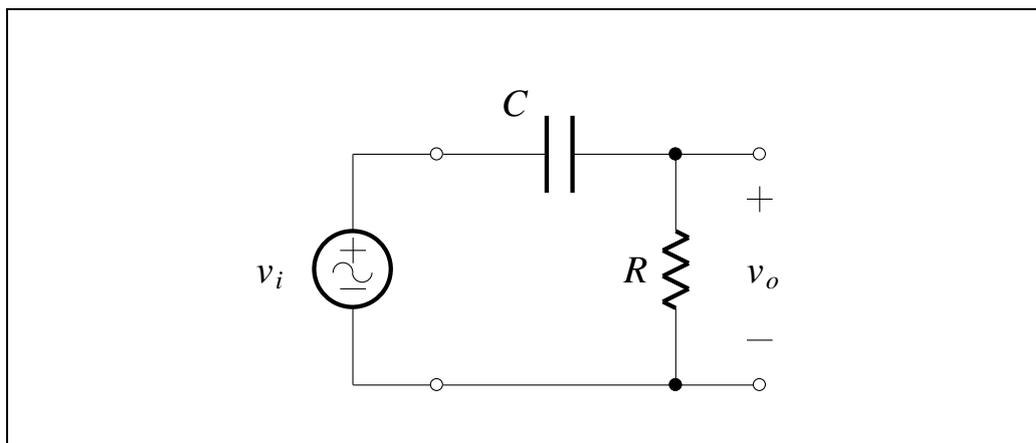


Figure 1.11

Show that the transfer function is:

$$\mathbf{T}(s) = \frac{s}{s + 1/RC} \quad (1.16)$$

Letting $1/RC = \omega_0$ again, and with $s = j\omega$, we obtain:

$$\mathbf{T}(j\omega) = \frac{j\omega/\omega_0}{1 + j\omega/\omega_0} \quad (1.17)$$

The magnitude function of this equation, at the three frequencies given in Eq. (1.15), is:

$$\begin{aligned} |\mathbf{T}(j0)| &= 0 \\ |\mathbf{T}(j\omega_0)| &= \frac{1}{\sqrt{2}} \approx 0.707 \\ |\mathbf{T}(j\infty)| &\rightarrow 1 \end{aligned} \quad (1.18)$$

1.16

The plot below shows the complete magnitude response of $T(j\omega)$ as a function of ω , and the circuit that produces it:

A simple highpass filter

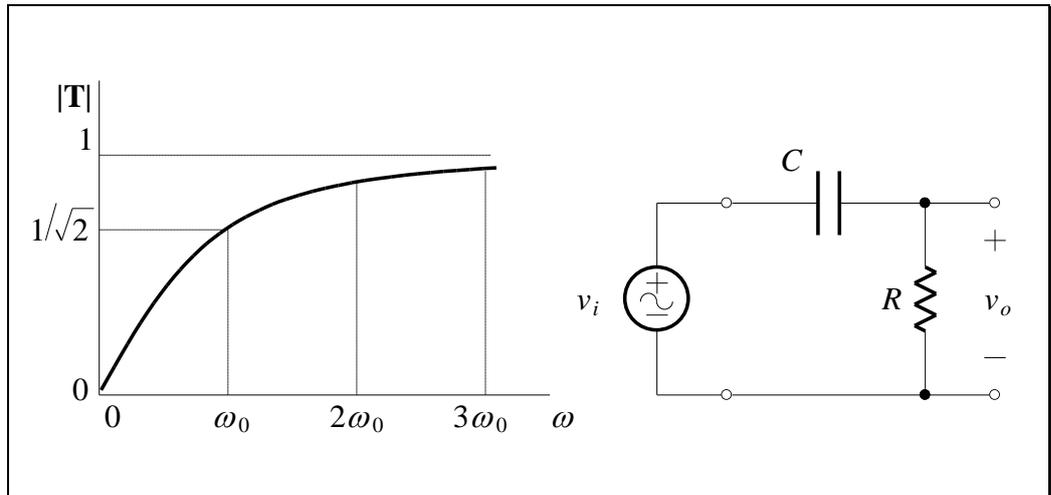


Figure 1.12

This filter is classified as a *highpass filter*. The ideal brick wall highpass filter is shown below:

An ideal highpass filter

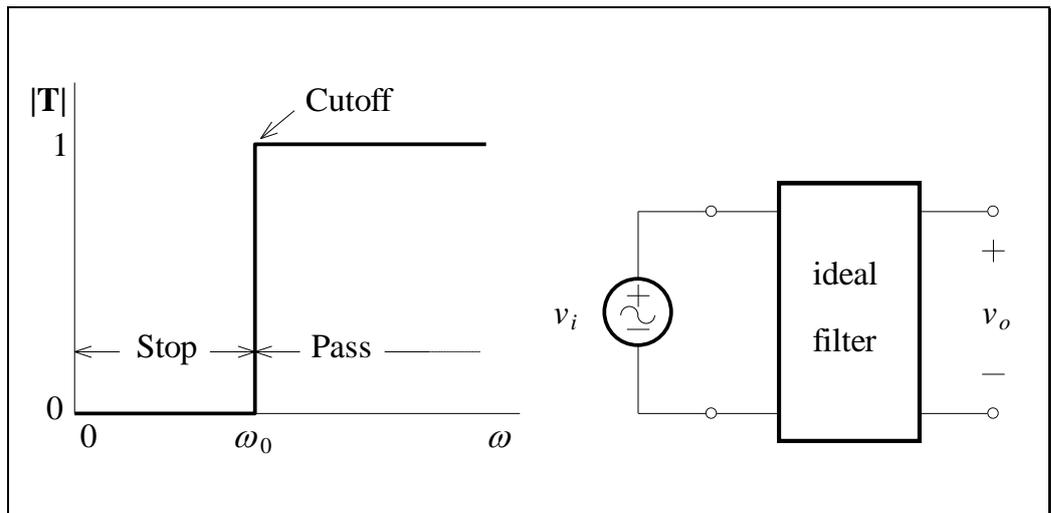


Figure 1.13

The cutoff frequency is ω_0 , as it was for the lowpass filter.

1.5 Phase Responses

Like magnitude responses, phase responses are only meaningful when we look at sinusoidal steady-state circuits. From Eq. (1.6), a frequency response can be expressed in polar form as:

Phase response is obtained in the sinusoidal steady state

$$\mathbf{T}(j\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{|\mathbf{V}_o| \angle \theta}{|\mathbf{V}_i| \angle 0} = |\mathbf{T}| \angle \theta \quad (1.19)$$

where the input is taken as reference (zero phase).

For the bilinear transfer function:

$$\mathbf{T}(j\omega) = K \frac{z + j\omega}{p + j\omega} \quad (1.20)$$

the phase is:

$$\theta = \angle K + \tan^{-1}\left(\frac{\omega}{z}\right) - \tan^{-1}\left(\frac{\omega}{p}\right)$$

(1.21) The phase of the bilinear transfer function

If K is positive, its phase is 0° , and if negative it is 180° . Assuming that it is positive, we have:

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{\omega}{z}\right) - \tan^{-1}\left(\frac{\omega}{p}\right) \\ &= \theta_z + \theta_p \end{aligned} \quad (1.22)$$

We use the sign of this phase angle to classify circuits. Those giving positive θ are known as *lead* circuits, those giving negative θ as *lag* circuits.

Lead and lag circuits defined

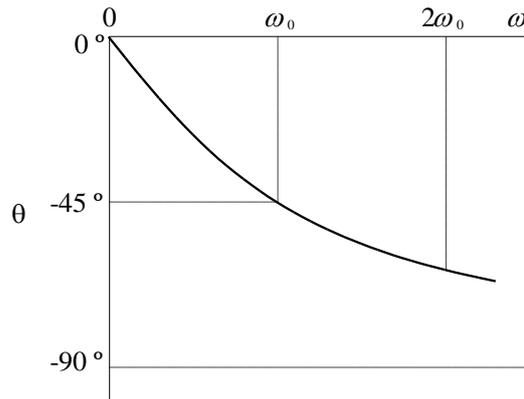
EXAMPLE 1.2 Phase Response of an RC Lowpass Circuit

For the simple RC circuit of Figure 1.7, we have already seen that:

$$\theta = -\tan^{-1}(\omega/\omega_0)$$

Since θ is negative for all ω , the circuit is a lag circuit. When $\omega = \omega_0$, $\theta = -\tan^{-1}(1) = -45^\circ$. A complete plot of the phase response is shown below:

Lagging phase response for a simple RC lowpass filter

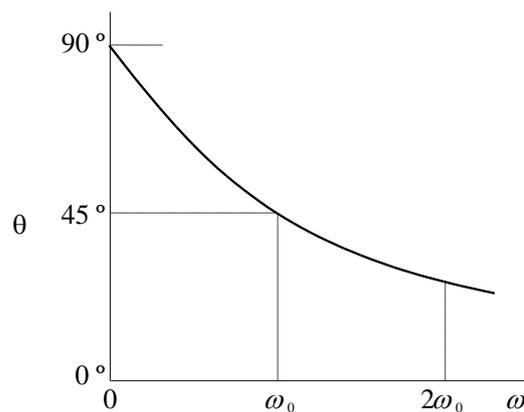
**EXAMPLE 1.3 Phase Response of an RC Highpass Circuit**

For the simple RC circuit of Figure 1.11, we can show that the phase is given by:

$$\theta = 90^\circ - \tan^{-1}(\omega/\omega_0)$$

The phase response is shown below:

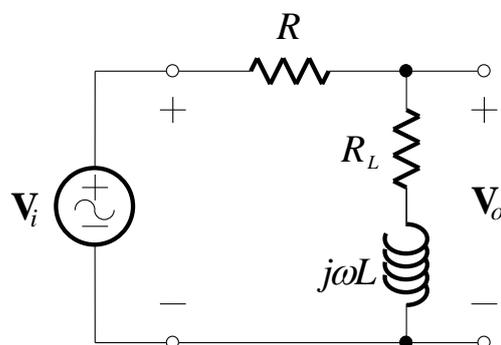
Leading phase response for a simple RC highpass filter



The angle θ is positive for all ω , and so the circuit is a lead circuit.

EXAMPLE 1.4 Frequency Response of a Real RL Highpass Circuit

Consider the RL circuit shown below:



Using the voltage divider rule, we obtain:

$$\mathbf{T}(j\omega) = \frac{R_L + j\omega L}{R + R_L + j\omega L}$$

If we write this in the standard form:

$$\mathbf{T}(j\omega) = K \frac{1 + j\omega/z}{1 + j\omega/p}$$

then:

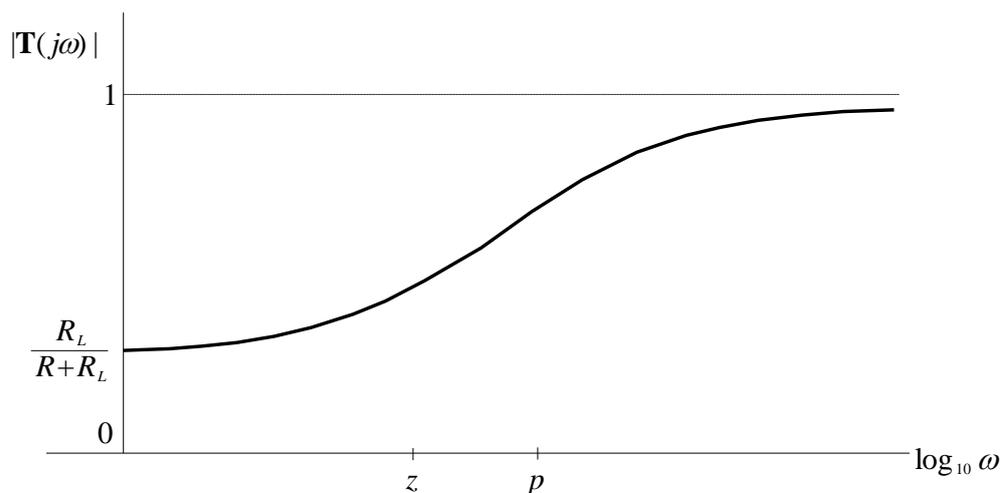
$$K = \frac{R_L}{R + R_L}, \quad z = \frac{R_L}{L} \quad \text{and} \quad p = \frac{R + R_L}{L}$$

so that $p > z$. We can see that:

$$\begin{aligned} |\mathbf{T}(j0)| &= K = \frac{R_L}{R + R_L} \\ |\mathbf{T}(j\infty)| &= K \frac{1/z}{1/p} = 1 \end{aligned}$$

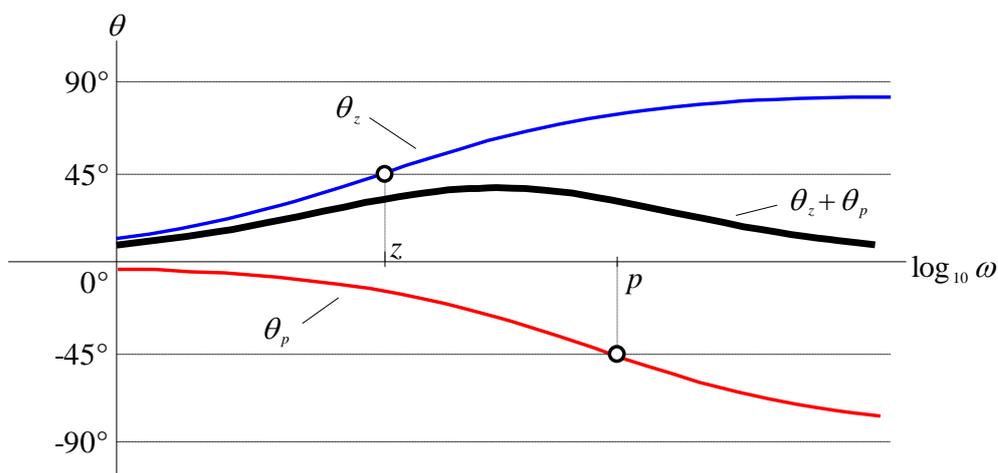
1.20

The magnitude response is:



This does not approximate the ideal of Figure 1.13 very well, but it is still known as a highpass filter.

From Eq. (1.22) we see that θ is characterized by the difference of two angles, the first a function of the zero numerator term, the second a function of the pole denominator term: $\theta = \theta_z + \theta_p$. For $p > z$ the phase function θ_z reaches $+45^\circ$ at a low frequency, while θ_p reaches -45° for a higher frequency. Therefore, the phase response looks like:



Thus, the circuit provides phase lead.

1.5.1 Summary of Bilinear Frequency Responses

We can summarize the magnitude and phase responses of $T(j\omega)$ for various values of z and p , where K is assumed to be positive, in the table below:

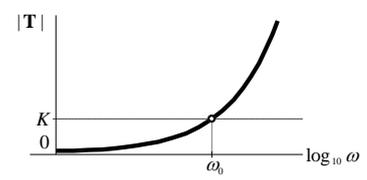
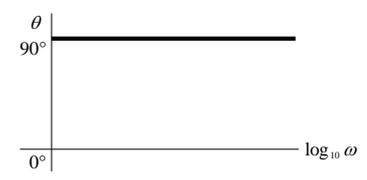
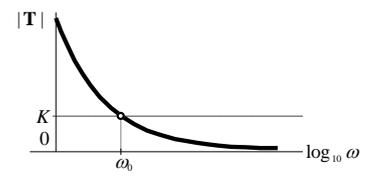
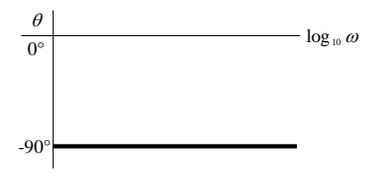
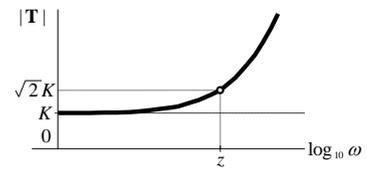
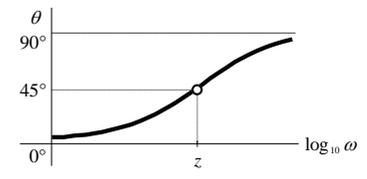
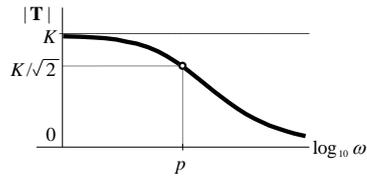
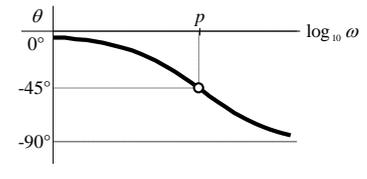
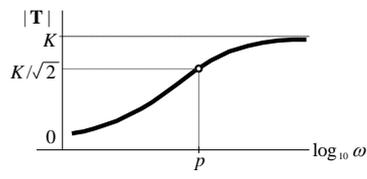
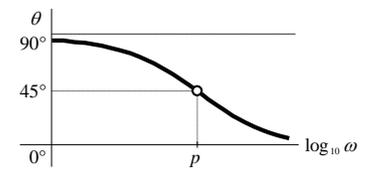
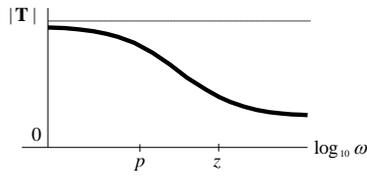
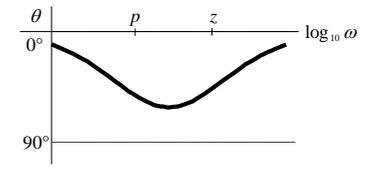
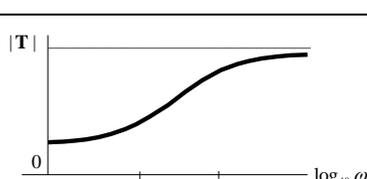
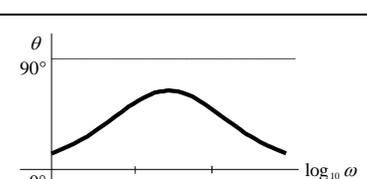
$T(j\omega)$	Magnitude Response	Phase Response
1 $K j\omega/\omega_0$		
2 $\frac{K}{j\omega/\omega_0}$		
3 $K(1 + j\omega/z)$		
4 $K \frac{1}{1 + j\omega/p}$		
5 $K \frac{j\omega/p}{1 + j\omega/p}$		
6 $K \frac{1 + j\omega/z}{1 + j\omega/p}$ $z > p$		
7 $K \frac{1 + j\omega/z}{1 + j\omega/p}$ $p > z$		

Table 1.1 – Summary of Bilinear Frequency Response

1.6 Bode* Plots

Bode plots defined

If we plot the magnitude A as in Eq. (1.14) as a function of ω with logarithmic coordinates, then the plot is known as a magnitude Bode plot. If we plot θ as a function of ω with logarithmic coordinates it is known as a phase Bode plot.

The advantages of Bode plots in the design of filters are:

1. Using a logarithmic scale for the magnitude of \mathbf{T} makes it possible to add and subtract rather than multiply and divide. This applies to phase also.
2. The slope of all asymptotic lines in magnitude plots for bilinear functions is 0 or ± 20 dB per decade.
3. Asymptotic plots serve to sketch out ideas in design - a kind of shorthand.
4. Because decades occur at equal *linear* distances, the shape of a Bode magnitude plot is maintained when frequency is scaled (see the next section).

The advantages of using Bode plots

The following figure shows how simple it is to design a particular response:

Composing a Bode plot from first-order parts

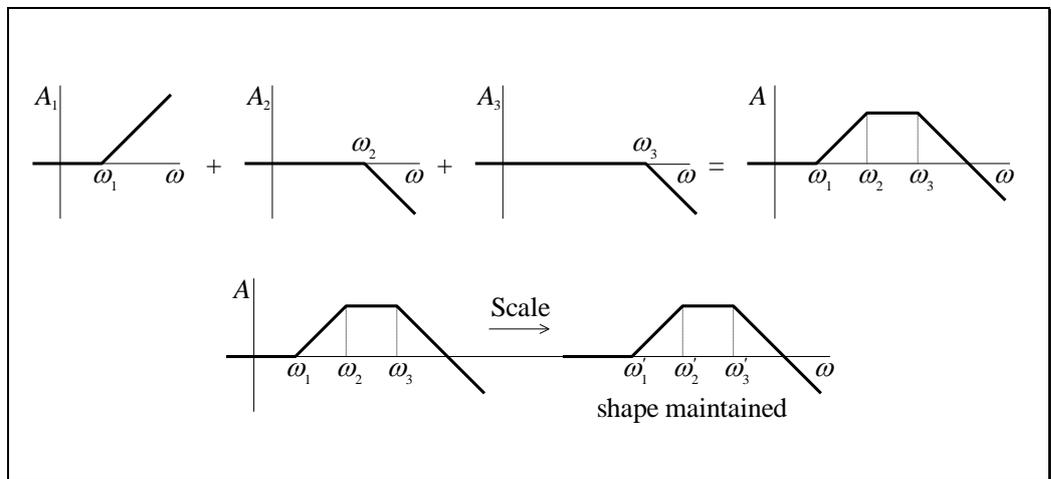


Figure 1.14

* Dr. Hendrik Bode grew up in Urbana, Illinois, USA, where his name is pronounced *boh-dee*. Purists insist on the original Dutch *boh-dah*. No one uses *bohd*.

1.7 Frequency and Magnitude Scaling

In filter design, it is common practice to *normalise* equations so that they have the same form. For example, we have seen the bilinear transfer function:

$$\mathbf{T}(s) = \frac{1/RC}{s + 1/RC} \quad (1.23)$$

expressed as:

$$\mathbf{T}(s) = \frac{\omega_0}{s + \omega_0} \quad (1.24)$$

We *normalise* the equation by setting $\omega_0 = 1$:

Normalising the cutoff frequency means setting it to 1

$$\mathbf{T}(s) = \frac{1}{s + 1} \quad (1.25)$$

Every equation in filter design will be normalised so that $\omega_0 = 1$. This is helpful, since every equation will be able to be compared on the same base. Also, tables of values such as pole locations can be easily made up for the unit circle.

The difficulty that now arises is *denormalising* the resulting equations, values or circuit designs.

1.7.1 Frequency Scaling (Denormalising)

and denormalising means setting the frequency back to its original value

Frequency scaling, or denormalising, means we want to change ω_0 from 1 back to its original value. To do this, we must change all frequency dependent terms in the transfer function, which also means frequency dependent elements in a circuit. Furthermore, the frequency scaling should not affect the magnitude of any impedance in the transfer function.

To scale the frequency by a factor k_f , for a capacitor, we must have:

$$|\mathbf{Z}_C| = \frac{1}{\omega C} = \frac{1}{(k_f \omega)(1/k_f)C} = \frac{1}{(k_f \omega)C_{\text{new}}} \quad (1.26)$$

Frequency scaling must keep the magnitude of the impedance the same

We must decrease the capacitance by the amount $1/k_f$, while increasing the frequency by the amount k_f if the magnitude of the impedance is to remain the same.

For an inductor, we must have:

$$|\mathbf{Z}_L| = \omega L = (k_f \omega) \frac{1}{k_f} L = (k_f \omega) L_{\text{new}} \quad (1.27)$$

Therefore, new element values may be expressed in terms of old values as follows:

$$L_{\text{new}} = \frac{1}{k_f} L_{\text{old}} \quad (1.28)$$

$$C_{\text{new}} = \frac{1}{k_f} C_{\text{old}} \quad (1.29)$$

$$R_{\text{new}} = R_{\text{old}} \quad (1.30)$$

The frequency scaling equations

1.7.2 Magnitude Scaling

Since a transfer function is always a ratio, if we increase the impedances in the numerator and denominator by the same amount, it changes nothing. We do this to obtain realistic values for the circuit elements in the implementation.

Magnitude scale to get realistic element values

If the impedance magnitudes are normally:

$$\mathbf{Z}_R = R, \quad |\mathbf{Z}_L| = \omega L, \quad |\mathbf{Z}_C| = \frac{1}{\omega C} \quad (1.31)$$

then after magnitude scaling with a constant k_m they will be:

$$\begin{aligned} k_m \mathbf{Z}_R &= k_m R, \\ k_m |\mathbf{Z}_L| &= k_m \omega L, \\ k_m |\mathbf{Z}_C| &= \frac{1}{\omega C / k_m} \end{aligned} \quad (1.32)$$

Therefore, new element values may be expressed in terms of old values as follows:

$$L_{\text{new}} = k_m L_{\text{old}} \quad (1.33)$$

$$C_{\text{new}} = \frac{1}{k_m} C_{\text{old}} \quad (1.34)$$

The magnitude scaling equations

$$R_{\text{new}} = k_m R_{\text{old}} \quad (1.35)$$

An easy-to-remember rule in scaling R 's and C 's in electronic circuits is that RC products should stay the same. For example, for the lowpass RC filter, the cutoff frequency is given by $\omega_0 = 1/RC$. Therefore, if the resistor value goes up, then the capacitor value goes down by the same factor – and vice versa.

1.8 Cascading Circuits

How can we create circuits with higher than first-order transfer functions by “cascading” first-order circuits? Consider the following circuit:

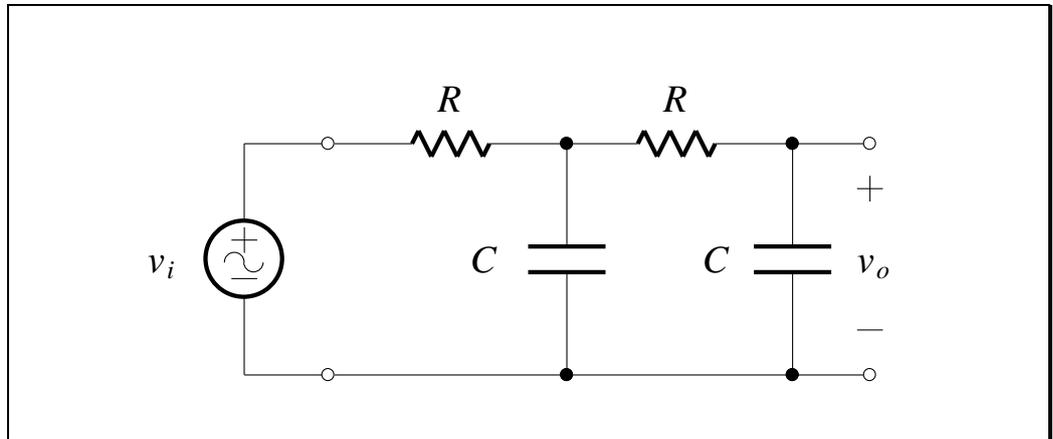


Figure 1.15

Show that the transfer function for the above circuit is:

$$\frac{V_o}{V_i} = \frac{(1/RC)^2}{s^2 + (3/RC)s + (1/RC)^2} \quad (1.36)$$

Compare with the following circuit:

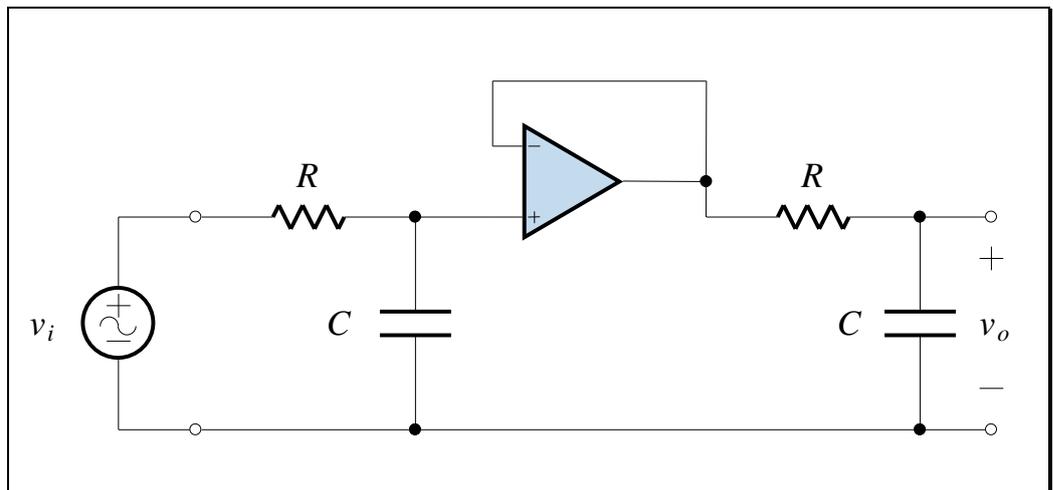


Figure 1.16

which has the transfer function:

$$\begin{aligned}\frac{\mathbf{V}_o}{\mathbf{V}_i} &= \frac{1/RC}{\mathbf{s} + 1/RC} \frac{1/RC}{\mathbf{s} + 1/RC} \\ &= \frac{(1/RC)^2}{\mathbf{s}^2 + (2/RC)\mathbf{s} + (1/RC)^2}\end{aligned}\quad (1.37)$$

We can cascade circuits if the “outputs” of the circuits present a low impedance to the next stage, so that each successive circuit does not “load” the previous circuit. Op-amp circuits of both the inverting and non-inverting type are ideal for cascading.

We can only cascade circuits if they are buffered

1.9 Inverting Bilinear Op-Amp Circuit

The transfer function of the inverting op-amp circuit is:

$$\mathbf{T}(\mathbf{s}) = -\frac{\mathbf{Z}_2}{\mathbf{Z}_1}\quad (1.38)$$

Since we are considering only the bilinear transfer function, we have:

$$\mathbf{T} = -\frac{\mathbf{Z}_2}{\mathbf{Z}_1} = -K \frac{\mathbf{s} + z}{\mathbf{s} + p}\quad (1.39)$$

The inverting op-amp circuit is one way to implement a bilinear transfer function

The specifications of the design problem are the values K , z and p . These may be found from a Bode plot - the break frequencies and the gain at some frequency - or obtained in any other way. The solution to the design problem involves finding a circuit and the values of the elements in that circuit. Since we are using an active device - the op-amp - inductors are excluded from our circuits. Therefore, we want to find values for the R 's and the C 's. Once found, these values can be adjusted by any necessary frequency scaling, and then by magnitude scaling to obtain convenient element values.

1.28

For the general bilinear transfer function, we can make the following identification:

$$K \frac{s + z}{s + p} = \frac{1/(s/K + p/K)}{1/(s + z)} = \frac{\mathbf{Z}_2}{\mathbf{Z}_1} \quad (1.40)$$

Therefore:

$$\mathbf{Z}_2 = \frac{1}{s/K + p/K} = \frac{1}{C_2 s + 1/R_2} \quad (1.41)$$

The impedances for the inverting op-amp circuit to implement a bilinear transfer function

$$\mathbf{Z}_1 = \frac{1}{s + z} = \frac{1}{C_1 s + 1/R_1} \quad (1.42)$$

The design equations become:

Element values for the inverting op-amp circuit in terms of the pole, zero and gain

$$R_1 = \frac{1}{z}, \quad C_1 = 1, \quad R_2 = \frac{K}{p}, \quad C_2 = \frac{1}{K} \quad (1.43)$$

One realisation of the bilinear transfer function is then:

An inverting op-amp circuit that implements a bilinear transfer function

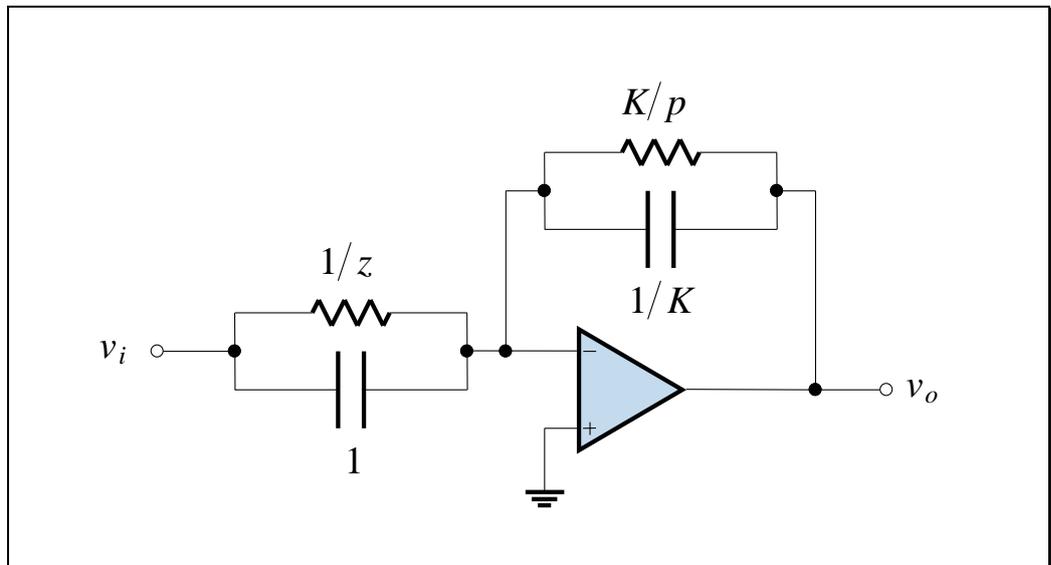


Figure 1.17

1.10 Inverting Op-Amp Circuits

The approach we took in obtaining a circuit to implement the bilinear frequency response can be applied to other forms of $T(j\omega)$ to give the entries in the table below:

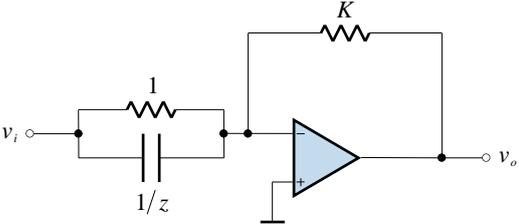
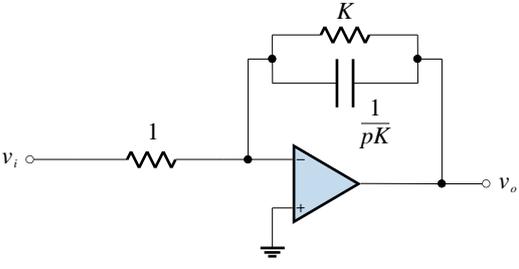
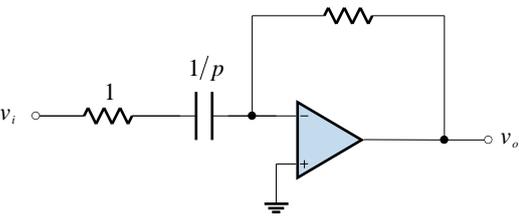
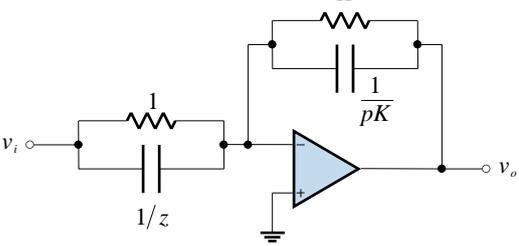
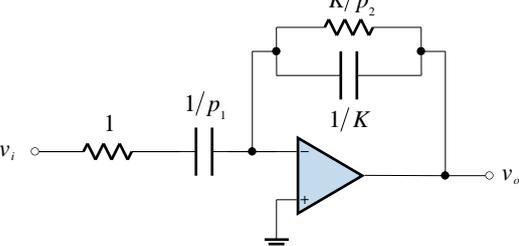
Frequency Response $T(j\omega)$	Circuit
1 $-K(1 + j\omega/z)$	
2 $-K \frac{1}{1 + j\omega/p}$	
3 $-K \frac{j\omega/p}{1 + j\omega/p}$	
4 $-K \frac{1 + j\omega/z}{1 + j\omega/p}$	
5 $-K \frac{j\omega/p_1 p_2}{(1 + j\omega/p_1)(1 + j\omega/p_2)}$	

Table 1.2 – Summary of Inverting Bilinear Op-Amp Circuits

1.30

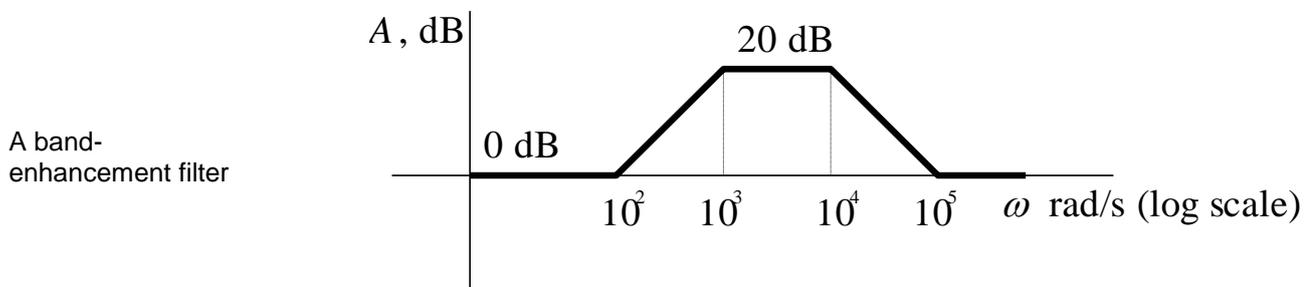
The last entry in the table illustrates an important point. If we use a series RC connection for Z_1 but a parallel RC connection for Z_2 , then the frequency response becomes one of second-order. So the manner in which the capacitors are connected in the circuit determines the order of the circuit.

1.11 Cascade Design

We can make use of cascaded modules, each of first order, to satisfy specifications that are more complicated than the bilinear function.

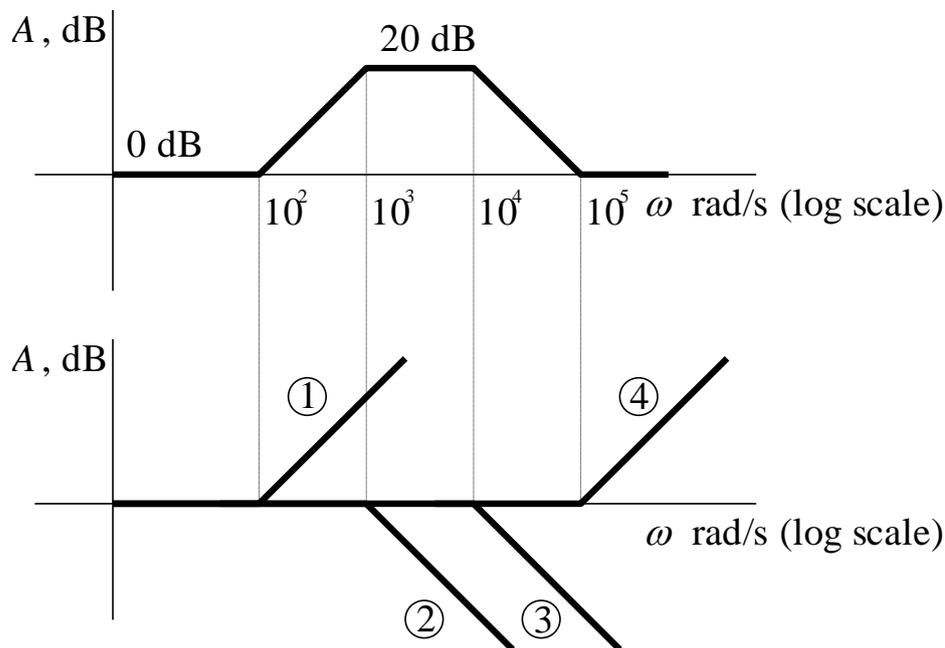
EXAMPLE 1.5 Band Enhancement Filter Design

The asymptotic Bode plot shown below is for a band-enhancement filter:



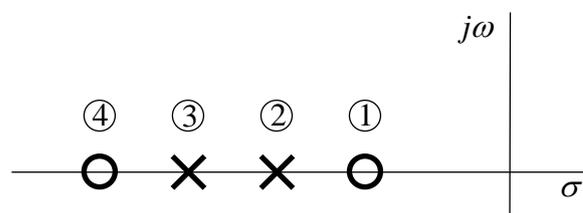
We wish to provide additional gain over a narrow band of frequencies, leaving the gain at higher and lower frequencies unchanged. We wish to design a filter to these specifications and the additional requirement that all capacitors have the value $C = 10$ nF.

The composite plot may be decomposed into four first-order factors as shown below:



Decomposing a Bode plot into first-order factors

Those marked 1 and 4 represent zero factors, while those marked 2 and 3 are pole factors. The pole-zero plot corresponding to these factors is shown below:



The pole-zero plot corresponding to the Bode plot

From the break frequencies given, we have:

$$\mathbf{T}(j\omega) = \frac{(1 + j\omega/10^2)(1 + j\omega/10^5)}{(1 + j\omega/10^3)(1 + j\omega/10^4)}$$

Substituting s for $j\omega$ gives the transfer function:

$$\mathbf{T}(s) = \frac{(s + 10^2)(s + 10^5)}{(s + 10^3)(s + 10^4)}$$

The transfer function corresponding to the Bode plot

1.32

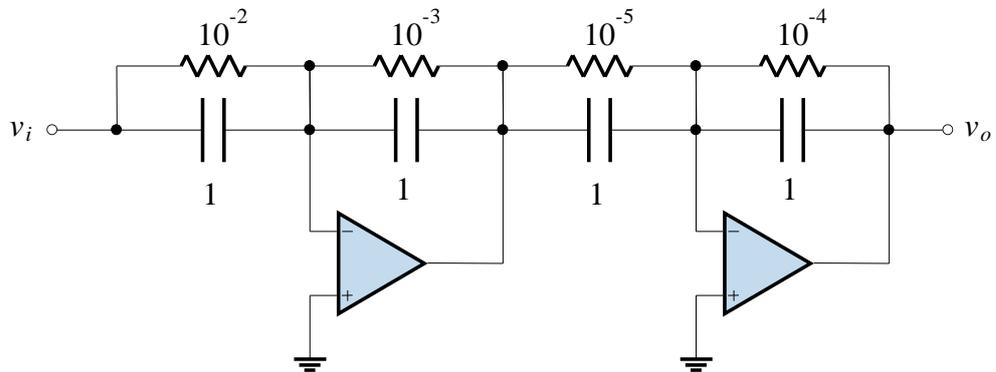
We next write $\mathbf{T}(s)$ as a product of bilinear functions. The choice is arbitrary, but one possibility is:

The transfer function as a cascade of bilinear functions

$$\mathbf{T}(s) = \mathbf{T}_1(s)\mathbf{T}_2(s) = \frac{s + 10^2}{s + 10^3} \times \frac{s + 10^5}{s + 10^4}$$

For a circuit realisation of \mathbf{T}_1 and \mathbf{T}_2 we decide to use the inverting op-amp circuit of Figure 1.17. Using the formulas for element values given there, we obtain the realisation shown below:

A realisation of the specifications



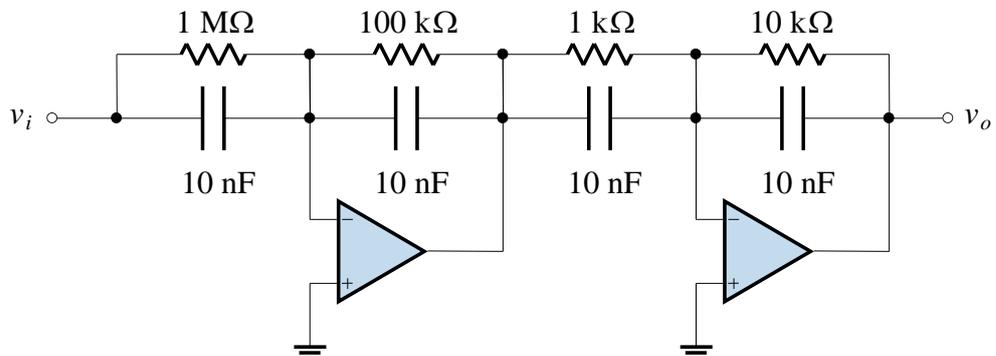
Frequency scaling is not required since we have worked directly with specified frequencies. The magnitude scaling of the circuit is accomplished with the equations:

Magnitude scaling is required to get realistic element values

$$C_{\text{new}} = \frac{1}{k_m} C_{\text{old}} \quad \text{and} \quad R_{\text{new}} = k_m R_{\text{old}}$$

Since the capacitors are to have the value 10 nF, this means $k_m = 10^8$. The element values that result are shown below and the design is complete:

A realistic implementation of the specifications



1.12 References

Van Valkenburg, M. E.: *Analog Filter Design*, Holt-Saunders, Tokyo, 1982.

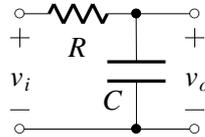
Williams, N.: *Hi-Fi: An Introduction*, Federal Publishing Company, Sydney, 1994.

1.34

Quiz

Encircle the correct answer, cross out the wrong answers. [one or none correct]

1.



The half power frequency is given by:

- (a) $\omega_0 C = R$ (b) $1/\omega_0 C = R$ (c) $\omega_0^2 = 1/RC$

2.

To frequency and magnitude scale a capacitor at the same time, the formula is:

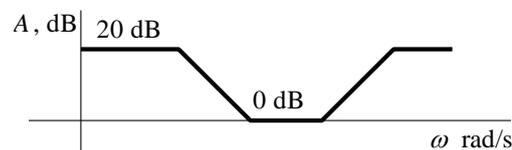
- (a) $C_{\text{new}} = \frac{k_m}{k_f} C_{\text{old}}$ (b) $C_{\text{new}} = \frac{k_f}{k_m} C_{\text{old}}$ (c) $C_{\text{new}} = \frac{1}{k_f k_m} C_{\text{old}}$

3.

The magnitude of a transfer function, in dB, is:

- (a) $A = 10 \log |\mathbf{T}(j\omega)|$ (b) $A = 20 \log |\mathbf{T}(j\omega)|$ (c) $A = 20 \ln |\mathbf{T}(j\omega)|$

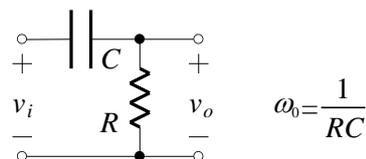
4.



The pole-zero plot corresponding to the asymptotic Bode plot is:

- (a) (b) (c)

5.



The magnitude function, $|\mathbf{T}(j\omega)|$, is:

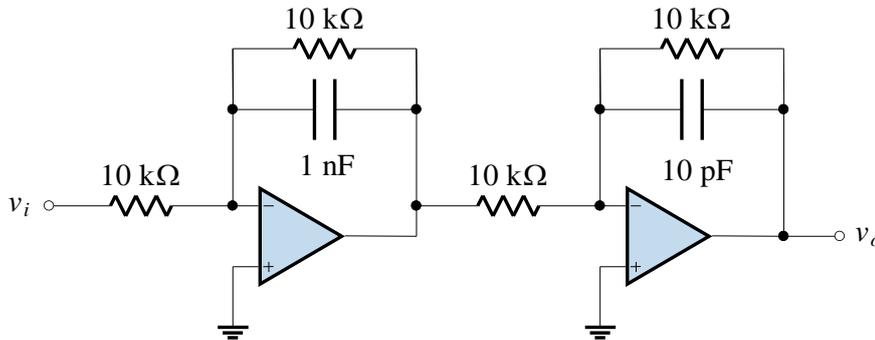
- (a) $1/\sqrt{1+(\omega/\omega_0)^2}$ (b) $1/\sqrt{\omega_0^2 + \omega^2}$ (c) $1/\sqrt{1 + \omega^2}$

Answers: 1. b 2. c 3. b 4. b 5. x

Exercises

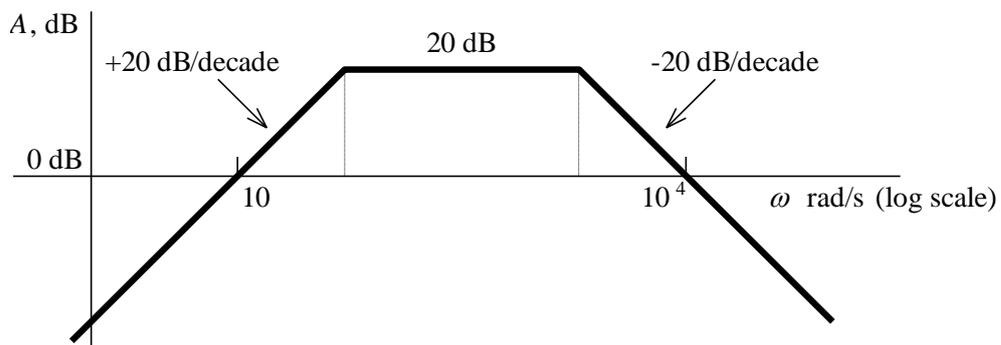
1.

For the circuit shown below, prepare the asymptotic Bode plot for the magnitude of $T(j\omega)$. Carefully identify all slopes and low and high frequency asymptotes.



2.

Design an RC op-amp filter to realise the bandpass response shown below.

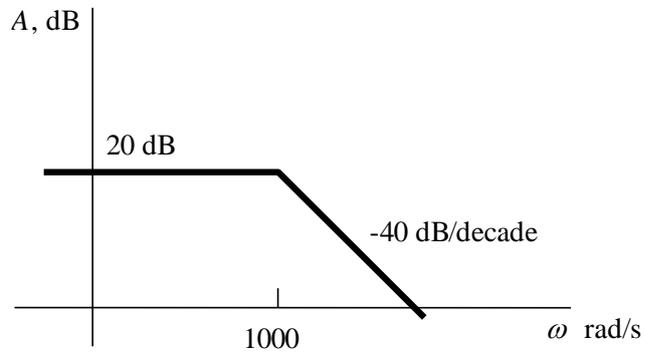


Use a minimum number of op-amps in your design, and scale so that the elements are in a practical range.

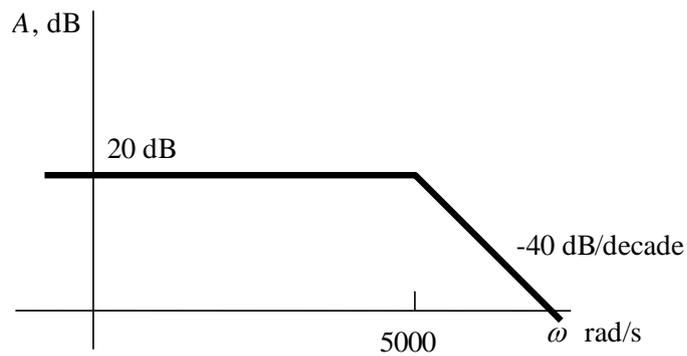
1.36

3.

The asymptotic Bode plot shown below represents a lowpass filter-amplifier with a break frequency of $\omega = 1000 \text{ rad/s}$.



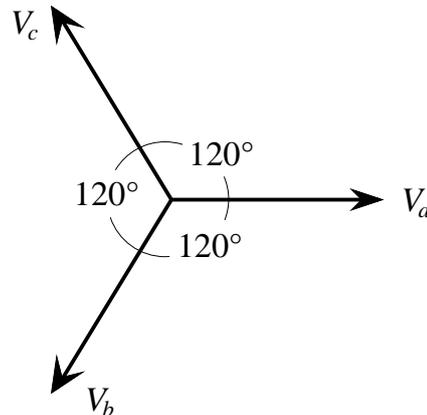
Design a circuit to be connected in cascade with the amplifier such that the break frequency is extended to $\omega = 5000 \text{ rad/s}$:



Problems

1. [Three-phase Voltage Generator]

Design a circuit to provide a set of three-phase 50 Hz voltages, each separated by 120° and equal in magnitude, as shown below.



These voltages will simulate those used in ordinary three-phase power distribution systems. Assume only one 50 Hz source is available.

2. [FM Radio Preemphasis-deemphasis]

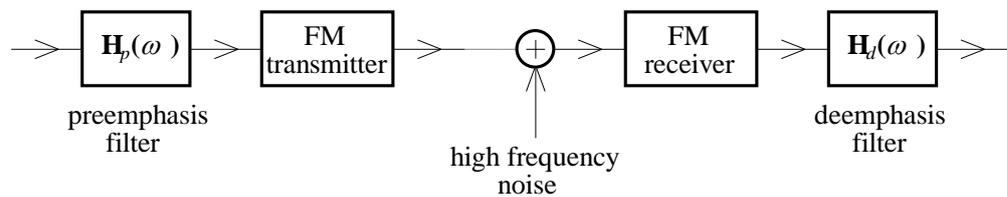
Radio broadcasting using frequency modulation (FM) uses a system of preemphasis-deemphasis (PDE) of the voice or music signal to increase the signal to noise ratio of high frequencies and thereby improve listening conditions.

For a radio broadcast, the signal $m(t)$ can sometimes reach a frequency of 15 kHz. Unfortunately, the noise at the receiver is strongest at this high frequency. If we boost the high-frequency components of the signal at the transmitter (preemphasis), and then attenuate them correspondingly at the FM receiver (deemphasis), we get back $m(t)$ undistorted.

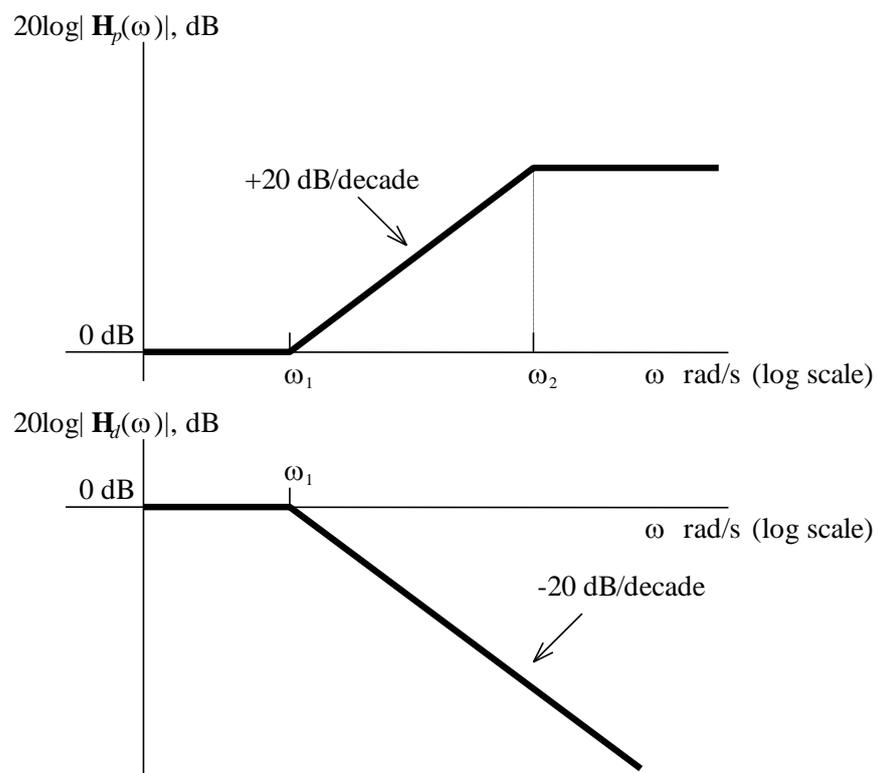
Using this scheme, the noise will be considerably weakened. This is because, unlike $m(t)$, the noise enters the system after the transmitter and is not boosted. It undergoes only deemphasis, or attenuation of high-frequency components, at the receiver.

1.38

The figure below shows a system with PDE filters as used in commercial FM broadcasting:



Filters $\mathbf{H}_p(\omega)$ and $\mathbf{H}_d(\omega)$ are shown below:



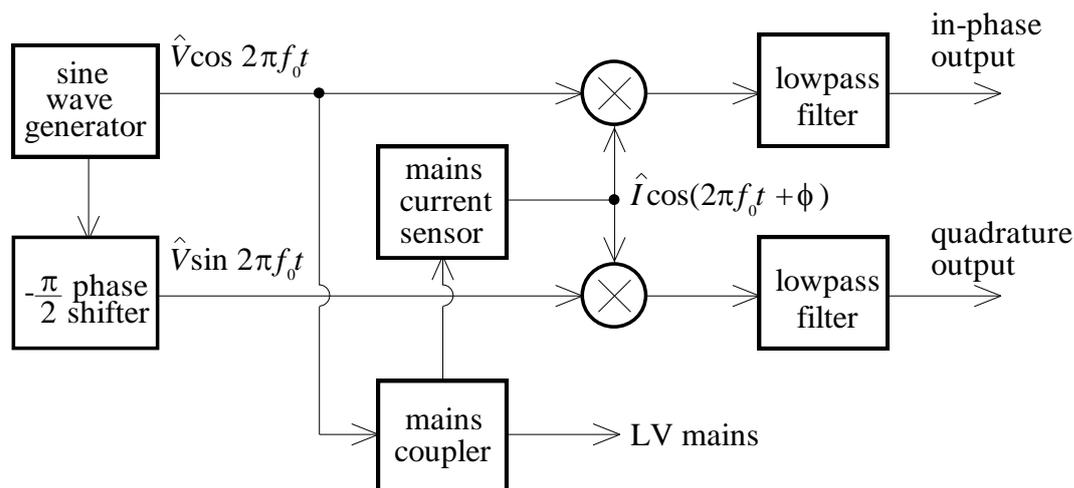
The frequency f_1 is 2.1 kHz, and f_2 is typically 30 kHz or more (well beyond audio range).

Find implementations for the preemphasis and deemphasis filters.

3. [Mains Impedance Tester]

Power line carrier (PLC) communication systems are used by electricity supply authorities for communication on the low voltage (230 V) network. PLC units are installed at the low voltage distribution centre and at each electricity meter. When transmitting, the PLC unit produces a sinusoidal voltage in the range 3 kHz to 5 kHz. It is important to know what the magnitude of these sinusoids is at the receivers. This depends on the power line's impedance which changes from minute to minute as load is switched on and off. Also, a power line's impedance varies not only in time, but also with location, being strongly dependent on the length of the underground cables and overhead conductors.

A mains impedance tester is therefore devised which has the following block diagram:



- (i) What are the outputs of the four-quadrant multipliers? (The mains coupler circuit severely attenuates the 50 Hz mains, and passes signals between 3 kHz and 5 kHz without attenuation).
- (ii) Design the output lowpass filters to produce a voltage readable by a DC meter, with any sinusoidal term above 1 kHz attenuated by at least 40 dB.
- (iii) How can we use the direct and quadrature outputs to determine the impedance of the power line at a particular frequency?

2 Butterworth Lowpass Filters

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2.2

2.1 Second-Order Parameters

Consider the following second-order circuit:

An *RLC* lowpass filter

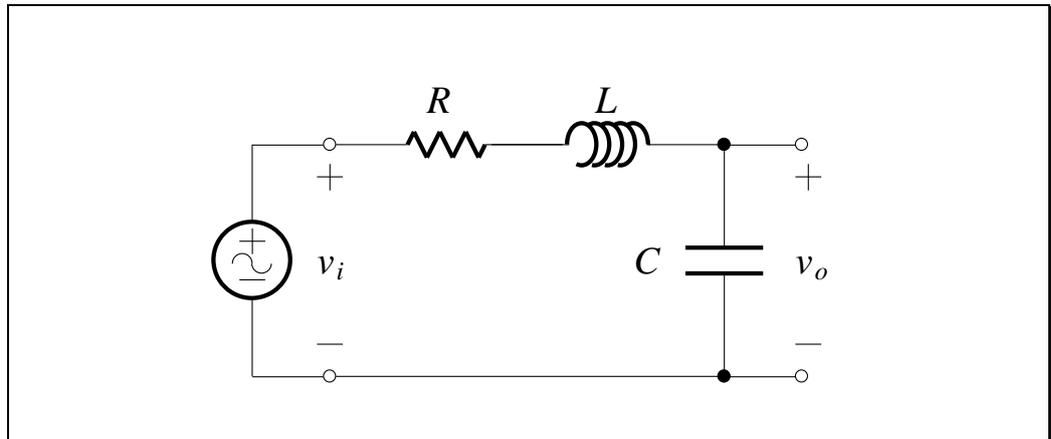


Figure 2.1

Show that the transfer function is given by:

$$\mathbf{T}(s) = \frac{1/LC}{s^2 + (R/L)s + 1/LC} \quad (2.1)$$

This can be put into a *standard form* by defining two new quantities.

When the circuit is lossless with $R = 0$, then the poles of the transfer function are:

$$\mathbf{s}_1, \mathbf{s}_2 = \pm j \frac{1}{\sqrt{LC}} = \pm j\omega_0 \quad (2.2)$$

This means the poles are on the imaginary axis and are conjugates.

The first parameter used in the standard form is therefore defined as:

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (2.3) \quad \omega_0 \text{ defined for an RLC circuit}$$

The other parameter we require originated in studies of lossy coils for which a *quality factor* Q_0 was defined as:

$$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}} \quad (2.4) \quad Q \text{ defined for an RLC circuit}$$

which is the ratio of reactance at the frequency ω_0 to resistance. The historical identification of Q_0 with lossy coils is no longer appropriate, since we can identify many kinds of circuits with the parameter Q_0 .

Eq. (2.4) can be solved for the ratio R/L which is used in Eq. (2.1):

$$\frac{R}{L} = \frac{\omega_0}{Q_0} \quad (2.5)$$

Substituting Eqs. (2.3) and (2.5) into (2.1) gives:

$$\mathbf{T}(s) = \frac{\omega_0^2}{s^2 + (\omega_0/Q_0)s + \omega_0^2} = \frac{\mathbf{N}(s)}{\mathbf{D}(s)} \quad (2.6) \quad \text{Standard form for a lowpass second-order transfer function}$$

This is the standard form for a lowpass second-order transfer function.

It is desirable to examine the pole locations of the second-order transfer function.

Let their pole locations be $-\alpha \pm j\beta$ so that:

$$\begin{aligned} \mathbf{D}(s) &= (s + \alpha + j\beta)(s + \alpha - j\beta) \\ &= s^2 + 2\alpha s + (\alpha^2 + \beta^2) \end{aligned} \quad (2.7)$$

2.4

Comparing with Eq. (2.6), we find that:

The real part of the second-order transfer function's poles

$$\alpha = \frac{\omega_0}{2Q_0} \quad (2.8)$$

and:

$$\omega_0^2 = \alpha^2 + \beta^2 \quad (2.9)$$

Combining this with Eq. (2.8) and solving for β gives:

The imaginary part of the second-order transfer function's poles

$$\beta = \omega_0 \sqrt{1 - \frac{1}{4Q_0^2}} \quad (2.10)$$

All of these relationships are shown below:

Rectangular and polar forms for specifying a complex pole location

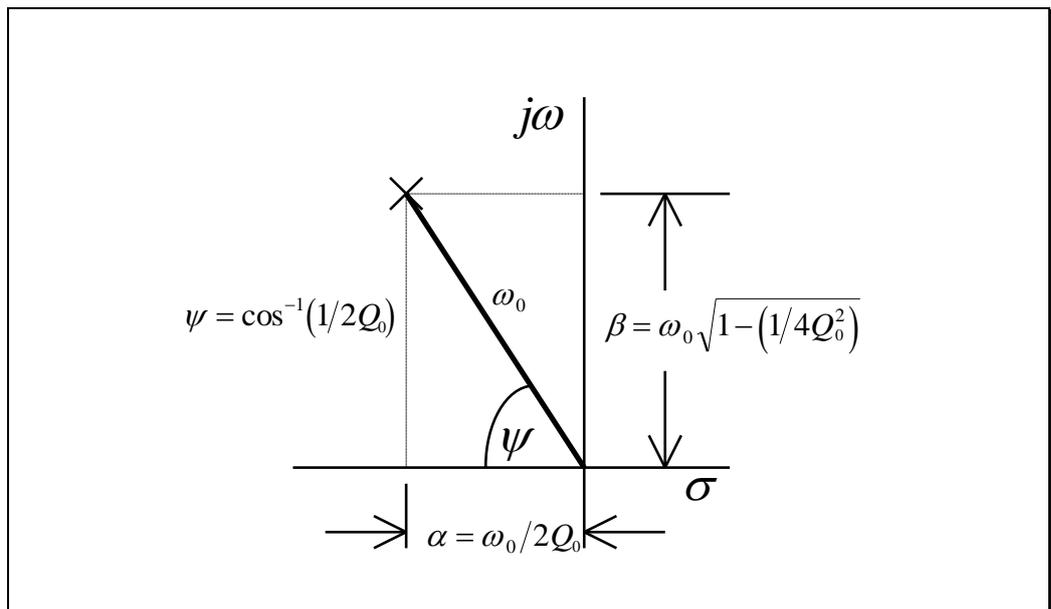


Figure 2.2

In Figure 2.2, we have also defined the angle ψ with respect to the negative real axis as:

$$\psi = \cos^{-1}\left(\frac{\alpha}{\omega_0}\right) = \cos^{-1}\left(\frac{1}{2Q_0}\right) \quad (2.11)$$

The two parameters Q_0 and ω_0 uniquely specify the standard form of a second-order transfer function as given by Eq. (2.6). We can now make the association of Q_0 and ω_0 with any second-order circuit, as suggested by the figure below:

ω_0 and Q uniquely specify a second-order transfer function

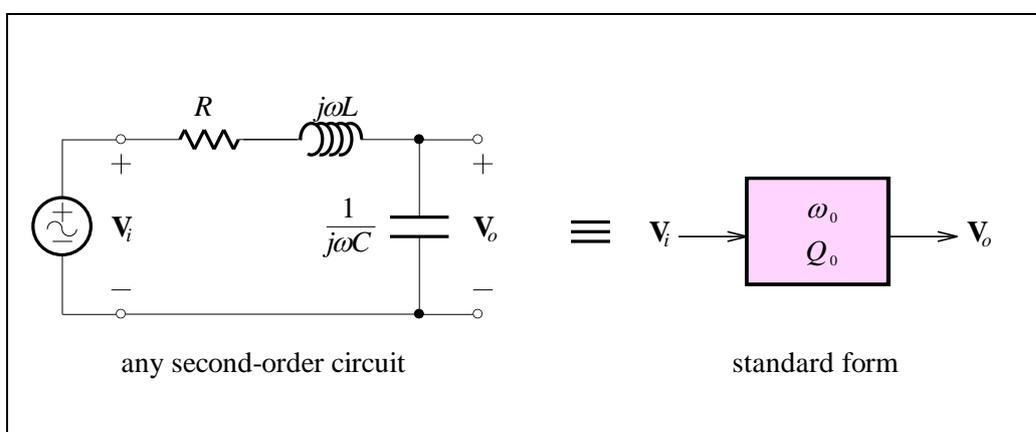


Figure 2.3

2.2 The Lowpass Biquad Circuit

The standard form of a lowpass second-order transfer function, as in Eq. (2.6), does not recognise the availability of gain that is possible with active circuits. Also, an active circuit may be inverting or non-inverting. A more general form for $\mathbf{T}(s)$ is therefore:

$$\mathbf{T}(s) = \frac{\pm H\omega_0^2}{s^2 + (\omega_0/Q_0)s + \omega_0^2} \quad (2.12)$$

Standard form of a lowpass second-order transfer function with gain

We seek a circuit that will implement this second-order transfer function, as well as any other “biquadratic” transfer function. (A biquadratic, or biquad transfer function is similar to the way a bilinear transfer function was defined - a biquadratic function is a ratio of second-order polynomials).

2.6

Normalising so that $\omega_0 = 1$, and anticipating an inverting realisation for the transfer function, we have:

Standard form of a normalised lowpass second-order transfer function with gain

$$\mathbf{T}(s) = \frac{-H}{s^2 + (1/Q_0)s + 1} = \frac{\mathbf{V}_o}{\mathbf{V}_i} \quad (2.13)$$

We can manipulate this equation so that it has a form that can be identified with simple circuits we have seen in Lecture 1. We first rewrite Eq. (2.13) as:

$$\left(s^2 + \frac{1}{Q_0}s + 1 \right) \mathbf{V}_o = -H \mathbf{V}_i \quad (2.14)$$

Dividing by $s(s + 1/Q_0)$, it becomes:

$$\left[1 + \frac{1}{s(s + 1/Q_0)} \right] \mathbf{V}_o = \frac{-H}{s(s + 1/Q_0)} \mathbf{V}_i \quad (2.15)$$

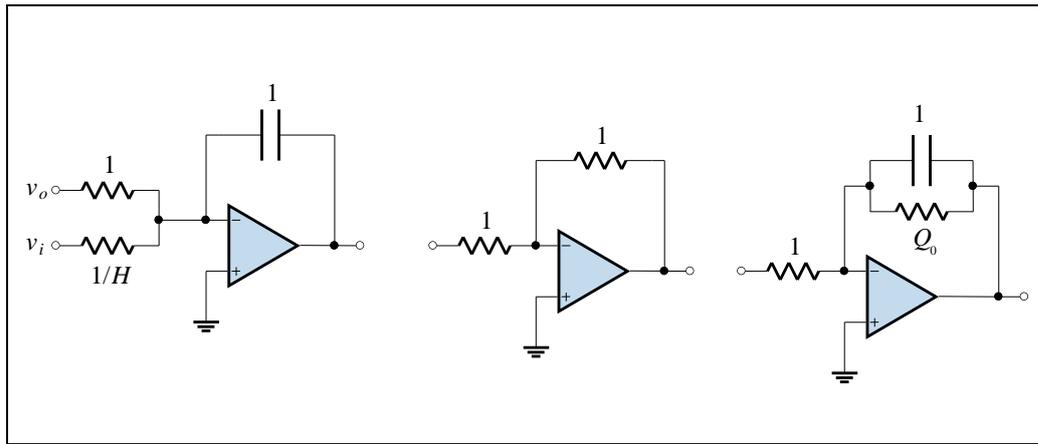
We can manipulate further to form:

Second-order transfer function made from first-order parts

$$\mathbf{V}_o = \left[\frac{-1}{s} \mathbf{V}_o + \frac{-H}{s} \mathbf{V}_i \right] \cdot (-1) \cdot \left(\frac{-1}{s + 1/Q_0} \right) \quad (2.16)$$

The (-1) term can be realised by an inverting circuit of gain 1. The factor $1/(s + 1/Q_0)$ is realised by a “lossy” inverting integrator. Two operations are indicated by the remaining factor. The circuit realisation must produce a sum of voltages, and it must have a transfer function of the form $(-1/s)$.

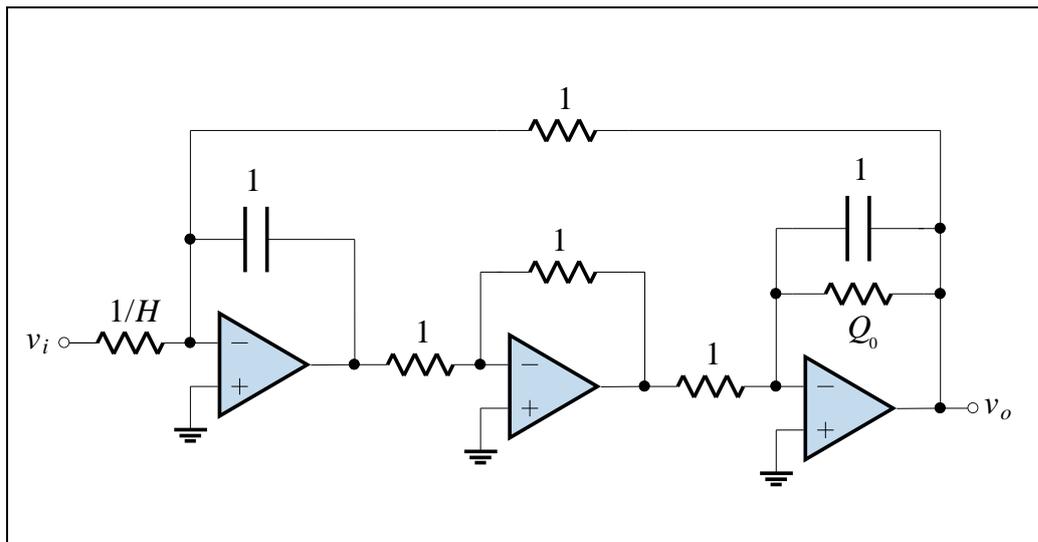
The three circuits that provide for these three operations are shown below:



The three first-order circuits that make a second-order circuit

Figure 2.4

If we connect the three circuits together, including a *feedback* connection of the output v_o to the input, the result is a scaled version of the *Tow-Thomas biquad circuit*:



The normalised Tow-Thomas biquad circuit

Figure 2.5

There are many circuits that implement biquadratic transfer functions. The Tow-Thomas circuit is one of them, the state-variable (KHN) circuit is another. For brevity, we will simply refer to the Tow Thomas biquad circuit as “the biquad”.

2.8

With the elements identified by R 's and C 's, we have:

The Tow-Thomas biquad circuit

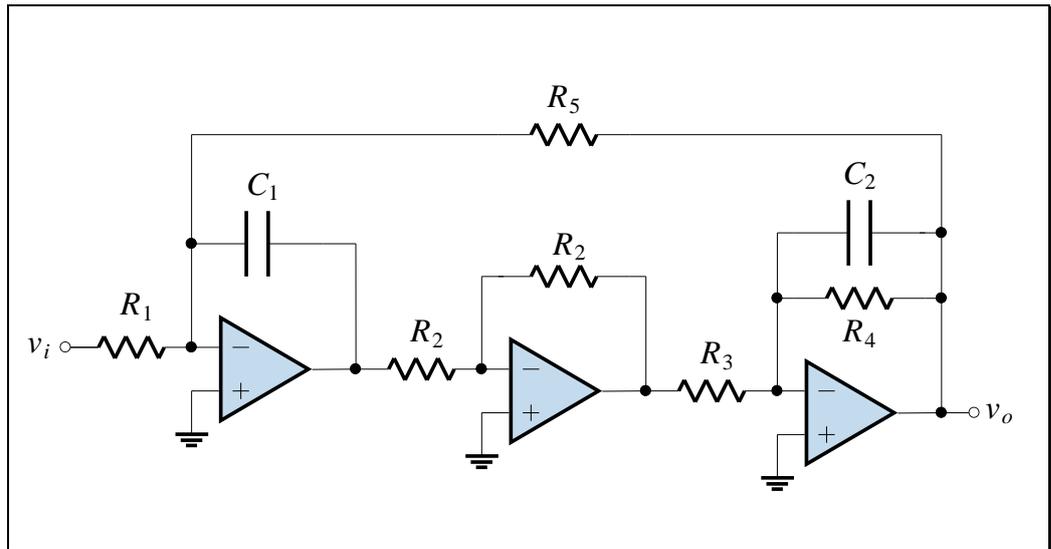


Figure 2.6

Show that the transfer function is:

The biquad's transfer function

$$\mathbf{T}(s) = \frac{-1/R_1 R_3 C_1 C_2}{s^2 + (1/R_4 C_2)s + 1/R_3 R_5 C_1 C_2} \quad (2.17)$$

Comparing this with Eq. (2.12), we have:

The biquad's design equations

$$\omega_0 = \frac{1}{\sqrt{R_3 R_5 C_1 C_2}} \quad (2.18a)$$

$$Q_0 = \sqrt{\frac{R_4^2 C_2}{R_3 R_5 C_1}} \quad (2.18a)$$

$$H = \frac{R_5}{R_1} \quad (2.18a)$$

An important property of the biquad is that it can be *orthogonally* tuned. Using the above equations, we can devise a *tuning algorithm*: The biquad can be orthogonally tuned

1. R_3 can be adjusted to the specified value of ω_0 .
2. R_4 can then be adjusted to give the specified value of Q_0 without changing ω_0 . The biquad's tuning algorithm
3. R_1 can then be adjusted to give the specified value of H without affecting either ω_0 or Q_0 .

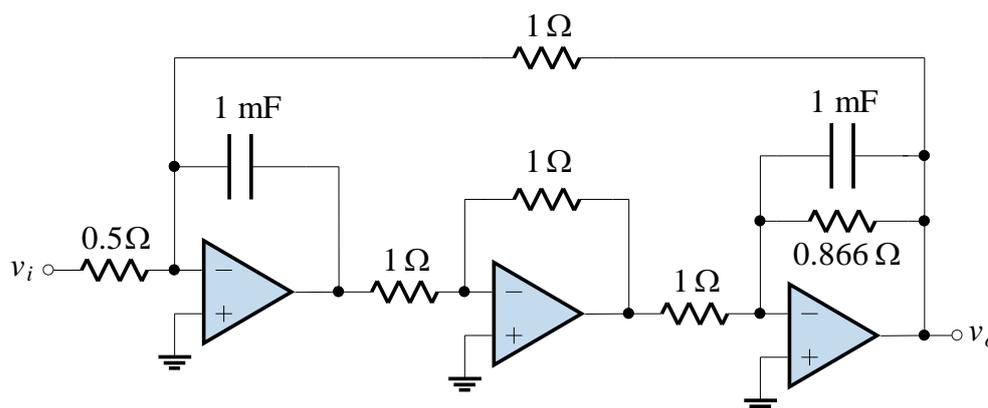
Other advantages of the circuit are:

- the input impedance is purely resistive
- there is effectively “pre-amplification” built-in to the topology via the gain setting resistor R_1 (the incoming signal amplitude is amplified and then filtered, which eliminates more “noise” than filtering and *then* amplifying).

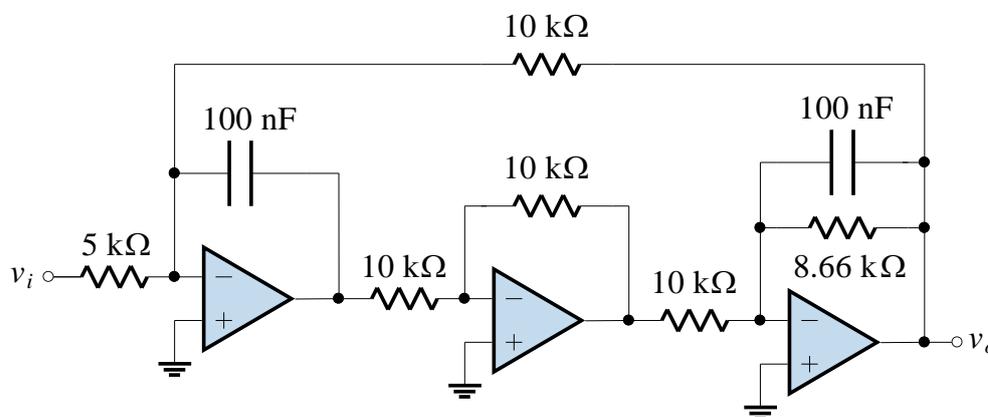
EXAMPLE 2.1 Second-Order Lowpass Filter Using a Biquad

We require a circuit that will provide an $\omega_0 = 1000 \text{ rads}^{-1}$, a $Q_0 = 0.866$ and a DC gain of $H = 2$. We set $\omega_0 = 1$ and use the biquad circuit of Figure 2.5 with the values of Q_0 and H specified above.

We then perform frequency scaling to meet the specifications, by setting $k_f = 1000$. The biquad circuit then becomes:



We then select $k_m = 10\,000$ to give convenient element values. A realistic circuit that meets the specifications is then:



2.3 Frequency Response of the Lowpass Biquad Circuit

To examine the frequency response of the biquad, we will set $H=1$ and frequency scale so that $\omega_0 = 1$. Then:

$$\mathbf{T}(j\omega) = \frac{1}{1 - \omega^2 + j\omega/Q_0} \quad (2.19)$$

The magnitude is:

$$|\mathbf{T}(j\omega)| = \frac{1}{\sqrt{(1 - \omega^2)^2 + (\omega/Q_0)^2}} \quad (2.20)$$

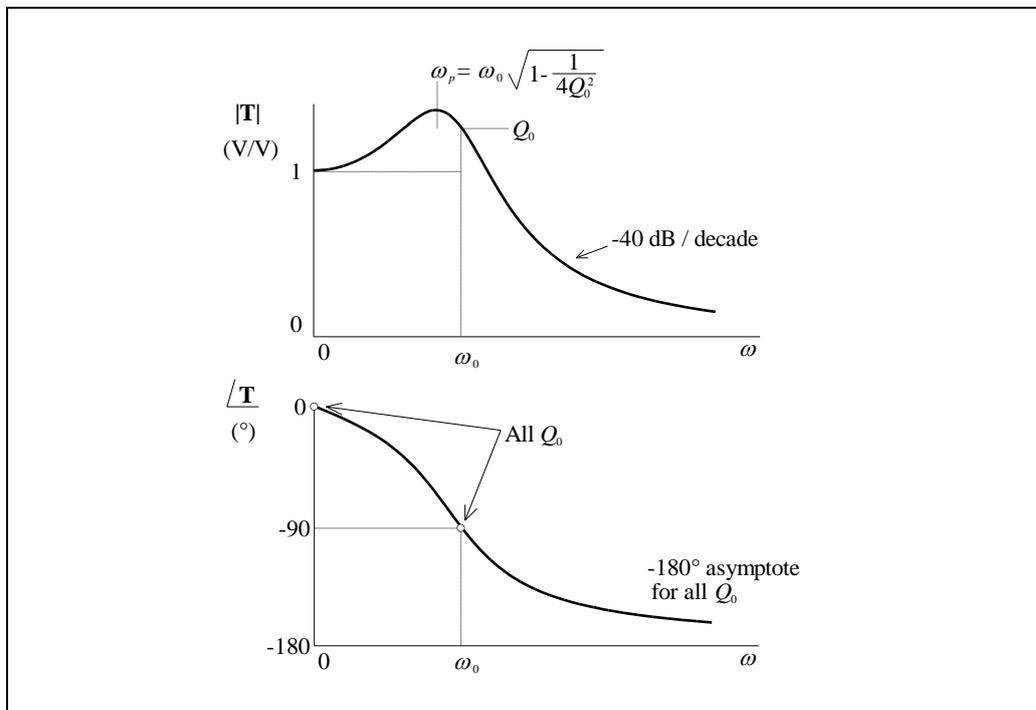
The magnitude response of a normalised lowpass second-order transfer function

and the phase is:

$$\theta = -\tan^{-1}\left(\frac{\omega/Q_0}{1 - \omega^2}\right) \quad (2.21)$$

The phase response of a normalised lowpass second-order transfer function

The magnitude and phase functions are plotted below for $Q_0 = 1.25$:



Typical magnitude and phase responses of a normalised lowpass second-order transfer function

Figure 2.7

2.12

For the magnitude function, from Eq. (2.20) we see that:

$$|\mathbf{T}(j0)| = 1, \quad |\mathbf{T}(j1)| = Q, \quad |\mathbf{T}(j\infty)| \rightarrow 0 \quad (2.22)$$

and for large ω , the magnitude decreases at a rate of -40 dB per decade, which is sometimes described as *two-pole rolloff*.

For the phase function, we see that:

$$\theta(j0) = 0^\circ, \quad \theta(j1) = -90^\circ, \quad \theta(j\infty) \rightarrow -180^\circ \quad (2.23)$$

These responses can be visualised in terms of the pole locations of the transfer function. Starting with:

Standard form for a normalised lowpass second-order transfer function

$$\mathbf{T}(s) = \frac{1}{s^2 + (1/Q_0)s + 1} \quad (2.24)$$

the poles are located on a circle of radius 1 and at an angle with respect to the negative real axis of $\psi = \cos^{-1}(1/2Q_0)$, as given by Eq. (2.11). These complex conjugate pole locations are shown below:

Pole locations for a normalised lowpass second-order transfer function

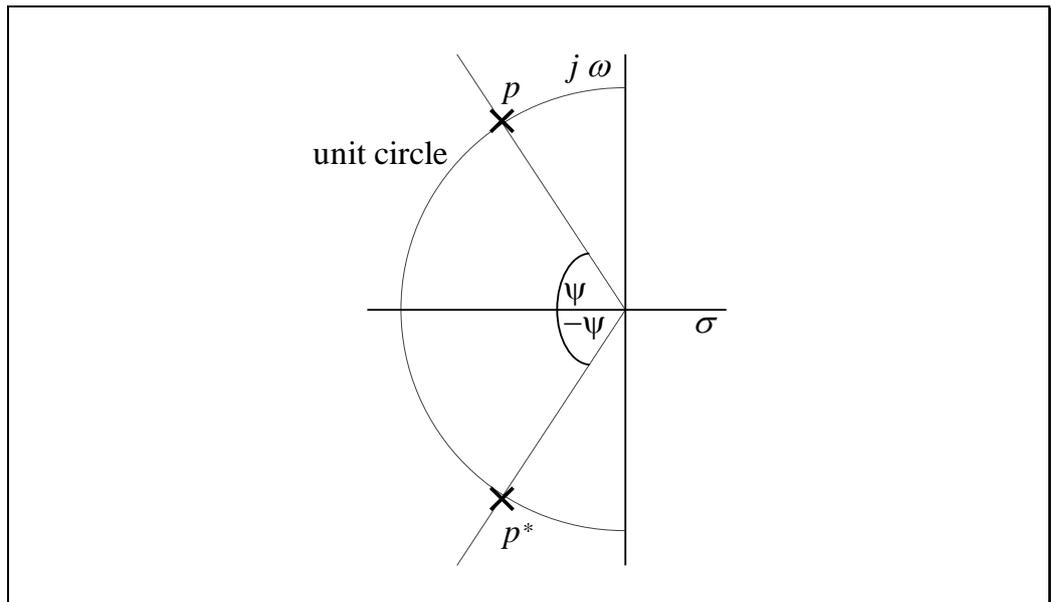


Figure 2.8

In terms of the poles shown in Figure 2.8, the transfer function is:

$$\mathbf{T}(s) = \frac{1}{(s - p)(s - p^*)} \quad (2.25)$$

Normalised lowpass
second-order
transfer function
using pole factors

With $s = j\omega$, the two factors in this equation become:

$$j\omega - p = m_1 \angle \phi_1 \quad \text{and} \quad j\omega - p^* = m_2 \angle \phi_2 \quad (2.26)$$

Polar representation
of the pole factors

In terms of these quantities, the magnitude and phase are:

$$|\mathbf{T}(j\omega)| = \frac{1}{m_1 m_2} \quad (2.27)$$

Magnitude function
written using the
polar representation
of the pole factors

and:

$$\theta = -(\phi_1 + \phi_2) \quad (2.28)$$

Phase function
written using the
polar representation
of the pole factors

2.14

Phasors representing Eq. (2.26) are shown below:

Determining the magnitude and phase response from the s plane

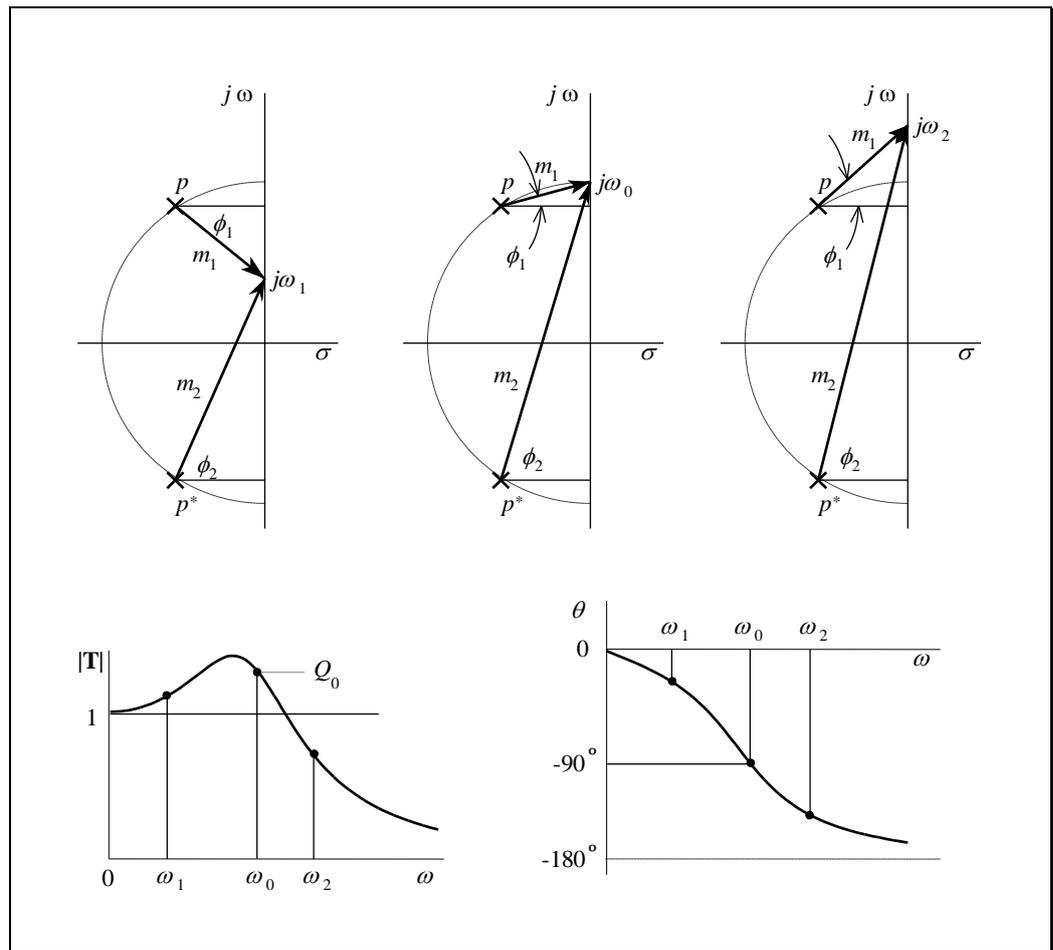


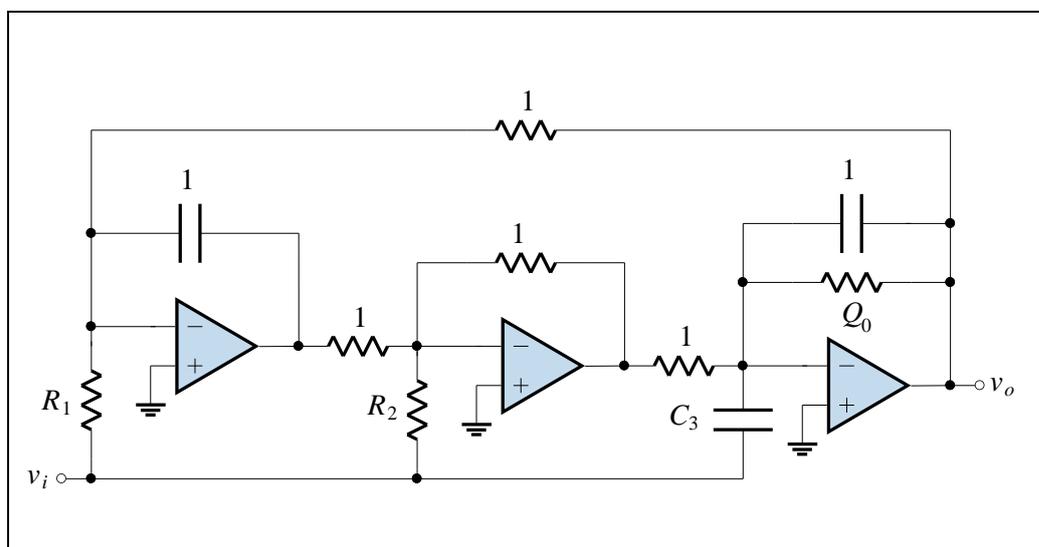
Figure 2.9

Figure 2.9 shows three different frequencies - one below ω_0 , one at ω_0 , and one above ω_0 . From this construction we can see that the short length of m_1 near the frequency ω_0 is the reason why the magnitude function reaches a peak near ω_0 . These plots are useful in visualising the behaviour of the circuit.

2.4 The Universal Biquad Circuit

By applying a feedforward scheme to the lowpass Tow-Thomas biquad circuit, a “universal” filter can be implemented. A universal filter is one that can be made either a lowpass, highpass, bandpass, notch or allpass filter by appropriate selection of component values.

A universal filter can implement any biquadratic transfer function



The normalised Tow-Thomas universal filter

Figure 2.10

In terms of the quantities in Figure 2.10, show that:

$$\frac{V_o}{V_i} = -\frac{C_3 s^2 - (1/R_2)s + 1/R_1}{s^2 + (1/Q)s + 1} \quad (2.29)$$

If we choose $C_3 = 1$ and $R_1 = R_2 = \infty$, then the second and third terms in the numerator vanish, leaving only the s^2 term. Writing this result in general, we have:

$$\mathbf{T}(s) = \frac{s^2}{s^2 + (\omega_0/Q_0)s + \omega_0^2} \quad (2.30)$$

The universal biquad circuit can implement a highpass second-order transfer function

2.16

If we normalise and let $s = j\omega$, then the magnitude is:

The normalised highpass second-order magnitude function

$$|\mathbf{T}(j\omega)| = \frac{\omega^2}{\sqrt{(1-\omega^2)^2 + (\omega/Q_0)^2}} \quad (2.31)$$

From this equation, we see the following:

$$|\mathbf{T}(j0)| = 0, \quad |\mathbf{T}(j1)| = Q_0, \quad |\mathbf{T}(j\infty)| = 1 \quad (2.32)$$

which means we have now created a highpass filter. A plot of the magnitude response is shown below:

The magnitude response of a highpass second-order transfer function

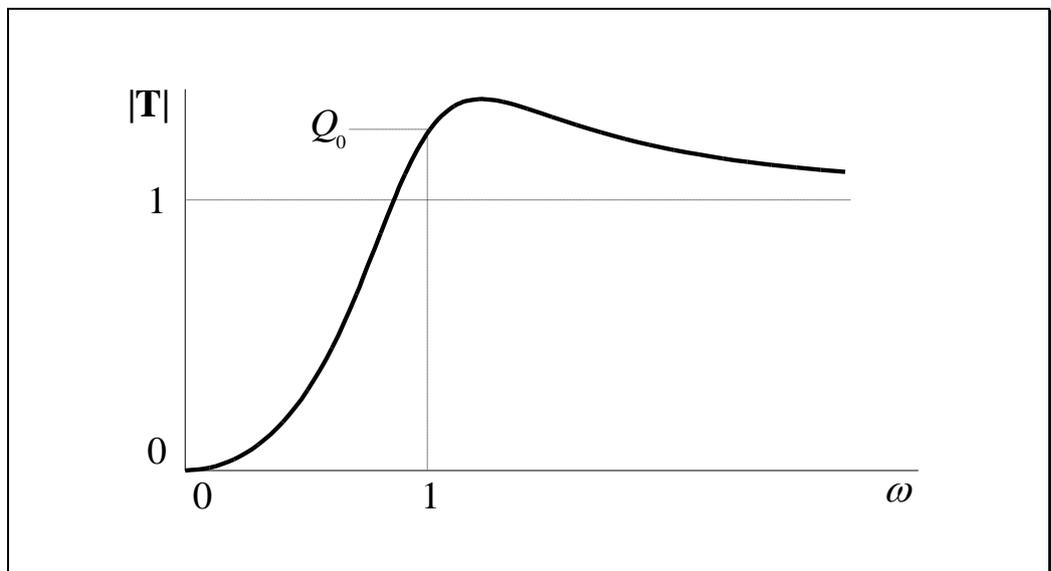
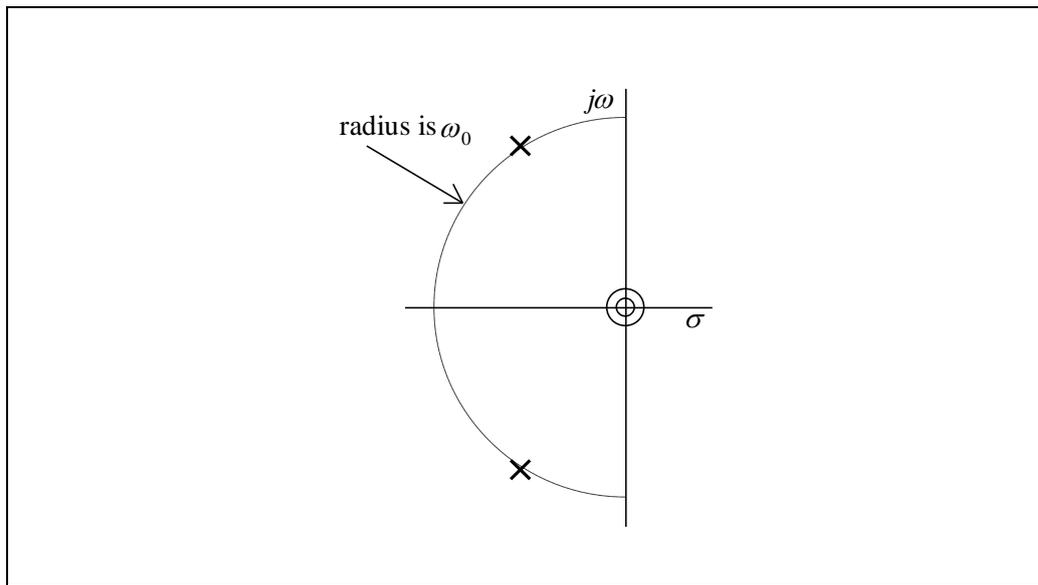


Figure 2.11

The locations of the poles and zeros for the highpass case are given by Eq. (2.30) and are shown below:



The pole-zero plot of a highpass second-order transfer function

Figure 2.12

We see in Figure 2.12 that there is a double zero at the origin of the s plane, with poles in the same position as in the lowpass case.

Starting with the universal biquad circuit, it is possible to realise a lowpass, highpass, bandpass, bandstop or allpass filter by making simple changes such as the removal of a resistor. The normalised design values for the various responses are given in the table below.

The universal biquad can implement many second-order transfer functions

Table 2.1 – Design Values for the Tow-Thomas Universal Filter

Filter Type	Design Values		
	R_1	R_2	C_3
Lowpass	$1/H$	∞	0
Bandpass	∞	Q_0/H	0
Highpass	∞	∞	H
Notch	$(\omega_0/\omega_n)^2/H$	∞	H
Allpass	$1/H$	Q_0/H	H

Table of design values for a universal filter

2.5 Approximating the Ideal Lowpass Filter

The features we want when approximating the ideal lowpass filter

The ideal lowpass filter characteristic is the brick wall filter. Although we cannot achieve the ideal, it provides a basis on which we can rate an approximation. We want $|T|$ to be as constant as possible in the passband, so that different low frequency signals are passed through the filter without a change in amplitude. In the stopband we require n -pole rolloff, where n is large, in contrast to the $n = 2$ rolloff for the biquad circuit. We want the transition from passband to stopband to be as abrupt as possible.

This is summarised in the figure below:

Approximating the ideal lowpass filter

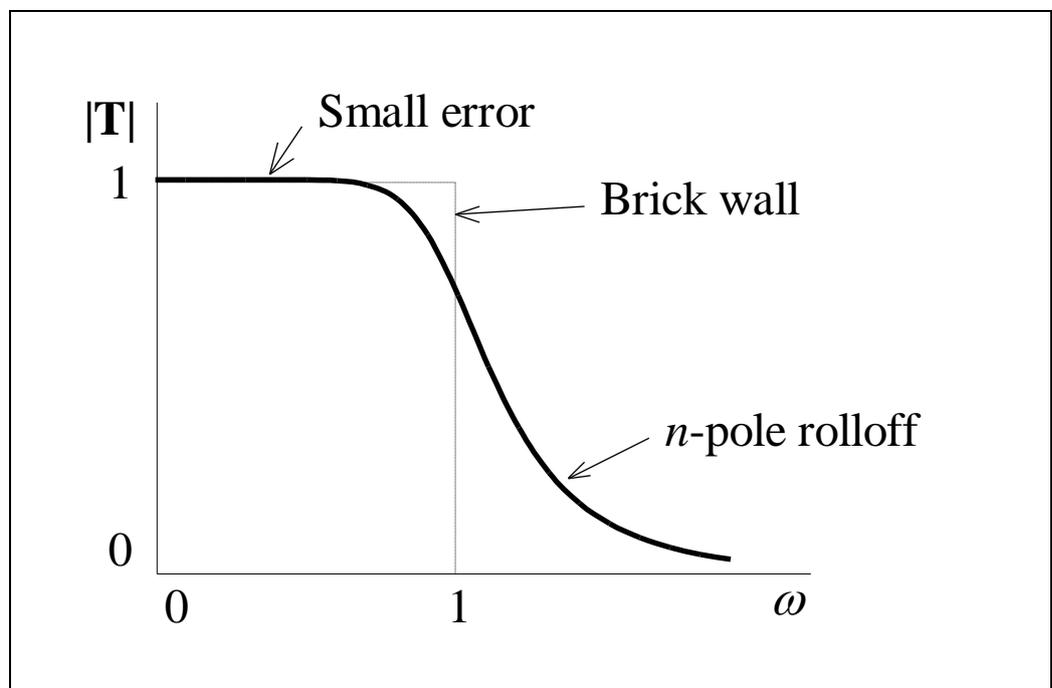
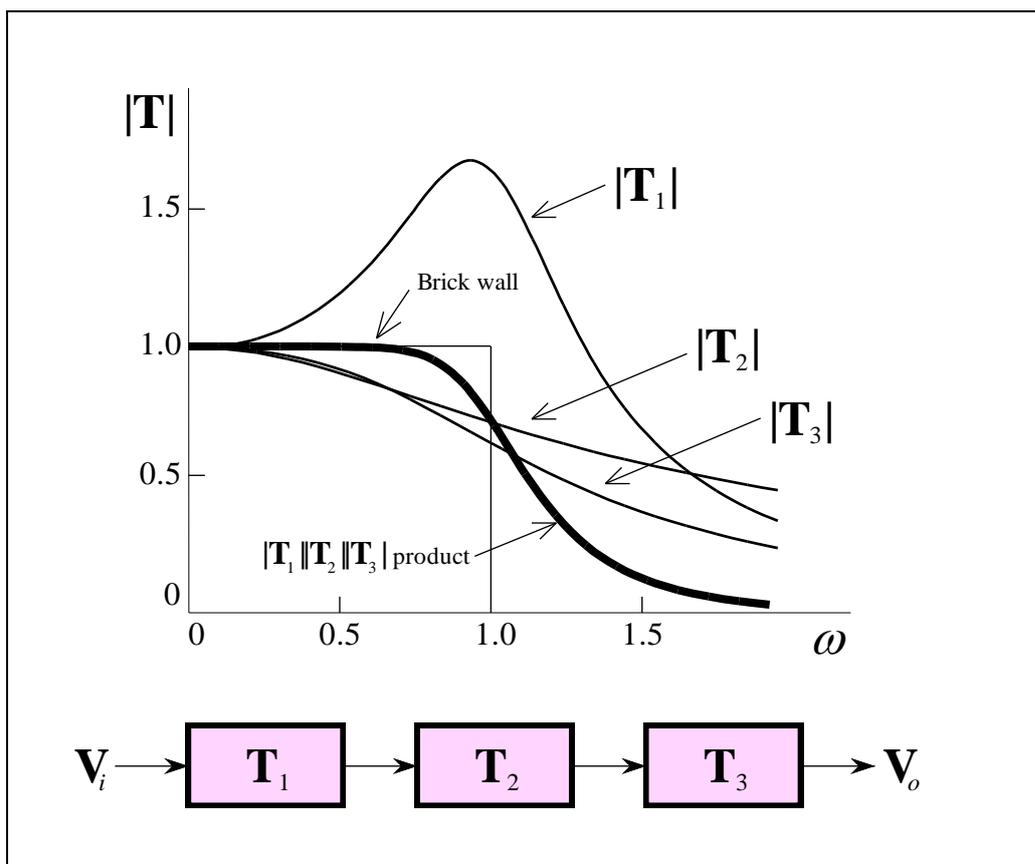


Figure 2.13

The method we will use in the approach to this problem is illustrated below:



We achieve the approximation to the ideal lowpass filter by cascading

Figure 2.14

We will connect modules in cascade such that the overall transfer function is of the form given in Figure 2.13. For the example in Figure 2.14, large values of $|T_1|$ are just overcome by the small values of $|T_2|$ and $|T_3|$ to achieve the approximation to the brick wall. The transfer functions have the same value of ω_0 , but different values of Q_0 .

The cascaded circuits have the same ω_0 but different Q_0

Determining the required *order* of the filter, n , and the values of ω_0 and Q_0 is a part of *filter design*. Note that the order of the filter, n , does not necessarily relate to the number of *stages* in the filter, since each stage may be either a 1st- order or 2nd-order circuit. For example, if $n = 5$, the realisation may be a cascade of three stages: two 2nd-order circuits and one 1st-order circuit.

2.6 The Butterworth Response

The Butterworth¹ response is the name given to the following magnitude function:

The Butterworth magnitude response defined

$$|\mathbf{T}_n(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_0)^{2n}}} \quad (2.33)$$

Normalising such that $\omega_0 = 1$ gives:

The normalised Butterworth magnitude response

$$|\mathbf{T}_n(j\omega)| = \frac{1}{\sqrt{1 + \omega^{2n}}} \quad (2.34)$$

From this equation we can observe some interesting properties of the Butterworth response:

Properties of the normalised Butterworth magnitude response

1. $|\mathbf{T}_n(j0)| = 1$ for all n .
1. $|\mathbf{T}_n(j1)| = 1/\sqrt{2} \approx 0.707$ for all n .
2. For large ω , $|\mathbf{T}_n(j\omega)|$ exhibits n -pole rolloff.
3. $|\mathbf{T}_n(j\omega)|$ has all derivatives but one equal to zero near $\omega = 0$. The response is known as *maximally flat*.

¹ Stephen Butterworth was a British engineer who described this type of response in connection with electronic amplifiers in his paper "On the Theory of Filter Amplifiers", *Wireless Eng.*, vol. 7, pp. 536-541, 1930.

These properties are shown below:

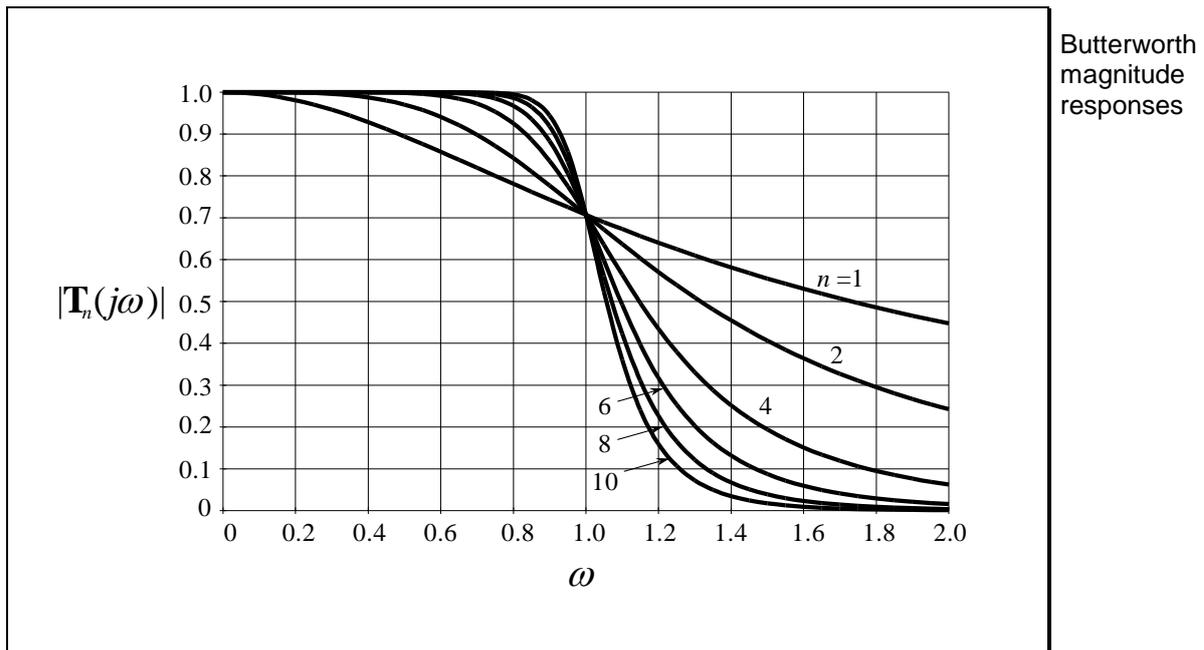


Figure 2.15

2.7 Butterworth Pole Locations

We now need the transfer function corresponding to the magnitude function given in Eq. (2.34) so that we can determine the pole locations. Once we have the pole locations, each complex pole pair can be assigned to a second-order circuit, and any real pole can be assigned to a first-order circuit. A cascade of such circuits will realise all the poles, and therefore the Butterworth response. Determining the pole locations is really just finding Q_0 for those poles, since we already know they lie on the unit circle.

The Butterworth response can be obtained by cascading circuits with different Q

We start by squaring Eq. (2.34):

$$|\mathbf{T}_n(j\omega)|^2 = \frac{1}{1 + \omega^{2n}} \quad (2.35)$$

and use the relationship:

$$|\mathbf{T}_n(j\omega)|^2 = \mathbf{T}_n(j\omega)\mathbf{T}_n^*(j\omega) = \mathbf{T}_n(j\omega)\mathbf{T}_n(-j\omega) \quad (2.36)$$

to get:

$$\mathbf{T}_n(\mathbf{s})\mathbf{T}_n(-\mathbf{s}) = \frac{1}{1 + (\mathbf{s}/j)^{2n}} = \frac{1}{1 + (-1)^n \mathbf{s}^{2n}} \quad (2.37)$$

The poles of Eq. (2.37) are the roots of the equation:

$$\mathbf{B}_n(\mathbf{s})\mathbf{B}_n(-\mathbf{s}) = 1 + (-1)^n \mathbf{s}^{2n} = 0 \quad (2.38)$$

Butterworth polynomial defined as the denominator of the Butterworth transfer function

where \mathbf{B}_n is designated the *Butterworth polynomial*.

For n odd, we have to solve the equation:

$$\mathbf{s}^{2n} = 1 \quad (2.39)$$

The roots of the Butterworth polynomial...

and for n even we solve:

$$\mathbf{s}^{2n} = -1 \quad (2.40)$$

...lie on the unit circle in the left half-plane and are equiangularly spaced

The solution of both of these equations should be familiar from complex variable theory. Both equations have as their solutions equiangularly spaced roots around the unit circle. Roots in the right half-plane correspond to an unstable system, so we select the roots in the left half-plane to associate with $\mathbf{B}_n(\mathbf{s})$ and $\mathbf{T}_n(\mathbf{s})$.

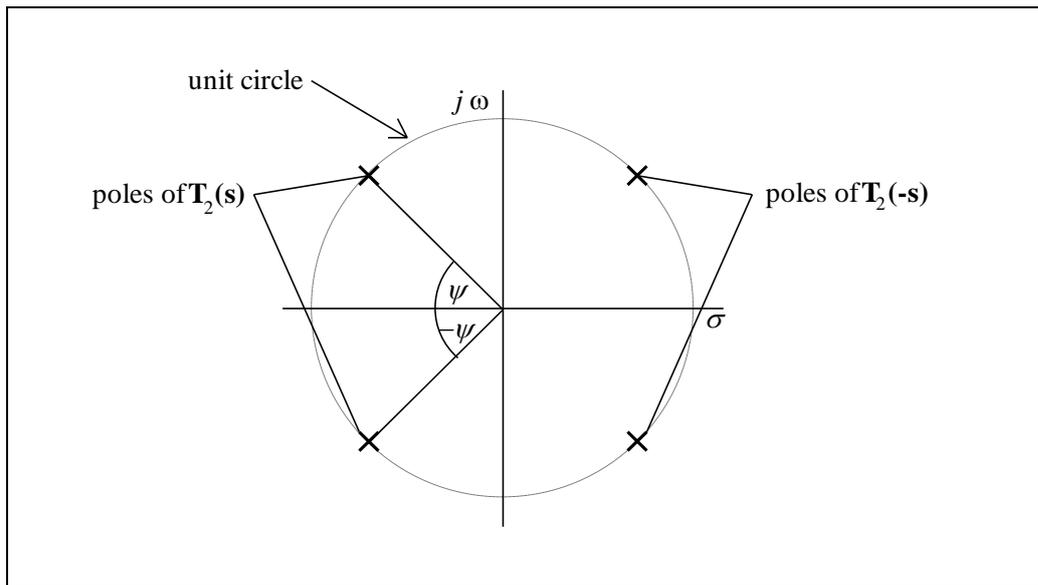
As an example, consider the case when $n = 2$. We then have to solve $\mathbf{s}^4 = -1$. Writing -1 in the polar form, we see that:

$$\mathbf{s}^4 = 1 \angle 180^\circ + k360^\circ = e^{j(\pi + 2\pi k)} \quad (2.41)$$

Taking the fourth root gives the pole locations as:

$$\mathbf{s}_k = e^{j(\pi + 2\pi k)/4} = 1 \angle 45^\circ + k90^\circ, \quad k = 0, 1, 2, 3 \quad (2.42)$$

A plot of these poles is shown below:



Pole locations for a second-order Butterworth transfer function

Figure 2.16

Show that the second-order polynomial associated with two complex conjugate roots on the unit circle is given by:

$$(s^2 + 2 \cos \psi s + 1) \quad (2.43)$$

For the case of $n=2$, the Butterworth polynomial is:

$$\mathbf{B}_2(s) = s^2 + \sqrt{2}s + 1 \quad (2.44)$$

and the transfer function that gives the Butterworth response is:

$$\mathbf{T}_2(s) = \frac{1}{\mathbf{B}_2(s)} = \frac{1}{s^2 + \sqrt{2}s + 1} \quad (2.45)$$

In general, the Butterworth polynomial is given by:

$$\mathbf{B}_n(s) = \frac{1}{(s+1)^k} \quad \text{or} \quad \prod_k (s^2 + 2 \cos \psi_k s + 1) \quad (2.46)$$

Butterworth polynomial defined in terms of pole locations

2.24

Two rules allow us to determine ψ_k quickly:

Determining Butterworth pole locations quickly

1. If n is odd, then there is a pole at $s = -1$; if n is even, then there are poles at $\psi = \pm 90^\circ/n$.
2. Poles are separated by $180^\circ/n$.

The value of Q_0 for each of the pole pairs is found using Eq. (2.11), and should be apparent from Eq. (2.46):

Determining Q_0 for each pole pair

$$Q_0 = \frac{1}{2 \cos \psi} \tag{2.47}$$

We can now tabulate values of Q_0 for any value of n .

Table 2.2 – Q_0 of Butterworth Poles

Table of Q_0 for Butterworth poles

n								
2	3	4	5	6	7	8	9	10
0.707	1.000	0.541	0.618	0.518	0.555	0.510	0.532	0.506
		1.307	1.618	0.707	0.802	0.601	0.653	0.561
				1.932	2.247	0.900	1.000	0.707
						2.563	2.879	1.101
								3.196

Note: For n odd, there is an additional first-order factor given by a pole at $s = -1$.

2.8 Lowpass Filter Specifications

Filter specifications are usually given by a plot as shown below:

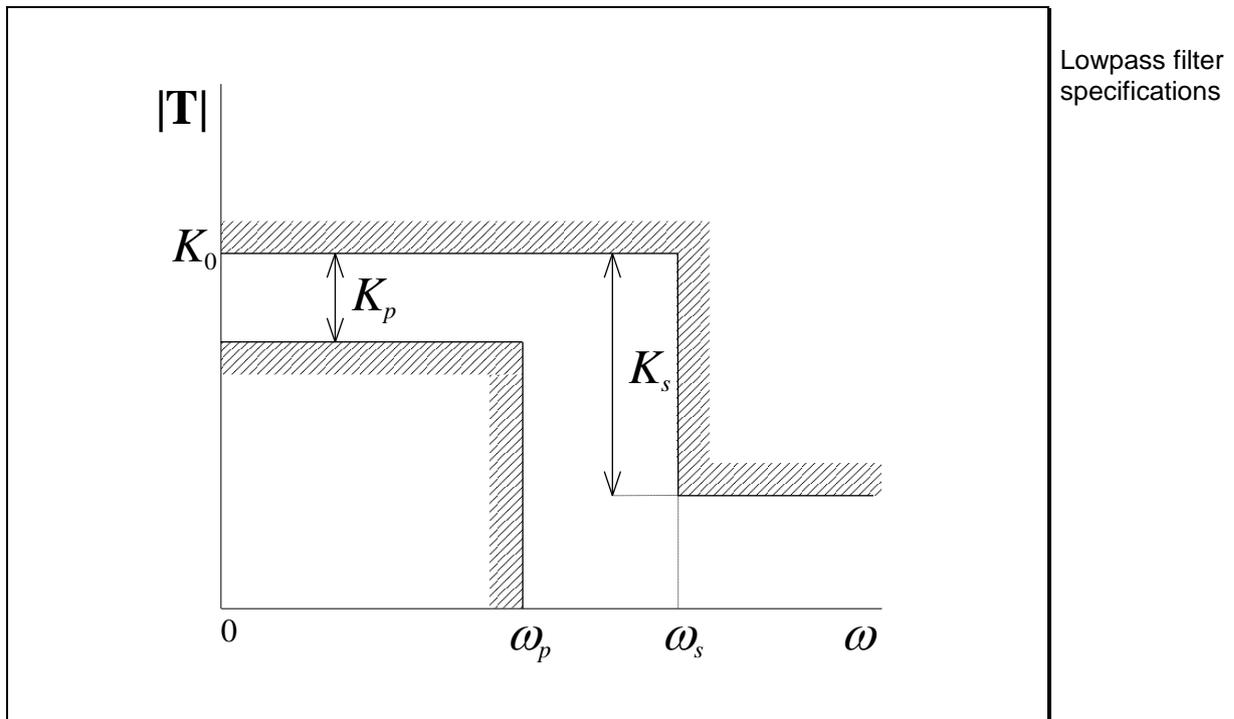


Figure 2.17

The gain at DC is K_0 dB. For the passband extending from $\omega = 0$ to $\omega = \omega_p$, the maximum deviation in the magnitude response is K_p dB. From ω_p to ω_s we have a *transition* band. Then the specifications indicate that from ω_s and for all higher frequencies, the minimum attenuation in the magnitude response is K_s dB. Given this information, we need to find n and ω_0 as applied to the Butterworth response.

Terms used in lowpass filter specifications

We begin with Eq. (2.33) and allow for the possibility of gain at DC:

$$|\mathbf{T}_n(j\omega)| = \frac{H}{\sqrt{1 + (\omega/\omega_0)^{2n}}}$$

(2.48) Butterworth magnitude response with gain

2.26

The gain at any frequency, in dB, is:

$$\begin{aligned} A &= 20 \log |\mathbf{T}_n(j\omega)| \\ &= 20 \log H - 10 \log \left[1 + (\omega/\omega_0)^{2n} \right] \end{aligned} \quad (2.49)$$

and since:

$K_0 = 20 \log H$

(2.50)

then:

$$K_0 - A = 10 \log \left[1 + (\omega/\omega_0)^{2n} \right] \quad (2.51)$$

Taking the antilogarithm, we have:

$$10^{0.1(K_0 - A)} = 1 + (\omega/\omega_0)^{2n} \quad (2.52)$$

To determine n , we substitute values of A and ω that go together, as in Figure 2.17. Then:

$$\left(\omega_p / \omega_0 \right)^{2n} = 10^{0.1K_p} - 1 \quad (2.53)$$

and:

$$\left(\omega_s / \omega_0 \right)^{2n} = 10^{0.1K_s} - 1 \quad (2.54)$$

Dividing these equations gives:

$$\left(\frac{\omega_s}{\omega_p} \right)^{2n} = \frac{10^{0.1K_s} - 1}{10^{0.1K_p} - 1} \quad (2.55)$$

Square rooting both sides:

$$\left(\frac{\omega_s}{\omega_p}\right)^n = \sqrt{\frac{10^{0.1K_s} - 1}{10^{0.1K_p} - 1}} \quad (2.56)$$

We now define:

$$M = \sqrt{\frac{10^{0.1K_s} - 1}{10^{0.1K_p} - 1}} \quad \text{and} \quad \Omega_s = \frac{\omega_s}{\omega_p} = \frac{f_s}{f_p} \quad (2.57)$$

M and Ω_s defined for use in finding n

Taking the logarithm of Eq. (2.56), together with the notation in Eq. (2.57) gives the desired result for the order of the filter:

$$n = \frac{\ln M}{\ln \Omega_s} \quad (2.58)$$

Finding n for a particular specification

This will ordinarily be a noninteger, so we *round up* to the next integer value and assign it to n . This ensures that we have an integer order, as well as meeting or exceeding the specifications.

Using this integer n , we can find ω_0 . We cannot meet the specifications exactly now since we are not using the noninteger n . Rearranging Eq. (2.52) gives:

$$\omega_0 = \frac{\omega}{\left(10^{0.1(K_0 - A)} - 1\right)^{1/2n}} \quad (2.59)$$

Since n is an integer, there will be a range of valid ω_0 's

If we use ω_p and the corresponding value of A in this equation, we get:

$$\omega_{0p} = \frac{\omega_p}{\left(10^{0.1K_p} - 1\right)^{1/2n}} \quad (2.60)$$

The ω_0 that meets the passband specifications

If we use this value of ω_0 then we will meet the specification at ω_p exactly, but have excess attenuation at ω_s . If we use ω_s and the corresponding value of A , we get:

The ω_0 that meets the stopband specifications

$$\omega_{0s} = \frac{\omega_s}{(10^{0.1K_s} - 1)^{1/2n}} \quad (2.61)$$

If we use this value for ω_0 then we will meet the specification at ω_s exactly, but have a smaller deviation in the passband. This dilemma is shown below:

Two choices of ω_0 that meet the specifications exactly

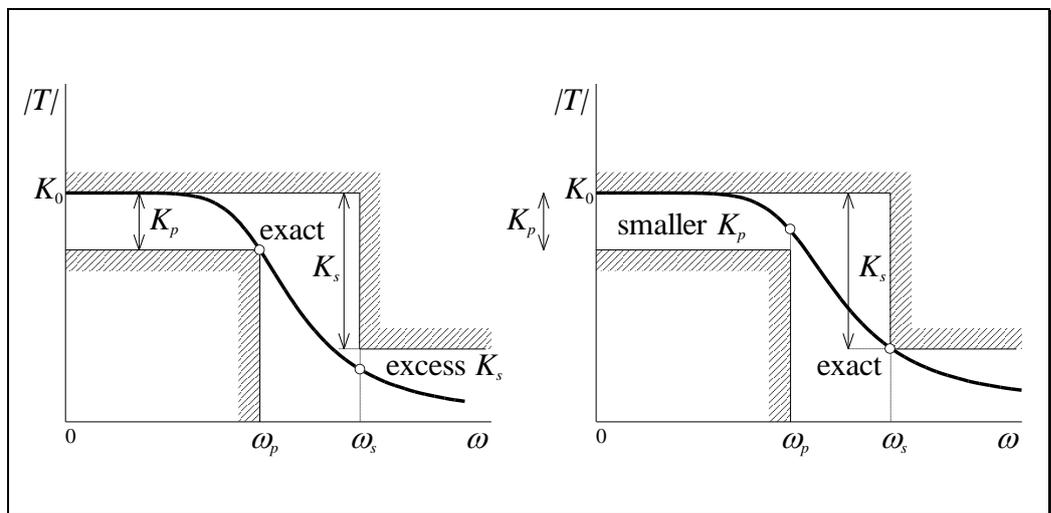


Figure 2.18

To make sure that we meet the specifications when the filter is realised, it is a good idea to exceed both of the specifications. That is, we want an ω_0 that is somewhere in between the two extremes shown above. We will choose the average between the two values *on a log scale*, so that:

We choose an ω_0 so that all specifications are exceeded

$$\log \omega_0 = \frac{\log \omega_{0p} + \log \omega_{os}}{2} \quad (2.62)$$

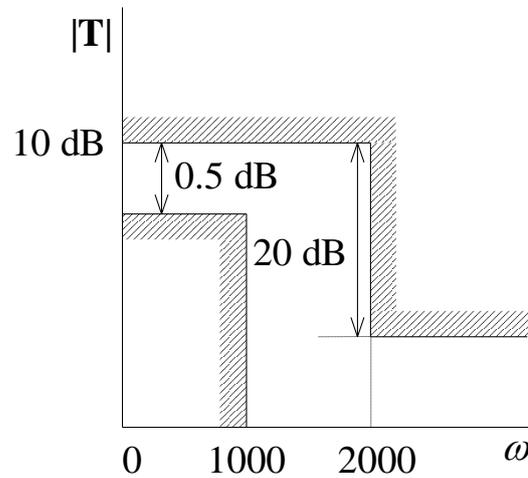
That is, we will choose the *geometric mean* of the two extremes, so that:

Finding ω_0 for a particular specification

$$\omega_0 = \sqrt{\omega_{0p} \omega_{os}} \quad (2.63)$$

EXAMPLE 2.2 Butterworth Lowpass Filter Design

We are required to realise the following specifications with a Butterworth response:



From the specifications, we have:

$$K_p = 0.5 \text{ dB}, \quad K_s = 20 \text{ dB},$$

$$\omega_p = 1000 \text{ rads}^{-1}, \quad \omega_s = 2000 \text{ rads}^{-1}$$

Substituting these values into Eqs. (2.57) and (2.58) we find that:

$$n = 4.832, \quad \text{round up to } n = 5$$

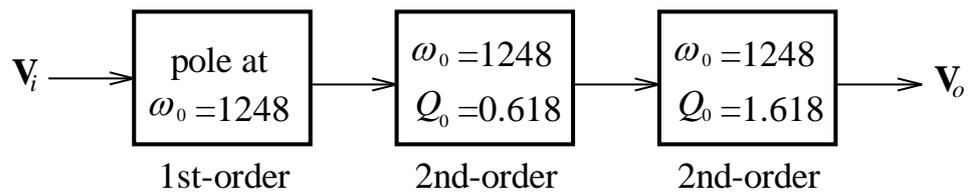
Using Eqs. (2.60), (2.61) and (2.63), we also find:

$$\omega_0 = 1248 \text{ rads}^{-1}$$

From Table 2.2, for $n = 5$ the required values of Q_0 are 0.618 and 1.618. There is also a first-order term with a pole at $-\omega_0$.

2.30

The realisation to meet these specifications can be shown in block diagram form:



Each second-order block could be realised using the biquad circuit. The first-order circuit can be realised with a simple buffered RC circuit. If we use the circuit of Figure 2.5 for the biquad, then we must frequency scale using $k_f = 1248$. Also, the gain at DC can be incorporated into one of the biquad circuits, with $H = 10^{10/20} = 3.162$. Alternatively, it could be provided by the first-order circuit, or be distributed among the three circuits.

2.9 References

Huelsman, L. P.: *Active and Passive Analog Filter Design*, McGraw-Hill, Singapore, 1993.

Sedra, A. & Smith, K.: *Microelectronic Circuits*, Saunders College Publishing, Sydney, 1991.

Van Valkenburg, M. E.: *Analog Filter Design*, Holt-Saunders, Tokyo, 1982.

Quiz

Encircle the correct answer, cross out the wrong answers. [one or none correct]

1.

The peak of the normalised lowpass second-order magnitude function occurs at:

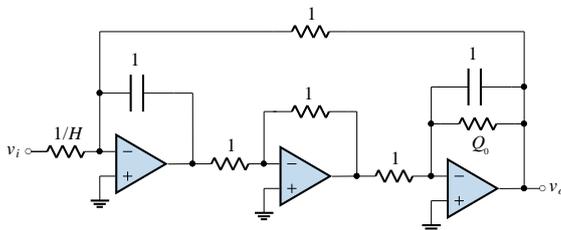
- (a) $\omega = \sqrt{1 - (1/4Q_0^2)}$ (b) $\omega = \sqrt{1 - (1/2Q_0^2)}$ (c) $\omega = 1$

2.

For the normalised lowpass Butterworth response, $|T_n(j1)|$ is:

- (a) $1/\sqrt{2}$ (b) $(1/\sqrt{2})^n$ (c) -6 dB

3.



For the normalised biquad circuit shown, the magnitude of the gain at $\omega = 1$ is:

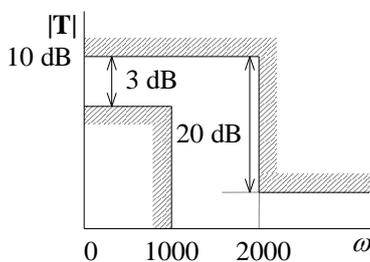
- (a) H (b) Q_0 (c) 1

4.

In terms of ω_0 and Q_0 , the left half-plane complex pole pairs of a lowpass second-order transfer function are at:

- (a) $-\omega_0/2Q_0 \pm j\omega_0\sqrt{1-1/4Q_0^2}$ (b) $-\omega_0/2Q_0 \pm j\omega_0\sqrt{1-4Q_0^2}$ (c) $-\omega_0/Q_0 \pm j\omega_0\sqrt{1-2Q_0^2}$

5.



A Butterworth filter meets the specifications shown. The gain A at $\omega = 1000$ should be:

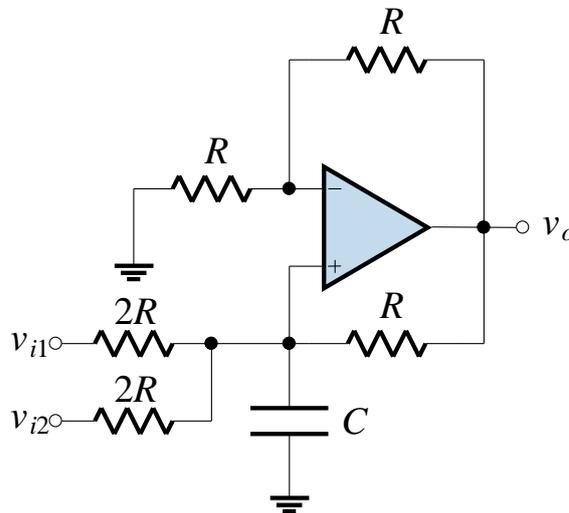
- (a) $A = 3 \text{ dB}$ exactly (b) $A < 7 \text{ dB}$ (c) $7 < A < 10 \text{ dB}$

Answers: 1. b 2. a 3. x 4. a 5. c

Exercises

1.

Consider the circuit shown below:



(a) Show that:

$$\mathbf{V}_o = \frac{1}{RCs} \mathbf{V}_{i1} + \frac{1}{RCs} \mathbf{V}_{i2}$$

(b) Show that the use of this circuit in the lowpass biquad circuit permits us to reduce the number of op-amps by one.

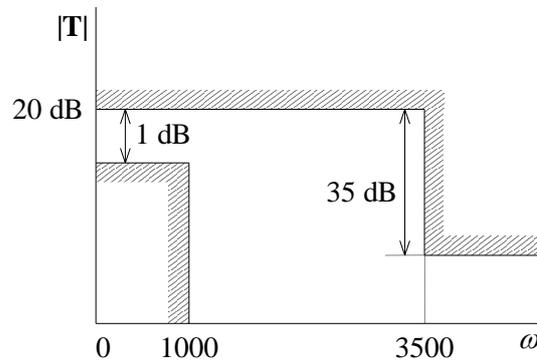
2.

(c) Design a lowpass second-order filter with $\omega_0 = 10\,000$, $Q_0 = 5$, $H=2$ using 1 nF capacitors.

(d) Design a highpass second-order filter with $\omega_0 = 5\,000$, $Q_0 = 8$, $H=1$ using 10 nF capacitors.

3.

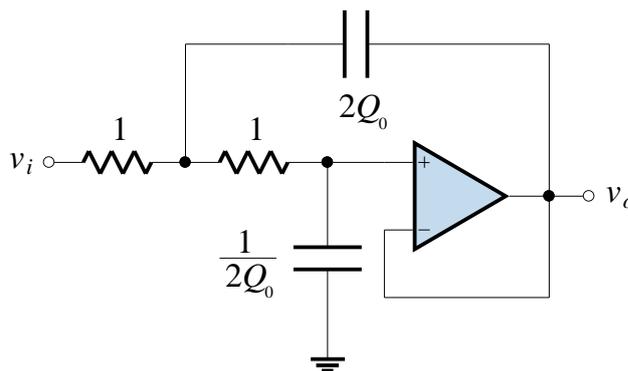
A lowpass filter is to have the following characteristics:



- Determine n , the required order of the Butterworth response.
- Determine the half power frequency, ω_0 .
- Determine the s plane location of the poles, and the Q_0 of each pole pair.
- Determine the actual gain/loss at the edge of the passband and the edge of the stopband.

4.

- Derive the transfer function of the so-called Sallen-Key circuit:



- This can be used instead of the biquad as a lowpass second-order circuit. Find a Sallen-Key realisation of your design in Exercise 3 and magnitude scale to obtain elements in a practical range.

Problems

1. [Sensitive Earth Fault Third Harmonic Remover]

An overcurrent protection relay is a device used by electricity supply authorities to detect faults on the system and to trip circuit breakers to clear these faults. Modern relays use microcontrollers to implement this function. Four currents are applied to the relay - phase A, phase B, phase C and earth. Ideally, in a balanced three-phase system, the earth current is zero. In practice, there is always some imbalance in the three phase currents that produces a small residual earth current.

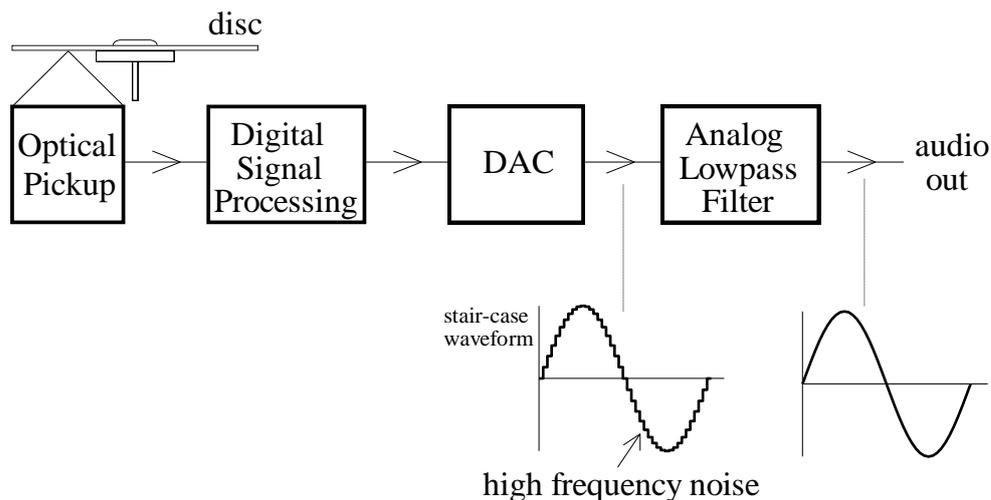
In addition, the non-linear nature of transformer magnetising characteristics, modern power electronic motor drives, and other electronically controlled loads all conspire to produce a third harmonic (150 Hz) earth current. These third harmonic currents are of considerable magnitude compared to the normal residual 50 Hz component and must be filtered out if the microcontroller relay is to detect small 50 Hz fault currents.

Accordingly, for “sensitive” earth faults (such as overhead wires down on a road) where the earth fault current is not large, an analog filter is required to remove third harmonics from the current measurement.

- (i) Design a filter so that at 50 Hz the output voltage of the filter is within 1% of the input voltage. At 150 Hz, the output voltage should be 20 times smaller than the input voltage.
- (ii) The microcontroller samples at a rate of 32 samples per cycle. By how many samples will the 50 Hz output of the filter be behind an unfiltered earth current input?

2. [CD Player Reconstruction Filter]

Audio CDs hold roughly 6 billion bits of information. A CD player reads these bits at 1.4112 Mbps and arranges them into 16-bit words. In first generation CD players each 16-bit word is fed to a digital to analog converter (DAC) at a rate of 44.1 kHz. Consequently, the output of the DAC is a stair-case waveform, which has to be smoothed somehow to reproduce the original analog sound:

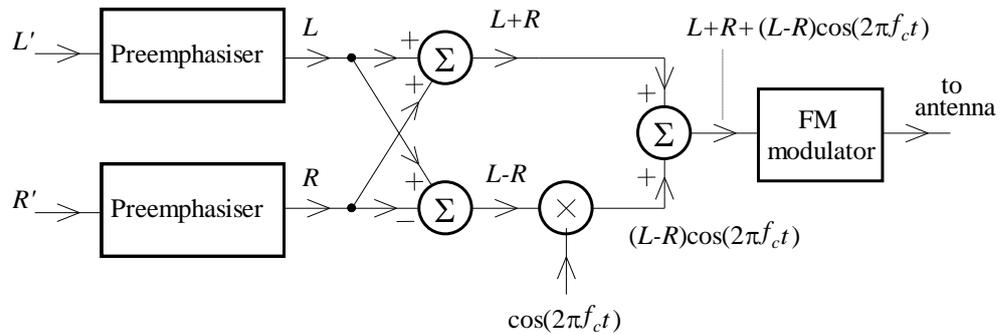


The audio range is from 20 Hz to 20 kHz, so in making the CD the digital recording restricts the maximum frequency to 20 kHz. We therefore need to smooth the DAC output to remove residual artefacts of the digitisation process.

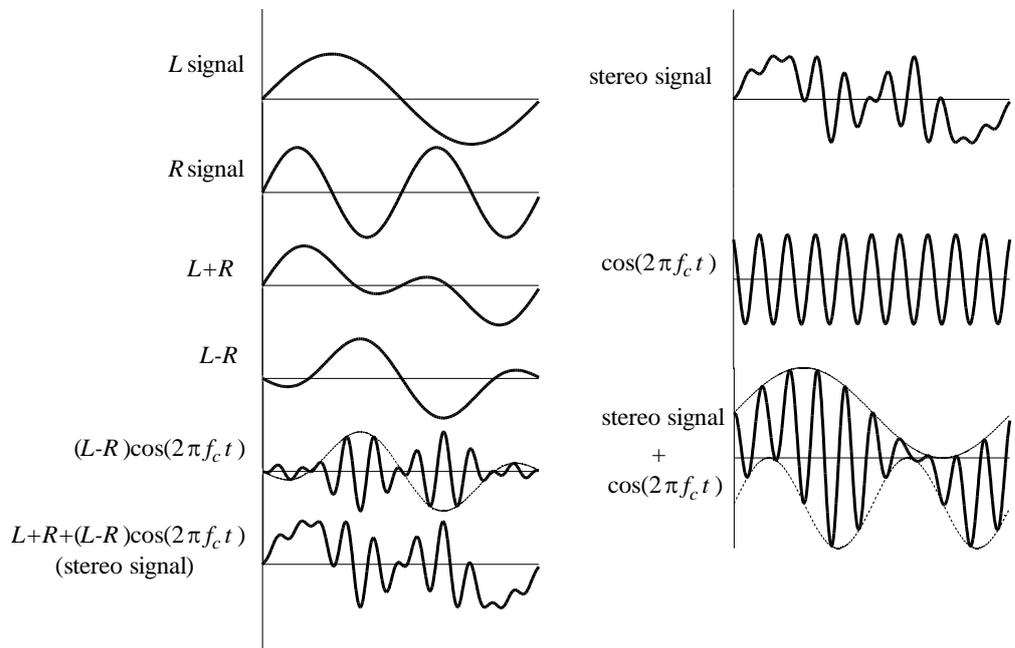
- (i) Design a fourth order analog lowpass filter (reconstruction filter) that has a 0 dB magnitude response at DC and a -0.5 dB magnitude response at 20 kHz. What is the attenuation (K_s) at 24.1 kHz? If 50 dB attenuation is normally required at 24.1 kHz, what order filter is necessary?
- (ii) How will the phase response of the reconstruction filter affect the audio (20 Hz to 20 kHz) output?

3. [Stereo FM Receiver]

Early FM broadcasts were monophonic. Stereophonic FM broadcasting, in which two audio signals L (left microphone) and R (right microphone) are used for a more natural effect, was proposed later. The stereo system had to be compatible with the original mono system. The scheme used is shown below:

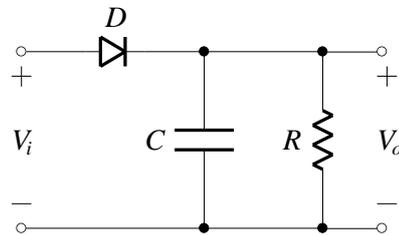


It is beneficial to graph the signals at various points in the above scheme. The left side of the diagram below shows how sinusoidal inputs on the left and right channel are converted into a stereo signal:

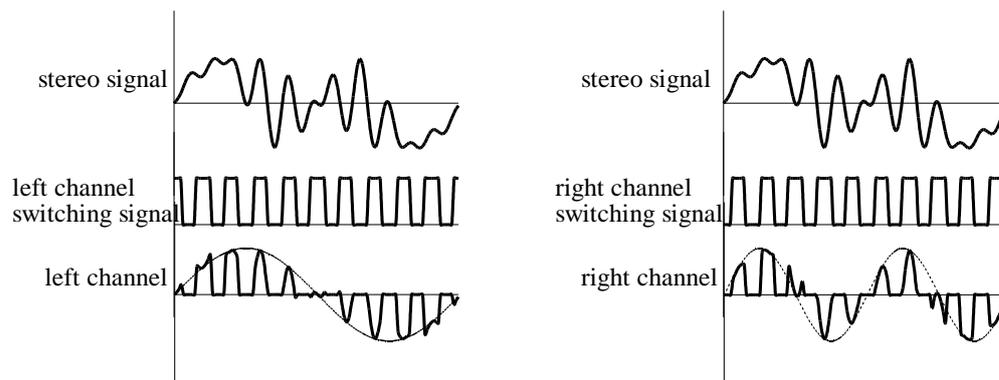


The right hand side of the above diagram shows how the stereo signal may be decoded at the receiver. The addition of the received signal with a sinusoid of frequency f_c results in a waveform that is easily separated into left and right channels. From the diagram, the left channel signal can be obtained by detecting

the positive “envelope”. The right channel can be obtained by detecting the negative “envelope”. Envelope detection can be done with a simple “lossy” peak detector:



The output of the above circuit would be the left channel. Another circuit, with the diode reversed, is required to detect the right channel. Alternatively, the diodes can be considered as switches which are turned on by alternate half-cycles of a sinusoid with frequency f_c . With this idea, the left and right components of the stereo signal are simply “steered” into the appropriate output channel. If such a switching scheme is used, the left and right channel outputs are:



Clearly, filtering is required. Design filters for the stereo FM output channels. They must introduce at most 0.5 dB attenuation between 0 and 15 kHz, and remove all frequency components above 30 kHz. Assume “remove” means attenuate by at least 20 dB. If the switching frequency is 38 kHz, by how much are switching residuals attenuated?

(For mono reception, the stereo signal can be simply passed through a 0-15 kHz brickwall lowpass filter).

3 Highpass and Bandpass Filters

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3.1 Negative Frequency

The concept of negative frequency is a very useful one. It finds application in frequency transformations and the complex representation of a Fourier series, to mention just two.

A negative frequency implies a clockwise rotating phasor

A negative frequency for a sinusoid just means that the phasor rotates in the clockwise direction instead of counterclockwise. For example, consider the sinusoid given by $x(t) = A \cos(\omega t + \phi)$. We can represent this in terms of complex exponentials as:

$$\begin{aligned} A \cos(\omega t + \phi) &= A \frac{e^{j(\omega t + \phi)} + e^{-j(\omega t + \phi)}}{2} \\ &= \frac{A}{2} e^{j\phi} e^{j\omega t} + \frac{A}{2} e^{-j\phi} e^{-j\omega t} \end{aligned} \quad (3.1)$$

If we define the phasor:

$$\mathbf{X} = \frac{A}{2} e^{j\phi} \quad (3.2)$$

then $A \cos(\omega t + \phi)$ expressed in terms of \mathbf{X} is:

A sinusoid is the sum of a forward rotating phasor and a backward rotating phasor

$$x(t) = A \cos(\omega t + \phi) = \mathbf{X} e^{j\omega t} + \mathbf{X}^* e^{-j\omega t} \quad (3.3)$$

That is, the sinusoid is the sum of a forward rotating phasor and a backward rotating phasor. The two phasors form a complex conjugate pair. The magnitude of each phasor is $A/2$.

The above result is shown graphically below:

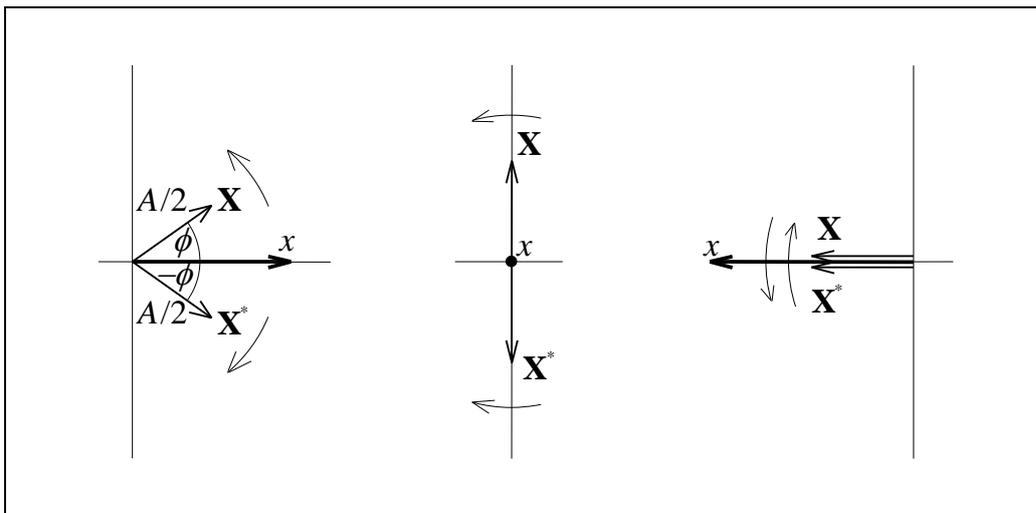
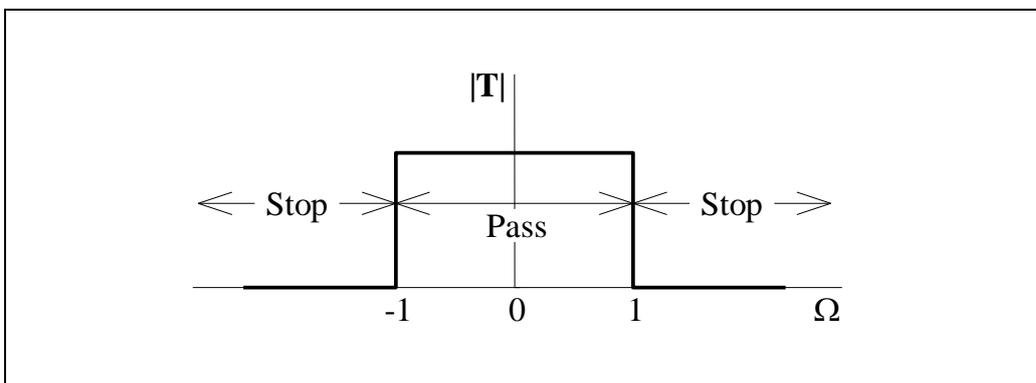


Figure 3.1

For frequency transformations, we will consider a sinusoid to be made up of a positive and negative frequency phasor.

3.2 Prototype Response

Frequency transformation begins with a lowpass magnitude response. We will represent all lowpass filter responses with a brick wall. We call this model lowpass response the *prototype response*. The prototype response, which includes positive and negative frequencies, is shown below:



The lowpass prototype response

Figure 3.2

The frequency for the prototype response is shown as Ω . The pass band extends from $\Omega = -1$ to $\Omega = 1$. All other frequencies constitute the stop bands.

The frequency Ω is the imaginary part of the complex frequency:

$$S = \Sigma + j\Omega$$

(3.4) The lowpass prototype's complex frequency

3.3 Frequency Transformations

We wish to transform frequencies of the prototype response into frequencies of a highpass response, or a bandpass response, or any other response we may wish to achieve. For example, to transform the prototype frequencies to highpass frequencies, we require that part of the imaginary axis extending from -1 to $+1$ to transform into the two segments of the imaginary axis from ω_0 to ∞ and from $-\omega_0$ to $-\infty$, as shown below:

An example of lowpass frequencies transforming to highpass frequencies

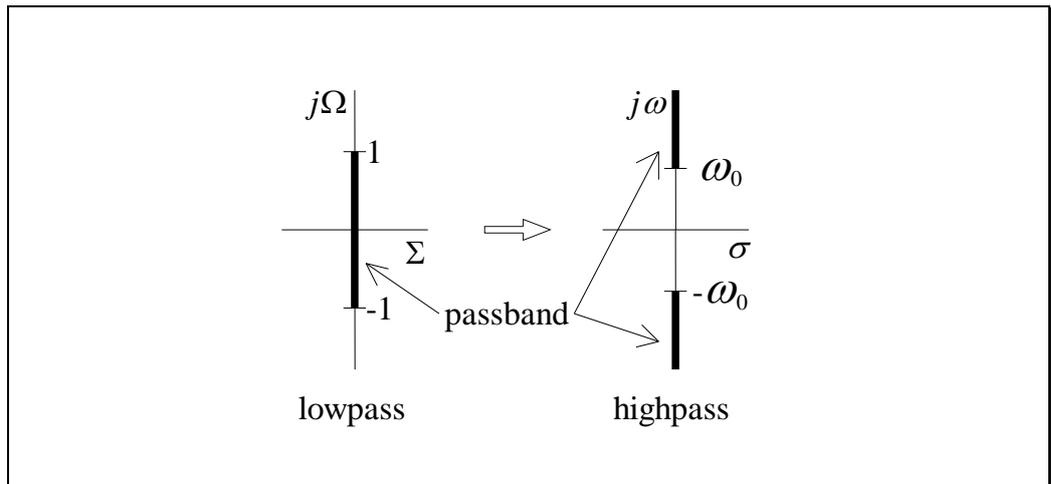


Figure 3.3

The frequency ω is the imaginary part of the complex frequency:

$$\mathbf{s} = \sigma + j\omega \tag{3.5}$$

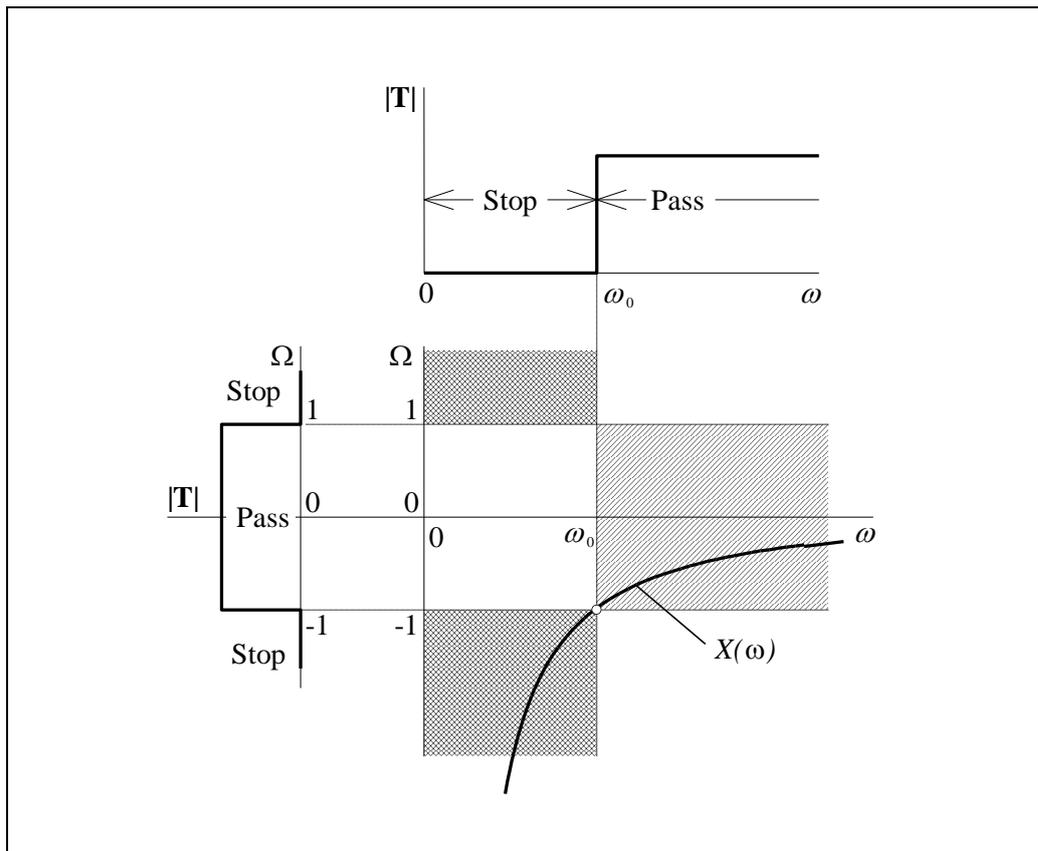
Therefore, the function that accomplishes the frequency transformation, designated as X , need only operate with the imaginary components of the complex frequency:

A frequency transformation

$$\mathbf{\Omega} = X(\omega) \tag{3.6}$$

When working out the function X , we superimpose on the $\Omega \sim \omega$ graph, the prototype pass and stop bands, and the desired response pass and stop bands.

An example of this is shown below for the case of a desired highpass response:



An example of a function that converts the lowpass prototype response into a highpass response

Figure 3.4

The regions that have the same requirements are cross-hatched, with double cross-hatching indicating the regions that specify stop bands and single cross-hatching for those regions that specify pass bands. The remaining regions, without cross-hatching, have requirements with respect to stop and pass that are in conflict. The function X must pass through the cross-hatched regions.

3.6

3.4 Foster Reactance Functions

Consider the class of functions:

$$\mathbf{A}(s) = H \frac{\mathbf{p}(s)}{\mathbf{q}(s)} \quad (3.7)$$

where H is a real and positive constant, and the poles and zeros have the following properties:

The Foster reactance function's properties

1. All poles and zeros are located on the imaginary axis. Since they must occur as conjugate imaginary terms, they will have the form:

$$(s + j\omega_0)(s - j\omega_0) = s^2 + \omega_0^2 \quad (3.8)$$

2. Poles and zeros alternate with increasing ω .
3. A pole or zero is located at the origin, $s = 0$.
4. A pole or zero is located at infinity. This means the degree of difference between $\mathbf{p}(s)$ and $\mathbf{q}(s)$ is 1.

Several functions that satisfy the requirements are:

$$\mathbf{A}_1(\mathbf{s}) = \frac{H}{\mathbf{s}} \quad (3.9) \quad \text{Several Foster reactance functions}$$

$$\mathbf{A}_2(\mathbf{s}) = H\mathbf{s} \quad (3.10)$$

$$\mathbf{A}_3(\mathbf{s}) = \frac{H(\mathbf{s}^2 + \omega_0^2)}{\mathbf{s}} \quad (3.11)$$

$$\mathbf{A}_4(\mathbf{s}) = \frac{H\mathbf{s}}{(\mathbf{s}^2 + \omega_0^2)} \quad (3.12)$$

$$\mathbf{A}_5(\mathbf{s}) = \frac{H(\mathbf{s}^2 + \omega_1^2)}{\mathbf{s}(\mathbf{s}^2 + \omega_2^2)} \quad (3.13)$$

When $\mathbf{s} = j\omega$, all the factors are real, except for the s in the numerator or denominator. Therefore, $\mathbf{A}(j\omega)$ is imaginary. The imaginary part is:

$$X(\omega) = \frac{1}{j} \mathbf{A}(j\omega) \quad (3.14)$$

3.8

Applying Eq. (3.14) to Eqs. (3.9)-(3.13), we have:

Several Foster reactance functions in terms of ω

$$X_1 = \frac{-H}{\omega} \quad (3.15)$$

$$X_2 = H\omega \quad (3.16)$$

$$X_3 = -H \frac{\omega_0^2 - \omega^2}{\omega} \quad (3.17)$$

$$X_4 = H \frac{\omega}{\omega_0^2 - \omega^2} \quad (3.18)$$

$$X_5 = \frac{-H}{\omega} \frac{\omega_1^2 - \omega^2}{\omega_2^2 - \omega^2} \quad (3.19)$$

The functions described above are known as reactance functions, and these properties were first described by Foster¹ in 1924. In recognition of Foster's work, we refer to them as *Foster reactance functions*.

¹ Ronald M. Foster was an American Telephone & Telegraph Company (later Bell Labs) mathematician whose work was of significance in designing electronic filters for use on telephone lines. The reactance functions appear in his paper "A Reactance Theorem", *Bell Sys. Tech. J.*, vol. 3, pp. 259-267, 1924.

These functions are shown below, together with the pole-zero plot for the corresponding $\mathbf{A(s)}$:

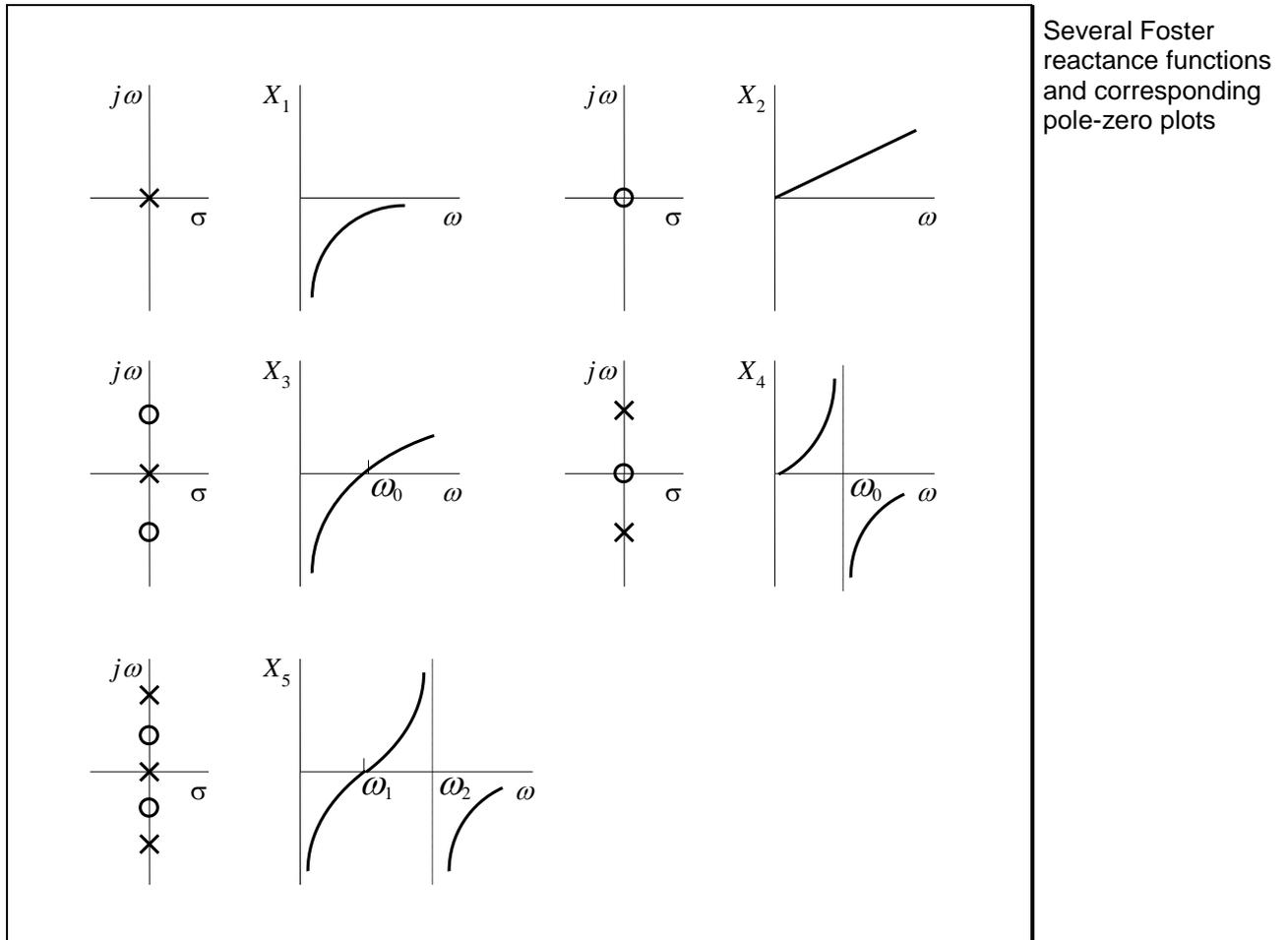


Figure 3.5

From Figure 3.5, we observe two properties of X :

1. $X(0)$ has either the value 0 or the value $-\infty$.
2. The slope of X with respect to ω is always positive.

More properties of the Foster reactance function

3.5 Highpass Transformation

The highpass transformation requirements are given below:

Highpass transformation requirements

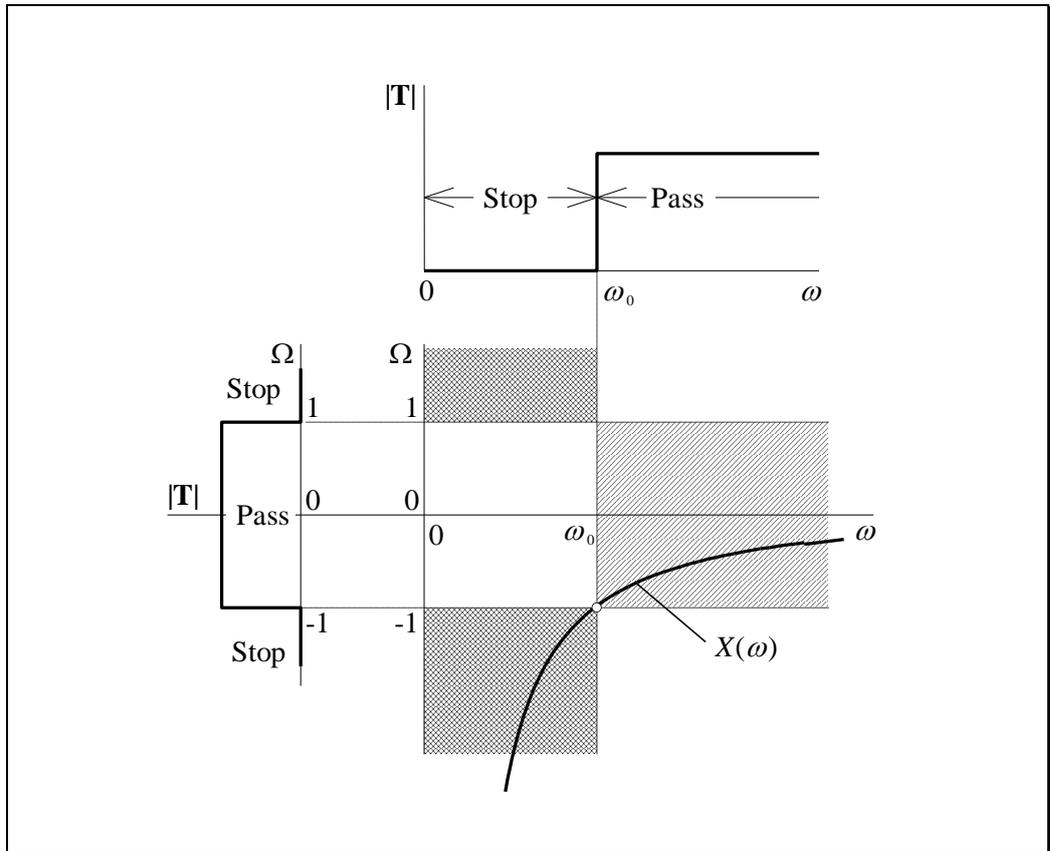


Figure 3.6

By property 1 above, when $\omega = 0$, $X(0)$ must be either 0 or $-\infty$. Since 0 is not in a cross-hatched region, we must have $X(0) = -\infty$. By property 2, X must be -1 at frequency ω_0 . The required form of the reactance function is shown in Figure 3.6 and described by Eq. (3.15):

The Foster reactance function for a highpass transformation

$$X(\omega) = \frac{-H}{\omega} \quad (3.20)$$

To evaluate H , we observe that:

$$X(\omega_0) = -1 = \frac{-H}{\omega_0} \quad (3.21)$$

which means $H = \omega_0$. Then:

$$jX(\omega) = \frac{\omega_0}{j\omega} \quad (3.22)$$

In terms of complex frequency s , we have:

$$\mathbf{S} = j\Omega = jX(\omega) = \frac{\omega_0}{j\omega} \Big|_{j\omega=s} \quad (3.23)$$

This evaluates to:

$$\mathbf{S} = \frac{\omega_0}{s}$$

(3.24) The highpass to lowpass prototype frequency transformation

which is the highpass to lowpass transformation.

3.6 Bandpass Transformation

The bandpass transformation requirements are given below:

Bandpass transformation requirements

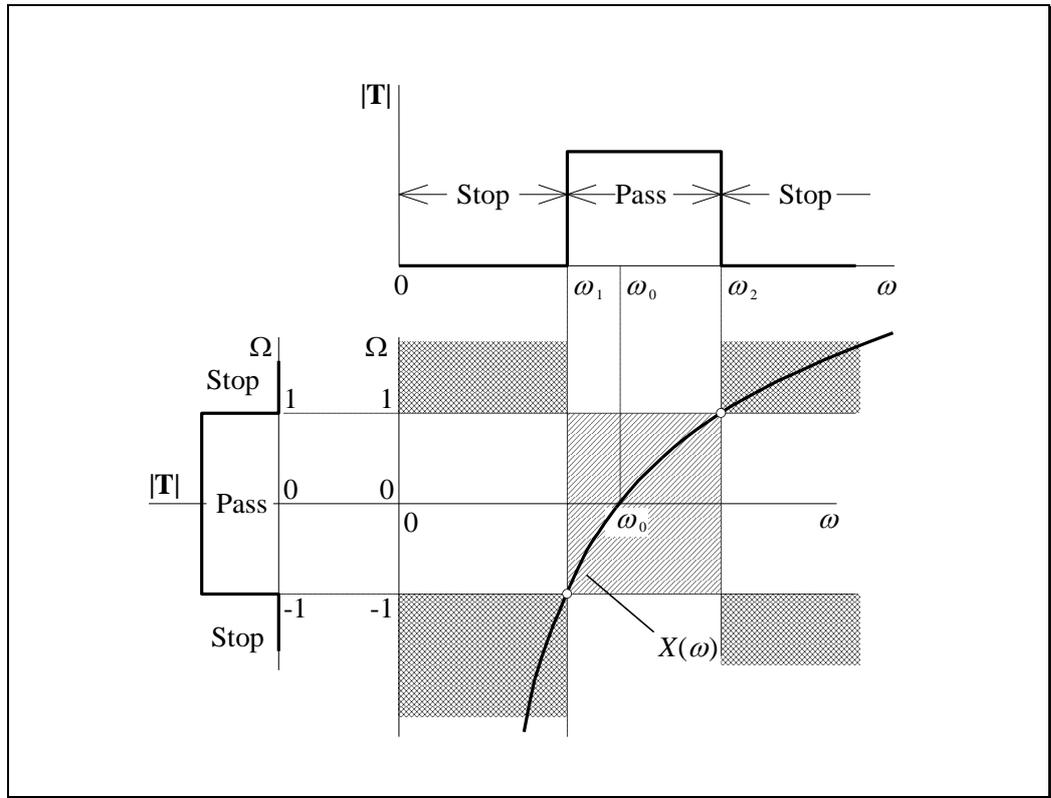


Figure 3.7

The frequency response at the top of Figure 3.7 is known as a bandpass frequency response. Frequencies between ω_1 and ω_2 are passed through this filter, while both high and low frequencies are stopped.

By property 1 of the reactance functions, we must have $X(0) = -\infty$. From property 2 of the reactance functions, the positive slope requirement implies that the only solution is that shown in Figure 3.7 and given by Eq. (3.17):

The Foster reactance function for a bandpass transformation

$$X(\omega) = -H \frac{\omega_0^2 - \omega^2}{\omega} \quad (3.25)$$

We can also come to this conclusion by observing that $X(\omega)$ has a pole at $\omega = 0$ and a zero at ω_0 .

To determine the value of H we substitute the known values of X at ω_1 and ω_2 .

Thus:

$$X(\omega_1) = -1 = -H \frac{\omega_0^2 - \omega_1^2}{\omega_1} \quad (3.26)$$

$$X(\omega_2) = +1 = -H \frac{\omega_0^2 - \omega_2^2}{\omega_2} \quad (3.27)$$

Dividing Eq. (3.26) by (3.27) gives:

$$\frac{\omega_2}{\omega_1} \frac{\omega_0^2 - \omega_1^2}{\omega_0^2 - \omega_2^2} = -1 \quad (3.28)$$

which can be manipulated so that:

$$\begin{aligned} \omega_2(\omega_0^2 - \omega_1^2) &= -\omega_1(\omega_0^2 - \omega_2^2) \\ \omega_2\omega_0^2 - \omega_2\omega_1^2 &= -\omega_1\omega_0^2 + \omega_1\omega_2^2 \\ \omega_0^2(\omega_1 + \omega_2) &= \omega_1\omega_2(\omega_1 + \omega_2) \\ \omega_0^2 &= \omega_1\omega_2 \end{aligned} \quad (3.29)$$

and we finally have the important relationship for a bandpass filter:

$\omega_0 = \sqrt{\omega_1\omega_2}$	(3.30) Specification centre frequency defined
--------------------------------------	---

Substituting this back into Eq. (3.26), we get:

$$-1 = -H \frac{\omega_1\omega_2 - \omega_1^2}{\omega_1} \quad (3.31)$$

3.14

which gives for H :

$$H = \frac{1}{\omega_2 - \omega_1} \quad (3.32)$$

For a bandpass filter, we define the bandwidth as:

Specification
bandwidth defined

$$\text{bw} = \omega_2 - \omega_1 \quad (3.33)$$

which is *different* to the 3 dB bandwidth, abbreviated B . With all these definitions, we now have:

$$jX(\omega) = \frac{j}{\text{bw}} \frac{\omega^2 - \omega_0^2}{\omega} \quad (3.34)$$

In terms of complex frequency s , we have:

$$\mathbf{S} = j\Omega = jX(\omega) = \frac{1}{\text{bw}} \frac{(j\omega)^2 + \omega_0^2}{j\omega} \Big|_{j\omega=s} \quad (3.35)$$

This evaluates to:

The bandpass to
lowpass prototype
frequency
transformation

$$\mathbf{S} = \frac{1}{\text{bw}} \frac{s^2 + \omega_0^2}{s} \quad (3.36)$$

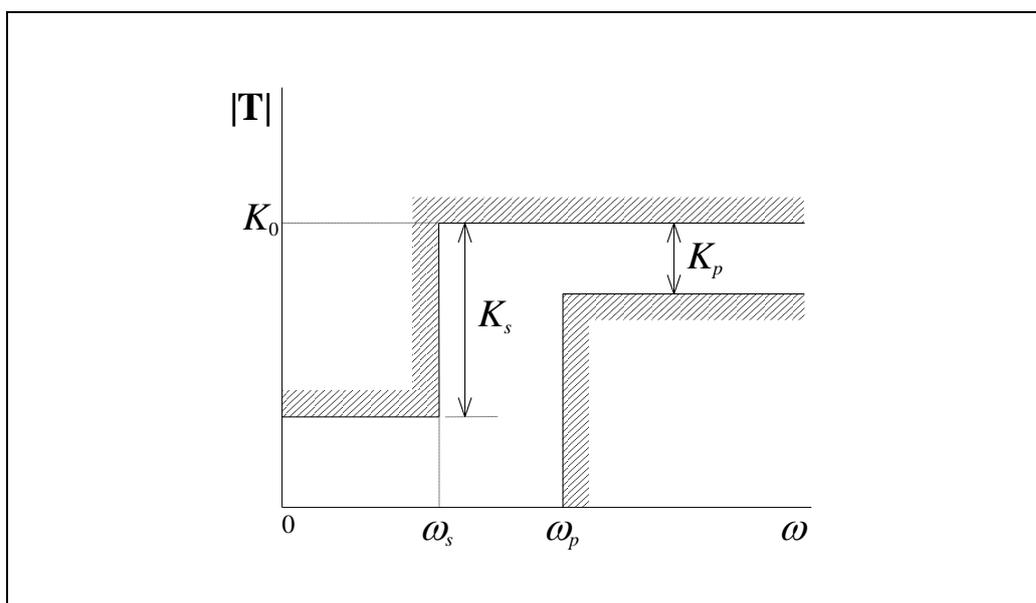
which is the bandpass to lowpass transformation.

3.7 Highpass Filter Design

We begin with highpass filter specifications and then employ a frequency transformation to obtain the equivalent lowpass prototype specifications. From these specifications the order of the filter is determined and also such quantities as the half-power frequency. The frequency transformation is then used once more to obtain the required highpass transfer function from which the filter may be designed.

The idea behind highpass filter design

A typical highpass filter specification is shown below:



Highpass filter specifications

Figure 3.8

The gain as $\omega \rightarrow \infty$ is K_0 dB. For the stopband extending from $\omega = 0$ to $\omega = \omega_s$, the minimum attenuation in the magnitude response is K_s dB. From ω_s to ω_p we have the *transition* band. Then the specifications indicate that from ω_p and for all higher frequencies, the maximum deviation in the magnitude response is K_p dB.

To this specification we will apply the frequency transformation:

$$\mathbf{S} = \frac{\omega_p}{\mathbf{s}} \quad (3.37)$$

3.16

With imaginary values for both \mathbf{S} and \mathbf{s} , this equation becomes:

$$\Omega = \frac{-\omega_p}{\omega} \tag{3.38}$$

The negative sign in this equation is simply a result of the negative frequencies for the lowpass prototype corresponding to positive frequencies for the highpass response (see Figure 3.6), shown below:

The relationship between highpass and lowpass prototype frequencies

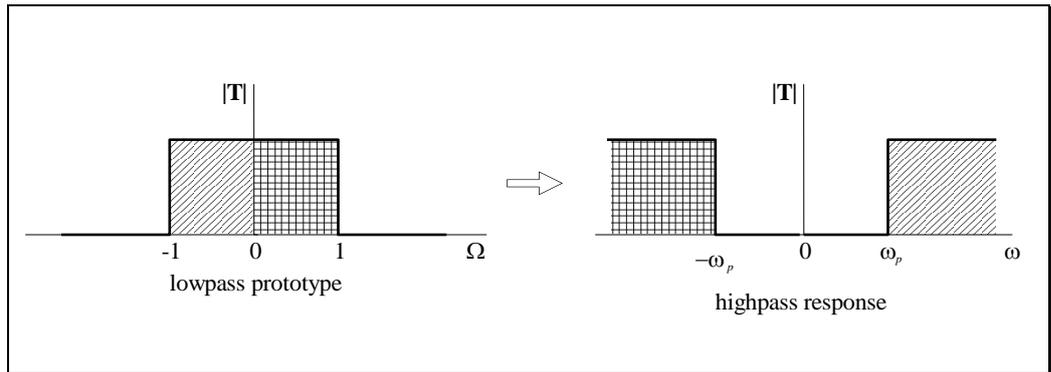


Figure 3.9

Since the responses have even symmetry when we include both positive and negative frequencies, we can drop the negative sign in Eq. (3.38) so that we work with positive frequencies in both the lowpass prototype and highpass cases.

The frequency transformation we use for specifications is therefore:

The highpass to lowpass prototype frequency transformation used for specifications

$$\Omega = \frac{\omega_p}{\omega} \tag{3.39}$$

We cannot use this transformation for complex frequency since the transformation would be $\mathbf{S} = -\omega_p/\mathbf{s}$. We cannot have a negative sign in the complex frequency transformation because this will cause some poles to be in the right half-plane. *Test this with the poles given by: $\mathbf{S}^2 + \mathbf{S} + 1$.*

From Eq. (3.39), we see that:

$\omega = \omega_p \text{ corresponds to } \Omega_p = 1$	(3.40)
--	--------

$\omega = \omega_s \text{ corresponds to } \Omega_s = \frac{\omega_p}{\omega_s}$	(3.41)	Lowpass prototype frequencies for the highpass case
--	--------	---

The specifications for the lowpass prototype filter are therefore:

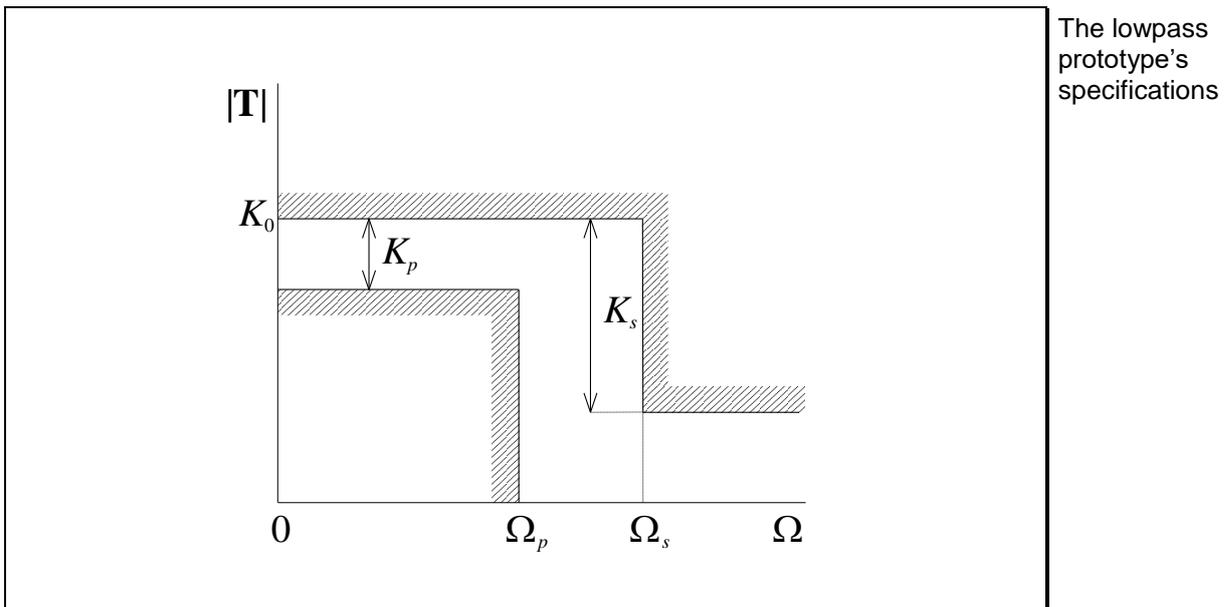


Figure 3.10

For a Butterworth filter, and in the same manner as for the lowpass filter, we define the following for the highpass filter:

$M = \sqrt{\frac{10^{0.1K_s} - 1}{10^{0.1K_p} - 1}}$	(3.42)	<i>M</i> for highpass filters in terms of lowpass prototype specifications
--	--------	--

In comparing this formula with that for the lowpass filter, note that Ω_s for the highpass case has been defined from the frequency transformation, Eq. (3.41).

3.18

We still have for the order:

The order of a
highpass
Butterworth filter

$$n = \frac{\ln M}{\ln \Omega_s} \quad (3.43)$$

The half-power frequency for the lowpass prototype that meets the passband specification is given by:

$$\Omega_{0p} = \frac{1}{\left(10^{0.1K_p} - 1\right)^{1/2n}} \quad (3.44)$$

The half-power frequency for the lowpass prototype that meets the stopband specification is given by:

$$\Omega_{0s} = \frac{\Omega_s}{\left(10^{0.1K_s} - 1\right)^{1/2n}} \quad (3.45)$$

and we take the half-power frequency for the lowpass prototype as:

The half-power
frequency of the
lowpass prototype
Butterworth filter

$$\Omega_0 = \sqrt{\Omega_{0p}\Omega_{0s}} \quad (3.46)$$

For the highpass filter, we use Eq. (3.38) to obtain the half-power frequency:

The half-power
frequency of the
highpass
Butterworth filter

$$\omega_0 = \frac{\omega_p}{\Omega_0} \quad (3.47)$$

Since we have found n , the normalised lowpass transfer function is known. Substituting the transformation for the normalised case:

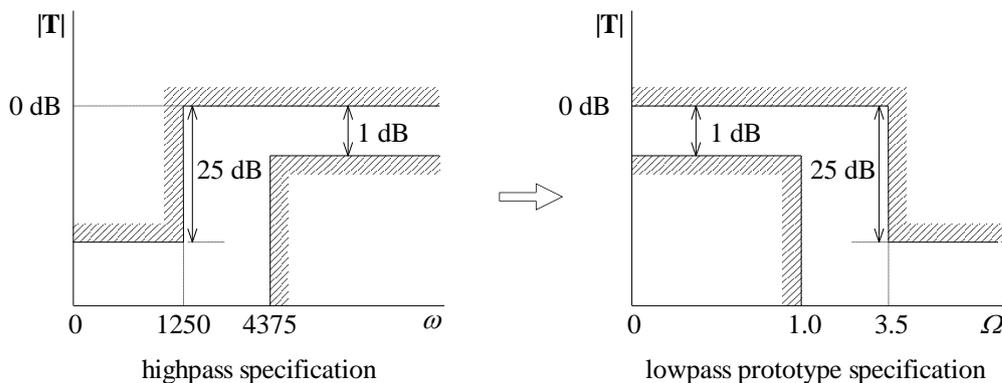
$$\mathbf{S} = \frac{1}{\mathbf{s}}$$

(3.48) The normalised highpass to lowpass transformation

gives the normalised highpass transfer function. The transfer function is implemented as a cascade of first- and second-order highpass circuits. The second-order circuits can be biquad circuits, or any other circuits that implement second-order highpass transfer functions.

EXAMPLE 3.1 Highpass Butterworth Filter Design

The specifications for a highpass filter are shown below, indicating that at least 25 dB of attenuation is required in the stop band and that there be less than 1 dB of deviation from 0 dB in the pass band. Also shown is the transformation to the lowpass prototype specification using Eqs. (3.40)-(3.41):



From Eq. (3.42), we find n as:

$$n = 2.836, \quad \text{round up to } n = 3$$

Using Eqs. (3.43), (3.44) and (3.45), we find the half-power frequency of the lowpass prototype:

$$\Omega_0 = 1.296 \text{ rad/s}^{-1}$$

3.20

For the highpass filter, the corresponding quantity is:

$$\omega_0 = \frac{\omega_p}{\Omega_0} = 3376 \text{ rads}^{-1}$$

Since we know that an $n = 3$ Butterworth response is required, the normalised lowpass transfer function is worked out using Table 2.2 from Topic 2:

$$\mathbf{T}(\mathbf{S}) = \frac{1}{(\mathbf{S}^2 + \mathbf{S} + 1)(\mathbf{S} + 1)}$$

Substituting the normalised frequency transformation given by Eq. (3.48) into the above equation gives:

$$\mathbf{T}(\mathbf{s}) = \frac{\mathbf{s}^3}{(\mathbf{s}^2 + \mathbf{s} + 1)(\mathbf{s} + 1)}$$

We then find a circuit realisation for this transfer function, frequency scale with $k_f = 3376$ and magnitude scale to get convenient element values.

3.8 Bandpass Response

Consider the following biquad circuit:

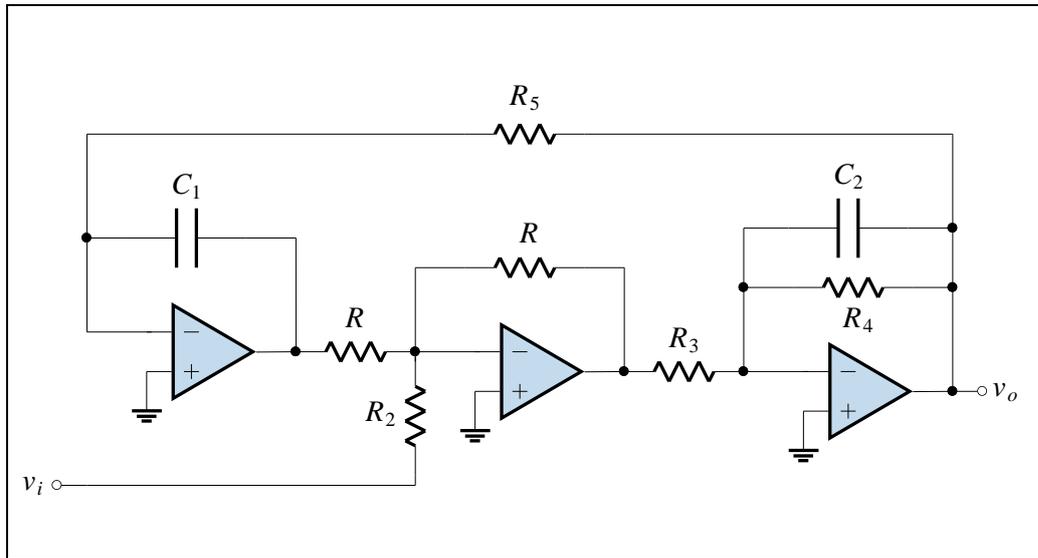


Figure 3.11

Show that the transfer function V_o/V_i is:

$$\frac{V_o}{V_i} = \frac{(R/R_2R_3C_2)s}{s^2 + (1/R_4C_2)s + 1/R_3R_5C_1C_2} \quad (3.49)$$

This differs from the biquad's lowpass transfer function:

$$\mathbf{T}(s) = \frac{-1/R_1R_3C_1C_2}{s^2 + (1/R_4C_2)s + 1/R_3R_5C_1C_2} \quad (3.50)$$

in that it has a zero at the origin. The denominators of the two transfer functions are the same.

3.22

The general transfer function, for which Eq. (3.49) is a special case, must be:

$$\mathbf{T}(s) = \frac{Ks}{s^2 + (\omega_0/Q_0)s + \omega_0^2} \quad (3.51)$$

We can convert Eq. (3.51) into a standard form by requiring that $|\mathbf{T}(j\omega_0)|$ have the value H , which is analogous to the DC gain for the lowpass case. By setting $s = j\omega_0$ in Eq. (3.51) we see that:

$$K = H \frac{\omega_0}{Q_0} \quad (3.52)$$

so that the standard form for a second-order bandpass transfer function is:

The standard form for a second-order bandpass transfer function

$$\mathbf{T}(s) = \frac{H(\omega_0/Q_0)s}{s^2 + (\omega_0/Q_0)s + \omega_0^2} \quad (3.53)$$

From Eq. (3.49) we see that:

Design equations for the biquad to implement a bandpass filter

$$\omega_0 = \frac{1}{\sqrt{R_3 R_5 C_1 C_2}} \quad (3.54)$$

$$Q_0 = \sqrt{\frac{R_4^2 C_2}{R_3 R_5 C_1}} \quad (3.55)$$

$$H = \frac{R R_4}{R_2 R_3} \quad (3.56)$$

These are the design equations for the biquad to implement a second-order bandpass transfer function.

The normalised form of Eq. (3.53), with $H = 1$, is:

$$\mathbf{T}(s) = \frac{(1/Q_0)s}{s^2 + (1/Q_0)s + 1} \quad (3.57)$$

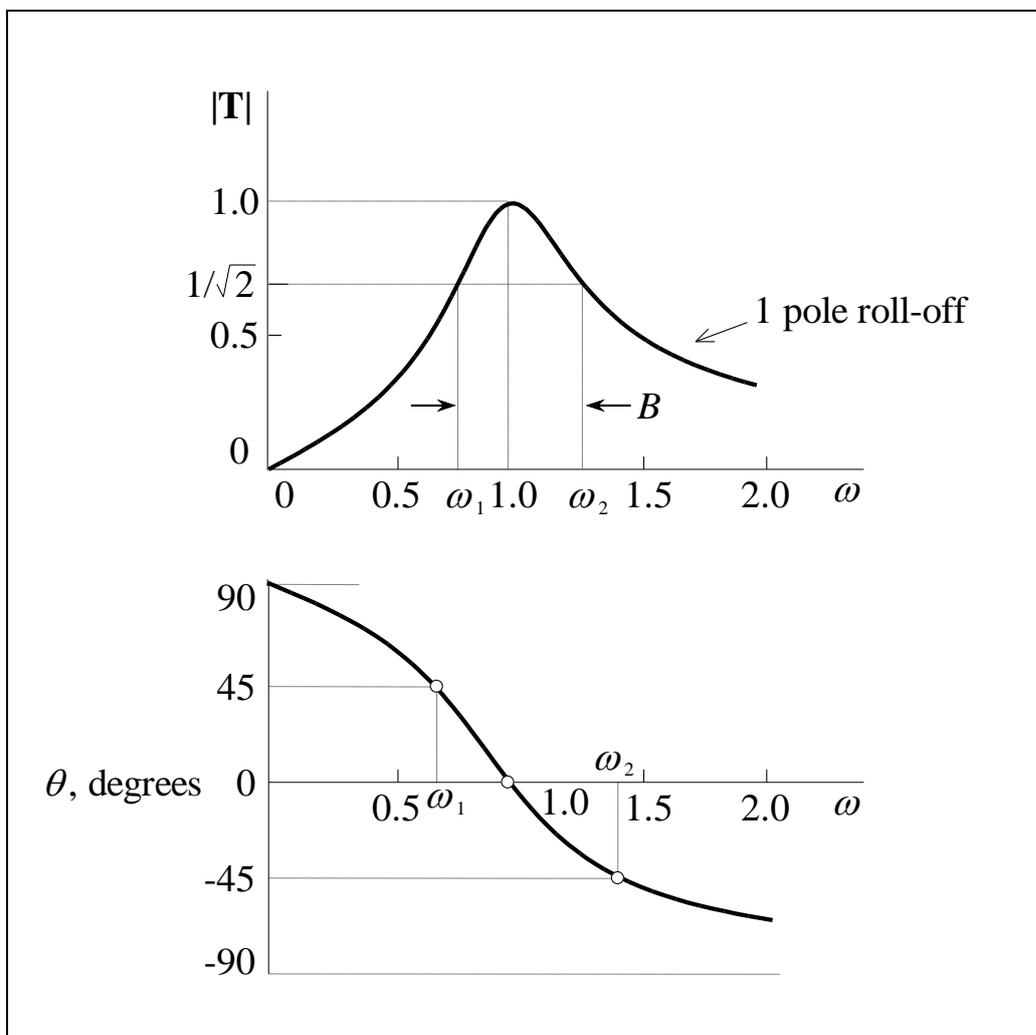
The standard form for a normalised second-order bandpass transfer function

The magnitude response of the normalised second-order bandpass transfer function is therefore:

$$|\mathbf{T}(j\omega)| = \frac{\omega/Q_0}{\sqrt{(1-\omega^2)^2 + (\omega/Q_0)^2}} \quad (3.58)$$

The magnitude response of a normalised second-order bandpass transfer function

A plot of the magnitude and phase response is shown below:



Typical magnitude and phase response of a normalised second-order bandpass transfer function

Figure 3.12

3.24

The locations of the poles and zero for the bandpass case are shown below:

Pole-zero constellation for a second-order bandpass transfer function

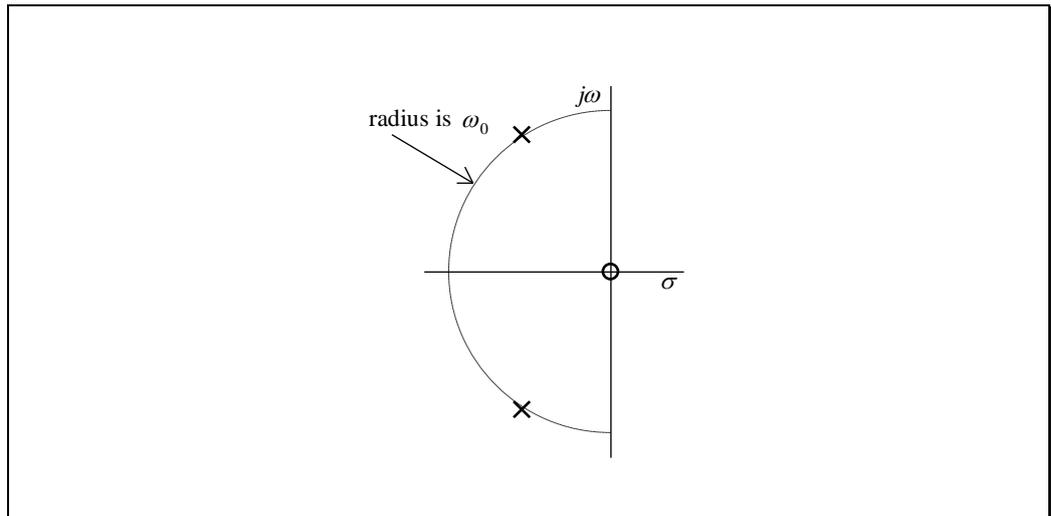


Figure 3.13

We see that there is a single zero at the origin of the s plane, with poles in the same position as in the lowpass case.

Show that the two half-power frequencies of the transfer function in Eq. (3.53) are given by:

Half-power frequencies defined for a second-order bandpass transfer function

$$\omega_2, \omega_1 = \omega_0 \left(\sqrt{1 + (1/2Q_0)^2} \pm 1/2Q_0 \right) \quad (3.59)$$

The product and difference of these two frequencies are:

Centre frequency defined for a second-order bandpass transfer function

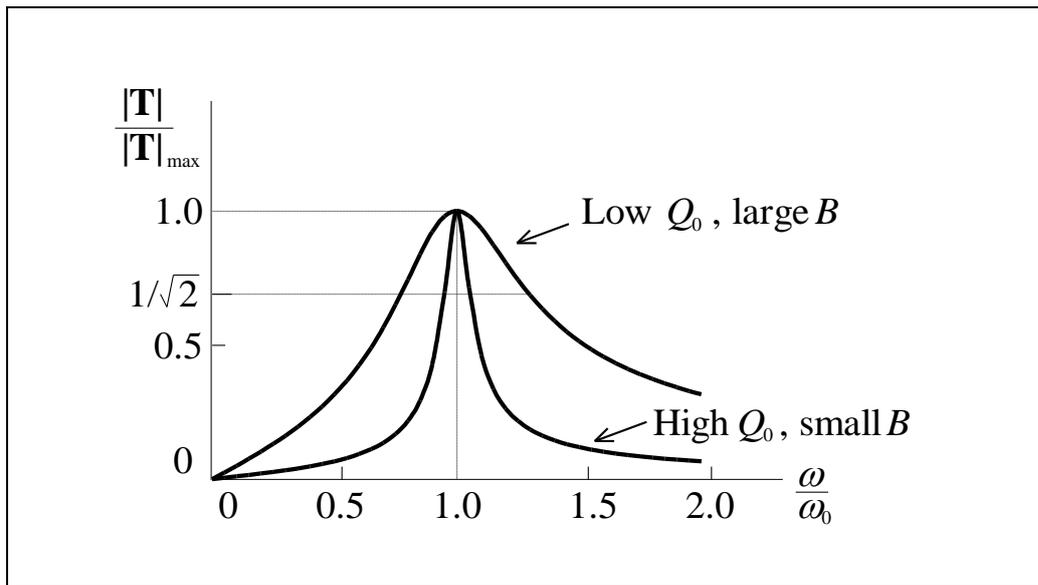
$$\omega_1 \omega_2 = \omega_0^2 \quad \text{and} \quad \omega_2 - \omega_1 = \frac{\omega_0}{Q_0} \quad (3.60)$$

This frequency difference is defined as the bandwidth, or B for short. Therefore we have the definition:

Bandwidth, B , defined for a second-order bandpass transfer function

$$B = \omega_2 - \omega_1 = \frac{\omega_0}{Q_0} \text{ rads}^{-1} \quad (3.61)$$

This equation tells us that Q_0 and B are inversely related, as shown below:



The inverse relationship between B and Q_0

Figure 3.14

3.9 Bandpass Filter Design

The idea behind bandpass filter design

Bandpass filter design is performed in the same manner as highpass filter design. We begin with bandpass filter specifications and then employ a frequency transformation to obtain the equivalent lowpass prototype specifications. From these specifications the order of the filter is determined and also such quantities as the half-power frequency. The frequency transformation is then used once more to obtain the poles of the bandpass transfer function from which the filter may be designed.

A typical bandpass filter specification is shown below:

Bandpass filter specifications

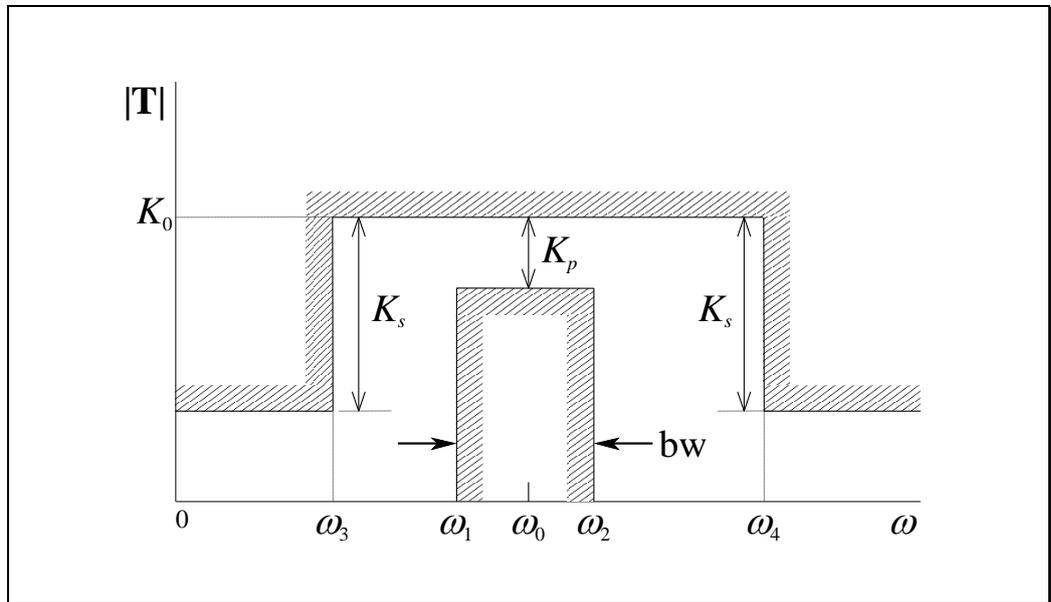


Figure 3.15

The gain at the centre frequency ω_0 is K_0 dB. For the passband extending from $\omega = \omega_1$ to $\omega = \omega_2$, of bandwidth bw , the maximum deviation in the magnitude response is K_p dB. From ω_3 to ω_1 and from ω_2 to ω_4 we have the *transition* bands. For the stopbands the specifications indicate that the minimum attenuation in the magnitude response is K_s dB.

To this specification we will apply the frequency transformation:

$$\mathbf{S} = \frac{1}{\mathbf{bw}} \frac{\mathbf{s}^2 + \omega_0^2}{\mathbf{s}} \quad (3.62)$$

When using this transformation, we must have:

$$\omega_1 \omega_2 = \omega_0^2 \quad \text{and} \quad \omega_3 \omega_4 = \omega_0^2 \quad (3.63)$$

Relationships that must be satisfied to use the bandpass transformation

and also:

$$\omega_2 - \omega_1 = \mathbf{bw} \quad (3.64)$$

With imaginary values in Eq. (3.62), the frequency transformation becomes:

$$\Omega = \frac{\omega^2 - \omega_0^2}{\omega(\omega_2 - \omega_1)} \quad (3.65)$$

The bandpass to lowpass prototype frequency transformation

Using this transformation, and Eq. (3.63), we see that:

$$\omega = \omega_2 \text{ corresponds to } \Omega_p = 1 \quad (3.66)$$

Lowpass prototype frequencies for the bandpass case

$$\omega = \omega_4 \text{ corresponds to } \Omega_s = \frac{\omega_4 - \omega_3}{\omega_2 - \omega_1} \quad (3.67)$$

The specifications for the lowpass prototype filter are therefore:

The lowpass prototype's specifications

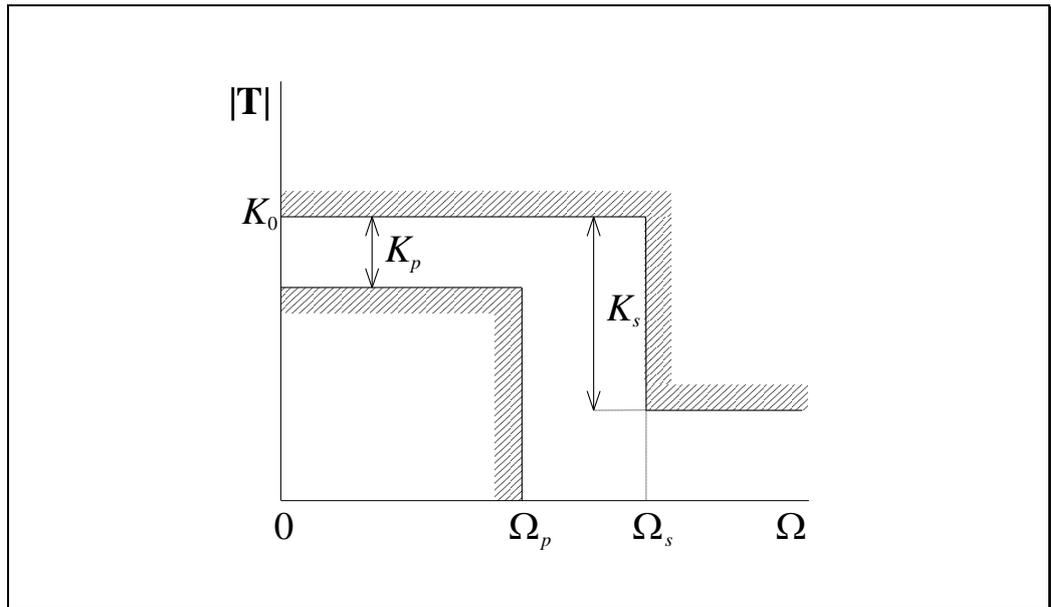


Figure 3.16

This is now the same as for the highpass case, and we use:

M for bandpass filters in terms of lowpass prototype specifications

$$M = \sqrt{\frac{10^{0.1K_s} - 1}{10^{0.1K_p} - 1}} \quad (3.68)$$

We still have for the order:

The order of a bandpass Butterworth filter

$$n = \frac{\ln M}{\ln \Omega_s} \quad (3.69)$$

The half-power frequency for the lowpass prototype that meets the passband specification is given by:

$$\Omega_{0p} = \frac{1}{\left(10^{0.1K_p} - 1\right)^{1/2n}} \quad (3.70)$$

The half-power frequency for the lowpass prototype that meets the stopband specification is given by:

$$\Omega_{0s} = \frac{\Omega_s}{\left(10^{0.1K_s} - 1\right)^{1/2n}} \quad (3.71)$$

and we take the half-power frequency for the lowpass prototype as:

$$\Omega_0 = \sqrt{\Omega_{0p}\Omega_{0s}} \quad (3.72)$$

The half-power frequency of the lowpass prototype Butterworth filter

For the bandpass filter, the half-power frequency of each stage is different. We will investigate this in the next section.

3.10 Bandpass Pole Locations

If the bandpass transformation, Eq. (3.62), is written as:

$$\mathbf{S} = \frac{\omega_0}{\text{bw}} \left(\frac{\mathbf{s}}{\omega_0} + \frac{\omega_0}{\mathbf{s}} \right) = q \left(\frac{\mathbf{s}}{\omega_0} + \frac{\omega_0}{\mathbf{s}} \right) \quad (3.73)$$

then we have defined a new quantity:

$$q = \frac{\omega_0}{\text{bw}} = \frac{\omega_0}{\omega_2 - \omega_1} \quad (3.74) \quad q \text{ defined}$$

The factor q is related to the specifications, and is not the same as the quality factor at resonance, Q_0 , which is related to B .

For the lowpass prototype, the Butterworth polynomial $\mathbf{B}_n(\mathbf{s})$ contains the factor $(\mathbf{s} + \Omega_0)$ for n odd, but otherwise is the product of second-order factors $(\mathbf{s}^2 + (\Omega_0/Q_0)\mathbf{s} + \Omega_0^2)$. We will examine what happens to these factors once the bandpass transformation is applied.

3.10.1 First-Order Factor

The first-order factor in the Butterworth polynomial, if it exists, undergoes the following transformation:

First-order lowpass prototype transfer function factors transform into second-order bandpass factors

$$\begin{aligned}
 \mathbf{S} + \Omega_0 &= q \left(\frac{\mathbf{s}}{\omega_0} + \frac{\omega_0}{\mathbf{s}} \right) + \Omega_0 \\
 &= \frac{\mathbf{s}^2 + (\Omega_0 \omega_0 / q) \mathbf{s} + \omega_0^2}{(\omega_0 / q) \mathbf{s}}
 \end{aligned}
 \tag{3.75}$$

We therefore see that a single real pole transforms into a typical bandpass pole-zero constellation, with two complex conjugate poles and one zero at the origin. Note that the gain at ω_0 for this factor will be $1/\Omega_0$.

By inspection of Eq. (3.75), the radius of the circle on which the poles lie is ω_0 , and their Q_0 is given by:

Q_0 of the resulting bandpass second-order factor

$$Q_0 = \frac{q}{\Omega_0}
 \tag{3.76}$$

This transformation is shown in the figure below:

Pole-zero plot of the transformation of a lowpass first-order factor into a bandpass second-order factor

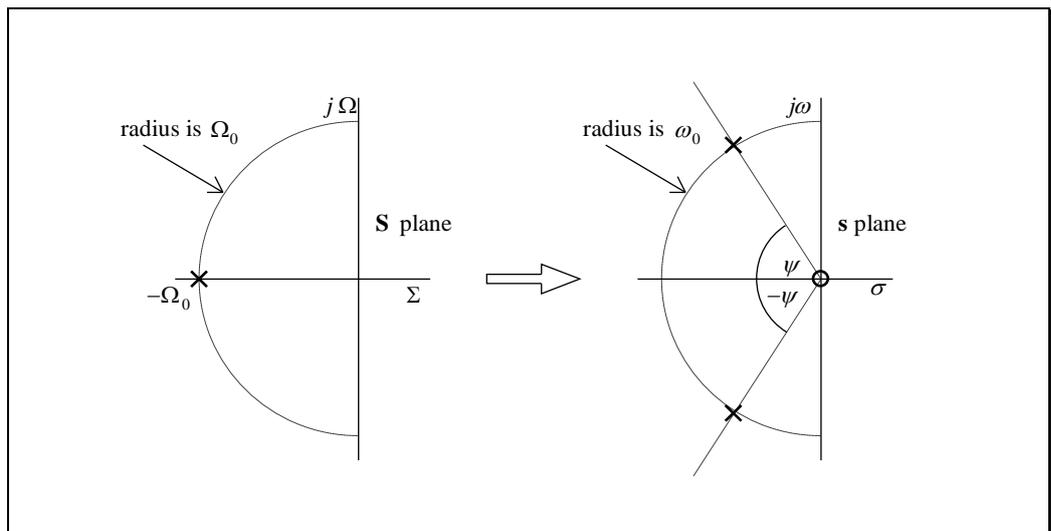


Figure 3.17

3.10.2 Second-Order Factors

A second-order factor in the Butterworth polynomial is:

$$\mathbf{S}^2 + \left(\Omega_0/Q_{0LP}\right)\mathbf{S} + \Omega_0^2 \quad (3.77)$$

where we have used the notation Q_{0LP} to indicate that it is for the lowpass prototype's second-order factor, and not for any eventual bandpass realisation.

Eq. (3.77) is transformed into:

$$q^2 \left(\frac{\mathbf{s}}{\omega_0} + \frac{\omega_0}{\mathbf{s}} \right)^2 + \frac{\Omega_0}{Q_{0LP}} q \left(\frac{\mathbf{s}}{\omega_0} + \frac{\omega_0}{\mathbf{s}} \right) + \Omega_0^2 \quad (3.78)$$

Show that this reduces to the fourth order equation:

$$\frac{\mathbf{s}^4 + \frac{\Omega_0\omega_0}{Q_{0LP}q}\mathbf{s}^3 + \left(2 + \frac{\Omega_0^2}{q^2}\right)\omega_0^2\mathbf{s}^2 + \frac{\Omega_0\omega_0^3}{Q_{0LP}q}\mathbf{s} + \omega_0^4}{(\omega_0/q)^2\mathbf{s}^2} \quad (3.79)$$

Second-order lowpass prototype transfer function factors transform into two second-order bandpass factors

Remembering that this equation is in the denominator of the transfer function, we see that we need to implement a fourth-order bandpass transfer function. To do this we have to cascade two second-order bandpass circuits. To find the ω_0 's and Q_0 's of these cascaded second-order transfer functions, we have to find the pole and zero locations given by Eq. (3.79). We can do this with a computer (using, for example, Matlab) or we can use *Geffe²'s algorithm*.

² P. R. Geffe, "Designer's Guide to Active Bandpass Filters", *EDN*, pp. 46-52, Apr. 5, 1974. The name is pronounced "geff-ee" with the g sounded as in "good".

Geffe showed that the Q_0 's of the two cascaded circuits are the same, but they have different ω_0 's. If we define:

Geffe's algorithm begins here

$$D = \frac{\Omega_0}{Q_{0LP}q} \quad (3.80)$$

$$E = 4 + \frac{\Omega_0^2}{q^2} \quad (3.81)$$

then the Geffe algorithm requires that the following calculations be made:

$$G = \sqrt{E^2 - 4D^2} \quad (3.82)$$

The Q_0 of the two second-order factors

$$Q_0 = \frac{1}{D} \sqrt{\frac{1}{2}(E + G)} \quad (3.83)$$

This is the value of Q_0 for the cascaded circuits. Then we do:

$$K = \frac{DQ_0}{2} \quad (3.84)$$

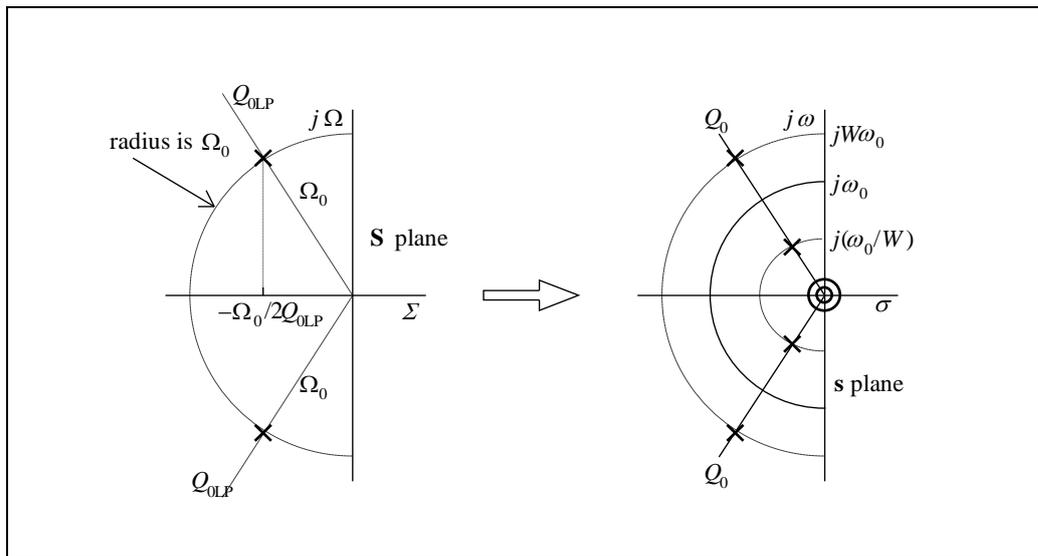
$$W = K + \sqrt{K^2 - 1} \quad (3.85)$$

Finally, we get the two ω_0 's for the cascaded circuits:

The ω_0 's of the two second-order factors

$$\omega_{01} = W\omega_0 \quad \text{and} \quad \omega_{02} = \frac{1}{W}\omega_0 \quad (3.86)$$

The transformation of the lowpass prototype's poles is shown in the figure below:

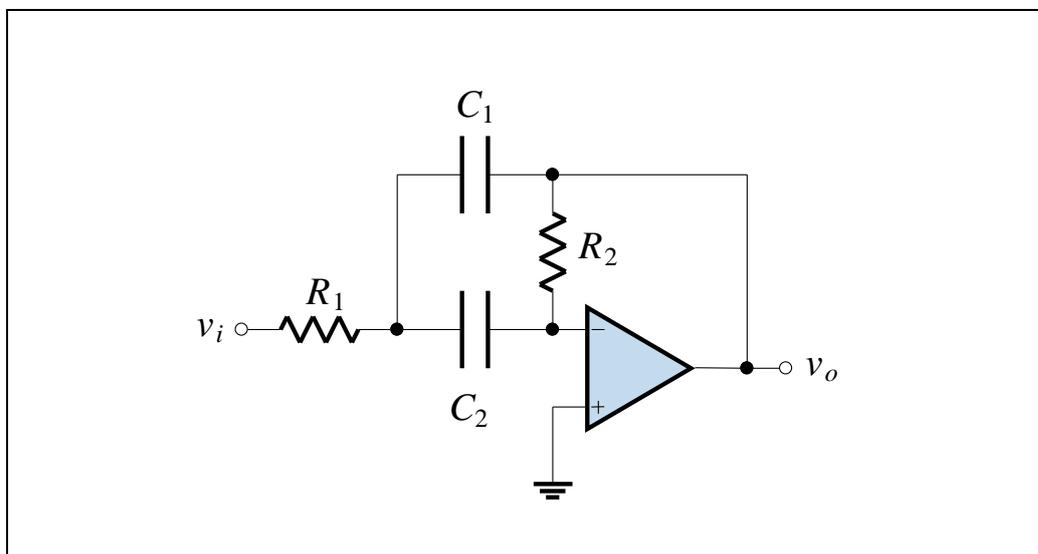


Pole-zero plot of the transformation of a lowpass second-order factor into two bandpass second-order factors

Figure 3.18

3.11 Friend Circuit

A useful bandpass circuit, other than the biquad, is the Friend circuit shown below:



The Friend circuit implements a second-order bandpass transfer function

Figure 3.19

3.34

Show that the transfer function of the Friend circuit is:

$$\frac{V_o}{V_i} = \frac{(-1/R_1 C_1)s}{s^2 + (1/R_2 C_1 + 1/R_2 C_2)s + 1/R_1 C_1 R_2 C_2} \quad (3.87)$$

Comparing this with the standard form for the bandpass response:

$$\frac{V_o}{V_i} = \frac{H(\omega_0/Q_0)s}{s^2 + (\omega_0/Q_0)s + \omega_0^2} \quad (3.88)$$

we have:

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \quad (3.89)$$

$$Q_0 = \frac{1}{C_1 + C_2} \sqrt{\frac{R_2 C_1 C_2}{R_1}} \quad (3.90)$$

$$H = \frac{-R_2}{R_1} \frac{C_2}{C_1 + C_2} \quad (3.91)$$

Design equations for
the Friend circuit

The design equations are considerably simplified if we choose $C_1 = C_2 = C$.

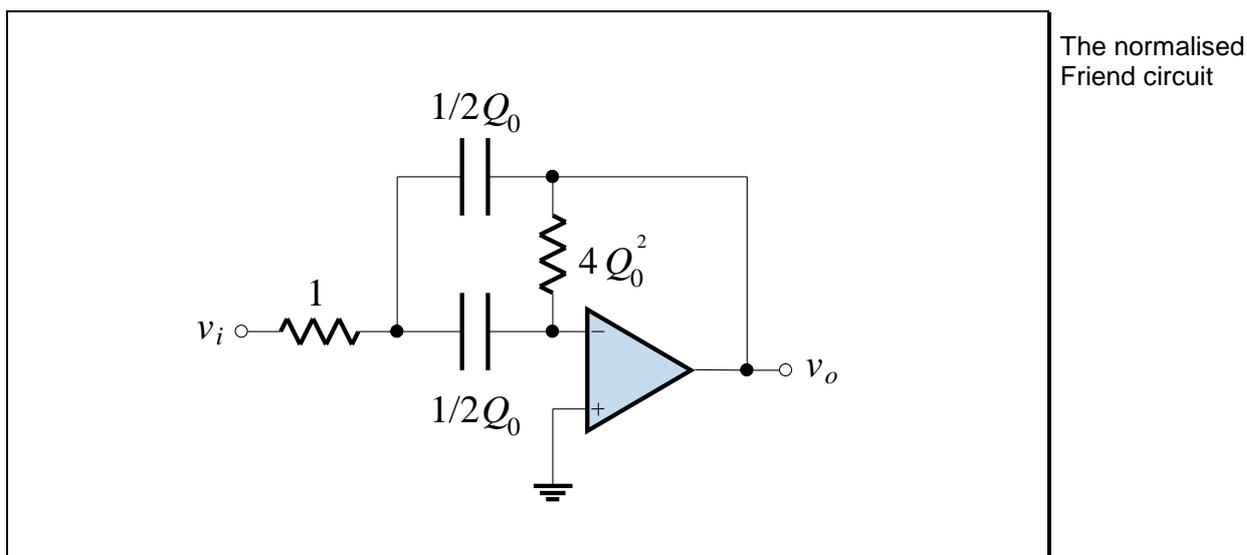
Show that with this choice of capacitor values, the Friend circuit can be orthogonally tuned.

Also, a normalised form of the circuit is obtained by choosing $R_1 = 1$ and $\omega_0 = 1$.

With this choice of values, show that:

$$R_2 = 4Q_0^2 \quad \text{and} \quad C = \frac{1}{2Q_0} \quad (3.92)$$

The normalised Friend circuit is shown below:



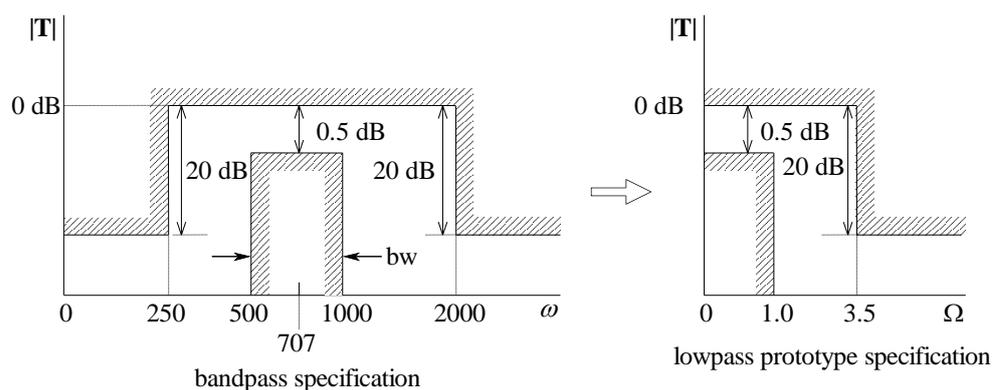
The normalised Friend circuit

Figure 3.20

One disadvantage of this circuit is that it has a mid-frequency gain of $H = -2Q_0^2$. That is, the gain is dependent upon Q_0 and cannot be changed easily.

EXAMPLE 3.2 Bandpass Butterworth Filter Design

The specifications for a bandpass filter are shown below, indicating that at least 20 dB of attenuation is required in the stop band and that there be less than 0.5 dB of deviation from 0 dB in the pass band. Also shown is the transformation to the lowpass prototype specification using Eqs. (3.66)-(3.67):



From Eq. (3.68), we find n as:

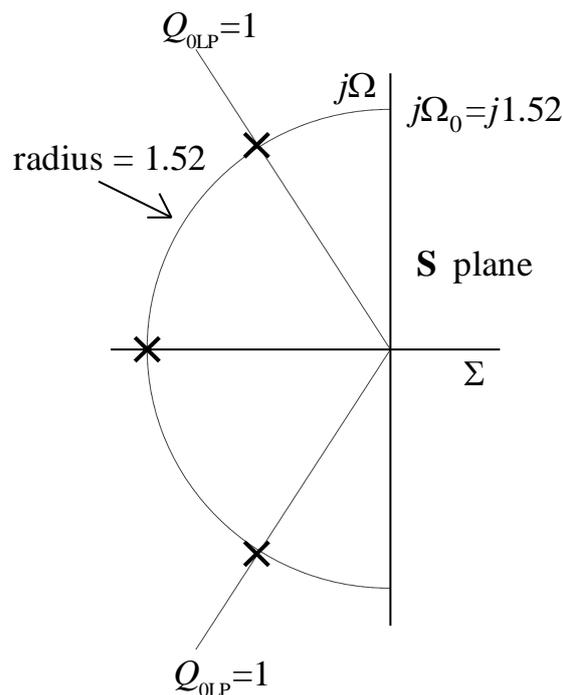
$$n = 2.674, \quad \text{round up to } n = 3$$

3.36

Using Eqs. (3.70)-(3.72), we find the half-power frequency of the lowpass prototype:

$$\Omega_0 = 1.520 \text{ rads}^{-1}$$

Therefore, we know that $n = 3$ is the required order of the Butterworth response, and that the poles of the lowpass prototype lie on a circle of radius $\Omega_0 = 1.52 \text{ rads}^{-1}$. This is shown below:



To these poles we apply the inverse frequency transformation to find the location of the bandpass poles and zeros.

For the real pole, using Eqs. (3.63) and (3.74), we have:

$$q = \omega_0 / \text{bw} = 707 / 500 = 1.414$$

For the lowpass prototype's real pole, the bandpass transformation creates two complex conjugate poles and a zero at the origin. The poles lie on a circle of radius ω_0 with a Q_0 given by Eq. (3.76) of:

$$Q_0 = q / \Omega_0 = 1.414 / 1.52 = 0.9303$$

For the lowpass prototype's conjugate poles, we follow the steps of the Geffe algorithm beginning with Eq. (3.80):

$$D = \frac{1.520}{1 \times 1.414} = 1.075$$

$$E = 4 + \frac{(1.520)^2}{(1.414)^2} = 5.155$$

$$G = \sqrt{(5.155)^2 - 4(1.075)^2} = 4.686$$

$$Q_0 = \frac{1}{1.075} \sqrt{\frac{1}{2}(5.155 + 4.686)} = 2.064$$

$$K = \frac{1.075 \times 2.064}{2} = 1.109$$

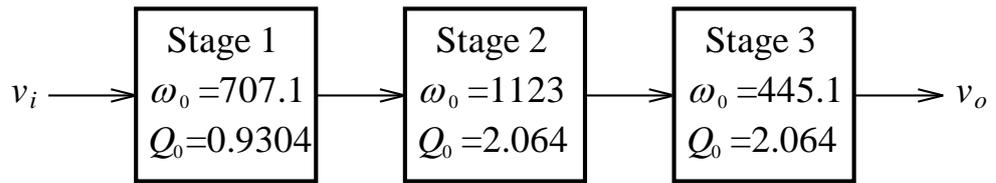
$$W = 1.109 + \sqrt{(1.109)^2 - 1} = 1.589$$

$$\omega_{01} = 1.589 \times 707.1 = 1123 \text{ rads}^{-1}$$

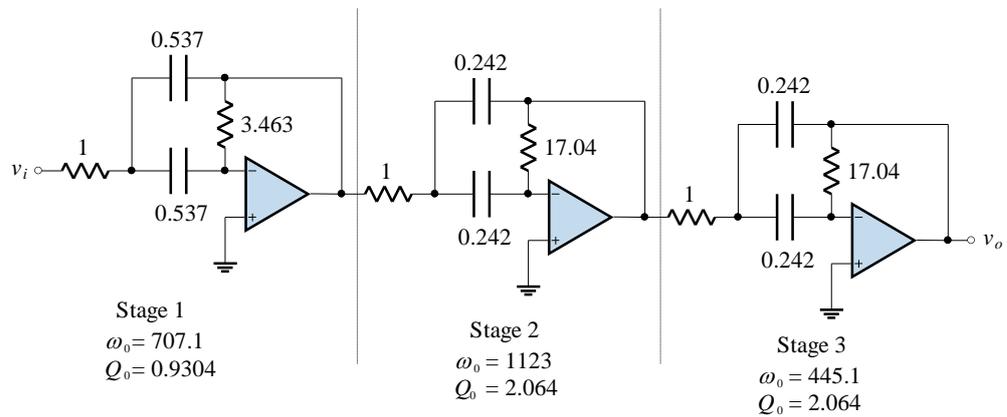
$$\omega_{02} = \frac{707.1}{1.589} = 445.1 \text{ rads}^{-1}$$

3.38

We therefore have the following cascade to achieve the desired response:



The implementation of the block diagram using the Friend circuit is shown below:



Bandpass filters can be stagger tuned

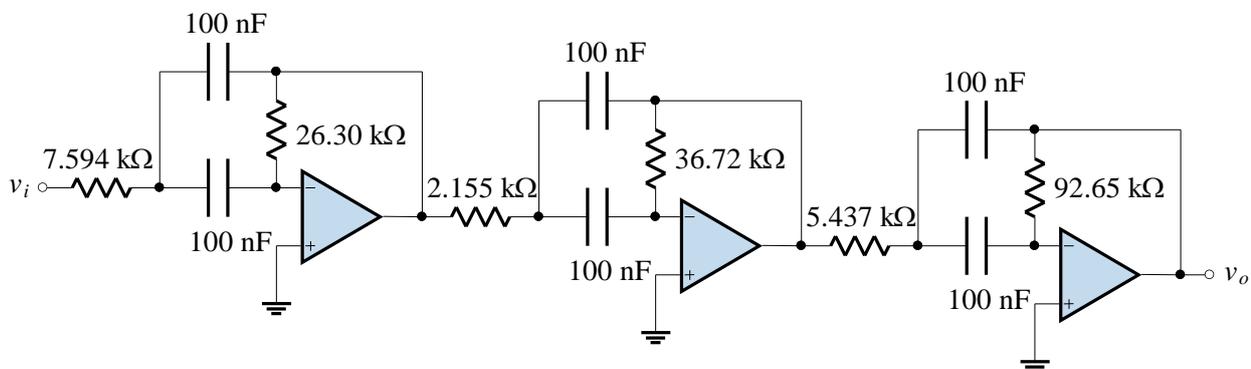
Because each stage is scaled to a different frequency, the combined circuit is said to be *stagger tuned*. Since no coupling exists between each stage, we can also choose a different magnitude scaling factor for each stage.

Suppose we decide that all capacitors are to have the value 100 nF. Calculations for the final element values can be carried out in the form shown in Table 3.1.

Table 3.1 – Design Values for the Bandpass Filter Example

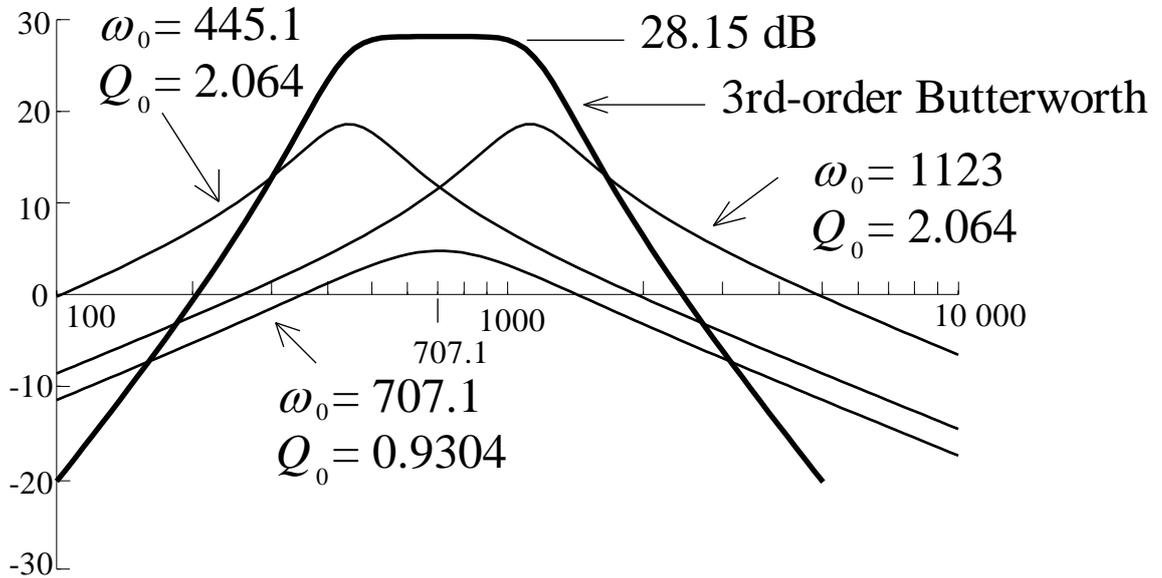
Stage	Element		
	$R_1 = 1$	$R_2 = 4Q_0^2$	$C_1 = C_2 = 1/2Q_0$
1. $Q_0 = 0.9304$ $k_f = 707.1$ $k_m = 7594$	1 Ω 7.594 k Ω	3.463 Ω 26.30 k Ω	0.537 F 100 nF
2. $Q_0 = 2.064$ $k_f = 1123$ $k_m = 2155$	1 Ω 2.155 k Ω	17.04 Ω 36.72 k Ω	0.242 F 100 nF
3. $Q_0 = 2.064$ $k_f = 445.1$ $k_m = 5437$	1 Ω 5.437 k Ω	17.04 Ω 92.65 k Ω	0.242 F 100 nF

The resulting scaled circuit is shown below:



3.40

A plot of the magnitude response for each of the three stages, along with the composite magnitude response found by summing the responses of each of the three stages, is shown below:

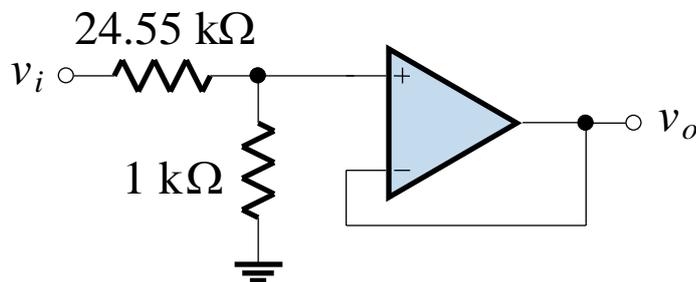


From the figure it is clear that the magnitude function reaches a maximum at the centre frequency $\omega_0 = 707.1$. The overall response at that frequency is:

$$|\mathbf{T}_1(j\omega_0)| |\mathbf{T}_2(j\omega_0)| |\mathbf{T}_3(j\omega_0)| = 1.731 \times (3.842)^2 = 25.55$$

If you use the Friend circuit to implement a bandpass response, you may have to provide attenuation

This value of gain corresponds to 28.15 dB. Since the combined circuit was to have a centre frequency gain of $K_0 = 0$ dB, we have to provide attenuation:



We can place this attenuation anywhere in the circuit. Sometimes, with low values of Q_0 , we have to provide gain.

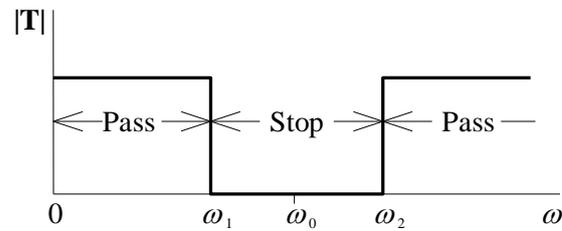
3.12 References

Van Valkenburg, M. E.: *Analog Filter Design*, Holt-Saunders, Tokyo, 1982.

Exercises

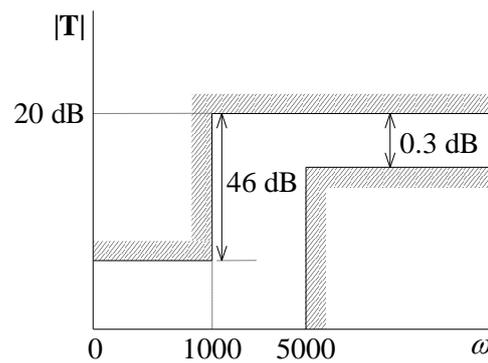
1.

Derive the complex frequency transformation for the *bandstop* response shown below. Express your answer in terms of ω_0 and $\text{bw} = \omega_2 - \omega_1$.



2.

Design a highpass filter with a Butterworth response to meet the specifications given below.

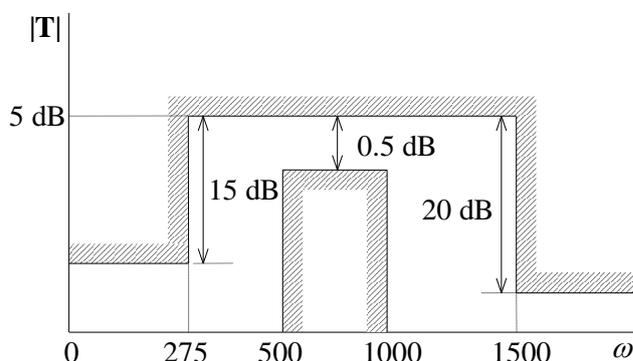


In addition, realise the filter using biquads where possible, making sure to use realistic element values.

3.44

3.

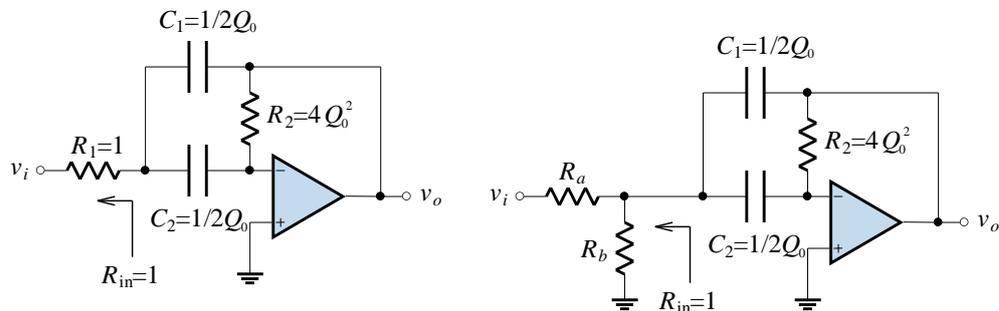
Design a Butterworth bandpass filter with the specifications given below.



Use the Friend circuit for the implementation and magnitude scale so that all capacitors in the filter have the same value.

4.

A bandpass filter using the Friend circuit often requires attenuation. We can provide this attenuation by turning the resistor R_1 into a voltage divider:



If we desire an attenuation of K (less than 1), derive values for R_a and R_b .

5.

Automate the filter design process using Matlab. Verify all your designs so far (problems from Lectures 1, 2 and 3) using a PSpice simulation.

Problems

1.[Power Line Carrier Receiver Filter]

A *power line carrier* system sends a number of sinusoids over the power lines for communication purposes. At the receiver the signal is to be filtered so that low frequencies (50 Hz and harmonics) and high frequencies (caused by motor drives) are attenuated.

The band of frequencies to be passed is from 3 kHz to 5 kHz. The receiver uses a *non-coherent* detection scheme, which means the phase response of the filter is not important. In the passband, the filter output magnitude should have less than 1% error compared to the input magnitude, and should be maximally flat. The attenuation at 50 Hz should be as large as possible.

Due to existing hardware restrictions, there are only *two* op-amps available for the implementation.

- (i) Using 50 Hz as the edge of the low frequency stopband, draw the bandpass filter specification. Give values for *all* frequencies and gains/attenuations that are used in the design.
- (ii) Draw a pole-zero plot for the lowpass prototype filter. Label the pole locations in terms of Ω_0 and Q_0 .
- (iii) Draw a block diagram of a cascade implementation of the bandpass filter. For each block, give the second-order bandpass specifications in terms of ω_0 and Q_0 .
- (iv) Using Friend circuits, draw the realisation of the filter. Use only 10 nF capacitors, and label *all* the elements in the circuit.
- (v) Plot the individual second-order responses of the Friend circuits, as well as the overall filter response, on the same gain (dB) versus frequency (Hz, log scale) graph.

4 Passive Components

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4.1 Introduction

A *resistor* is not a pure resistance of exact value. A real resistor has a series inductance and a parallel stray capacitance. These affect the behaviour of the resistor, particularly at high frequencies.

A resistor is not a pure resistance

A *capacitor* is not simply a capacitance, possibly with some losses. Both capacitance value and losses depend on frequency and temperature, sometimes quite significantly. A real capacitor has a series inductance, a series resistance and a parallel resistance. At higher frequencies capacitors behave as complex resonant systems. Above its series resonant frequency, a capacitor behaves like an inductance.

A capacitor is not a pure capacitance

The characteristics of capacitors are very different to one another since they depend on the dielectric and technology used to make them. It is extremely important to understand these material / technology specific characteristics, in order to choose the proper capacitor for a given application.

For the passive components we will:

- define the essential characteristics used to describe their performance;
- present the main types of devices, differentiated by the materials and technologies used in manufacturing them, as these determine the characteristics;
- compare the characteristics and recommend applications for different types of components.

4.2 Resistor Characteristics

Resistors are the most commonly used passive electronic component. The characteristics and the features to consider when choosing a resistor are: resistance value, tolerance on value, stability, temperature coefficient, voltage coefficient, humidity effects, power dissipation, frequency effects, noise, reliability, size and packaging, availability and cost. For insight into these factors, the materials and construction of the various resistor types must be considered.

4.2.1 Tolerance on value

Service variability of a resistor value is an *overall, long-term value tolerance* composed of factors such as purchase tolerance, ageing, stress and short-term excursions due to the local environment (temperature, humidity, etc). A resistor of, say, 10% *rated tolerance* can be selected to be within $\pm 1\%$ of the required value when installed in a circuit, but such a resistor might well drift within (and often even outside) the rated $\pm 10\%$ during its service. Hence, if either a *very accurate* or *stable* resistance value is required in a circuit, choose the right tolerance resistor when purchasing it, i.e. with a tolerance tighter than the required end-of-life tolerance.

Component tolerances are tolerances at time of purchase

4.2.2 Preferred Values and the Decade Progression

Fundamental standardization practices require the selection of *preferred* values within the ranges available. Standard values may at first sight seem to be strangely numbered. There is, however, a beautiful logic behind them, dictated by the tolerance ranges available.

The *decade progression* of preferred values is based on preferred numbers generated by a geometric progression, repeated in succeeding decades. In 1963, the International Electrotechnical Commission (IEC) standardized the preferred number series for resistors and capacitors (standard IEC 60063). It is based on the fact that we can linearly space values along a logarithmic scale so a percentage change of a value results in a linear change on the logarithmic scale.

Component values have been standardized by the IEC

Recall that for a geometric progression, each individual term is given by:

$$a_n = ar^{n-1} \quad (4.1)$$

where a_n is the n -th term, a is the scale factor, and r is the common ratio. If r is chosen to be the k -th root of 10, and the scale factor is set to 100, then:

$$a_n = 100 \left(\sqrt[k]{10} \right)^{n-1} \quad (4.2)$$

Thus, the selection of k determines how many values of the geometric progression there are in one decade. For example, if 6 values per decade are desired, $k = 6$ and the common ratio is $\sqrt[6]{10} \approx 1.468$. The six rounded-off values become 100, 150, 220, 330, 470, 680.

Component values are spaced equidistantly on a logarithmic scale

To figure out what tolerance is allowable on resistors with these values, we can let the tolerance be $\pm \delta$. Then if we require successive values of resistors to increase with *almost certainty*, then:

$$\begin{aligned} a_n (1 + \delta) &= a_{n+1} (1 - \delta) \\ \frac{a_{n+1}}{a_n} &= \frac{1 + \delta}{1 - \delta} \\ \sqrt[k]{10} &= \frac{1 + \delta}{1 - \delta} \\ \delta &= \frac{\sqrt[k]{10} - 1}{\sqrt[k]{10} + 1} \end{aligned} \quad (4.3)$$

4.6

For $k = 6$, we get $\delta = 18.95\%$. Thus, if we set the tolerance of resistors with 6 values per decade to $\delta = 20\%$, we get values that almost certainly increase, with minimal overlap:

$100 \pm 20\% = 80 \text{ to } 120$	$330 \pm 20\% = 264 \text{ to } 396$
$150 \pm 20\% = 120 \text{ to } 180$	$470 \pm 20\% = 376 \text{ to } 564$
$220 \pm 20\% = 176 \text{ to } 264$	$680 \pm 20\% = 544 \text{ to } 816$

4.2.3 The 'E' Series Values

The 'E' series values explained

The IEC set the number of values for resistors (and capacitors) per decade based on their tolerance. These tolerances are 0.5%, 1%, 2%, 5%, 10%, 20% and 40% and are respectively known as the E192, E96, E48, E24, E12, E6 and E3 series, the number indicating the quantity of values per decade in that series. For example, if resistors have a tolerance of 5%, a series of 24 values can be assigned to a single decade multiple (e.g. 100 to 999) knowing that the possible extreme values of each resistor overlap the extreme values of adjacent resistors in the same series.

Any of the numbers in a series can be applied to any decade multiple set. Thus, for instance, multiplying 220 by each decade multiple (0.1, 1, 10 100, 1000 etc.) produces values of 22, 220, 2 200, 22 000, 220 000 etc.

The 'E' series of preferred resistor and capacitor values according to IEC 60063 are reproduced in Table 4.1.

0.5% E192	1% E96	2% E48													
100	100	100	169	169	169	287	287	287	487	487	487	825	825	825	
101			172			291			493			835			
102	102		174	174		294	294		499	499		845	845		
104			176			298			505			856			
105	105	105	178	178	178	301	301	301	511	511	511	866	866	866	
106			180			305			517			876			
107	107		182	182		309	309		523	523		887	887		
109			184			312			530			898			
110	110	110	187	187	187	316	316	316	536	536	536	909	909	909	
111			189			320			542			920			
113	113		191	191		324	324		549	549		931	931		
114			196			328			556			942			
115	115	115	196	196	196	332	332	332	562	562	562	953	953	953	
117			198			336			569			965			
118	118		200	200		340	340		576	576		976	976		
120			203			344			583			988			
121	121	121	205	205	205	348	348	348	590	590	590				
123			208			352			597			5%	10%	20%	40%
124	124		210	210		357	357		604	604		E24	E12	E6	E3
126			213			361			612						
127	127	127	215	215	215	365	365	365	619	619	619	100	100	100	100
129			218			370			626			110			
130	130		221	221		374	374		634	634		120	120		
132			223			379			642			130			
133	133	133	226	226	226	383	383	383	649	649	649	150	150	150	
135			229			388			657			160			
137	137		232	232		392	392		665	665		180	180		
138			234			397			673			200			
140	140	140	237	237	237	402	402	402	681	681	681	220	220	220	220
142			240			407			690			240			
143	143		243	243		412	412		698	698		270	270		
145			246			417			706			300			
147	147	147	249	249	249	422	422	422	715	715	715	330	330	330	
149			252			427			723			360			
150	150		255	255		432	432		732	732		390	390		
152			259			437			741			430			
154	154	154	261	261	261	442	442	442	750	750	750	470	470	470	470
156			264			448			759			510			
158	158		267	267		453	453		768	768		560	560		
160			271			459			777			620			
162	162	162	274	274	274	464	464	464	787	787	787	680	680	680	
164			277			470			796			750			
165	165		280	280		475	475		806	806		820	820		
167			284			481			816			910			

Table 4.1 - IEC standard 'E' series of values in a decade

4.2.4 Marking Codes

The IEC also defines how manufacturers should mark the values of resistors and capacitors in the standard called IEC 60062. The colours used on fixed leaded resistors are shown below:

IEC labelling for leaded resistors

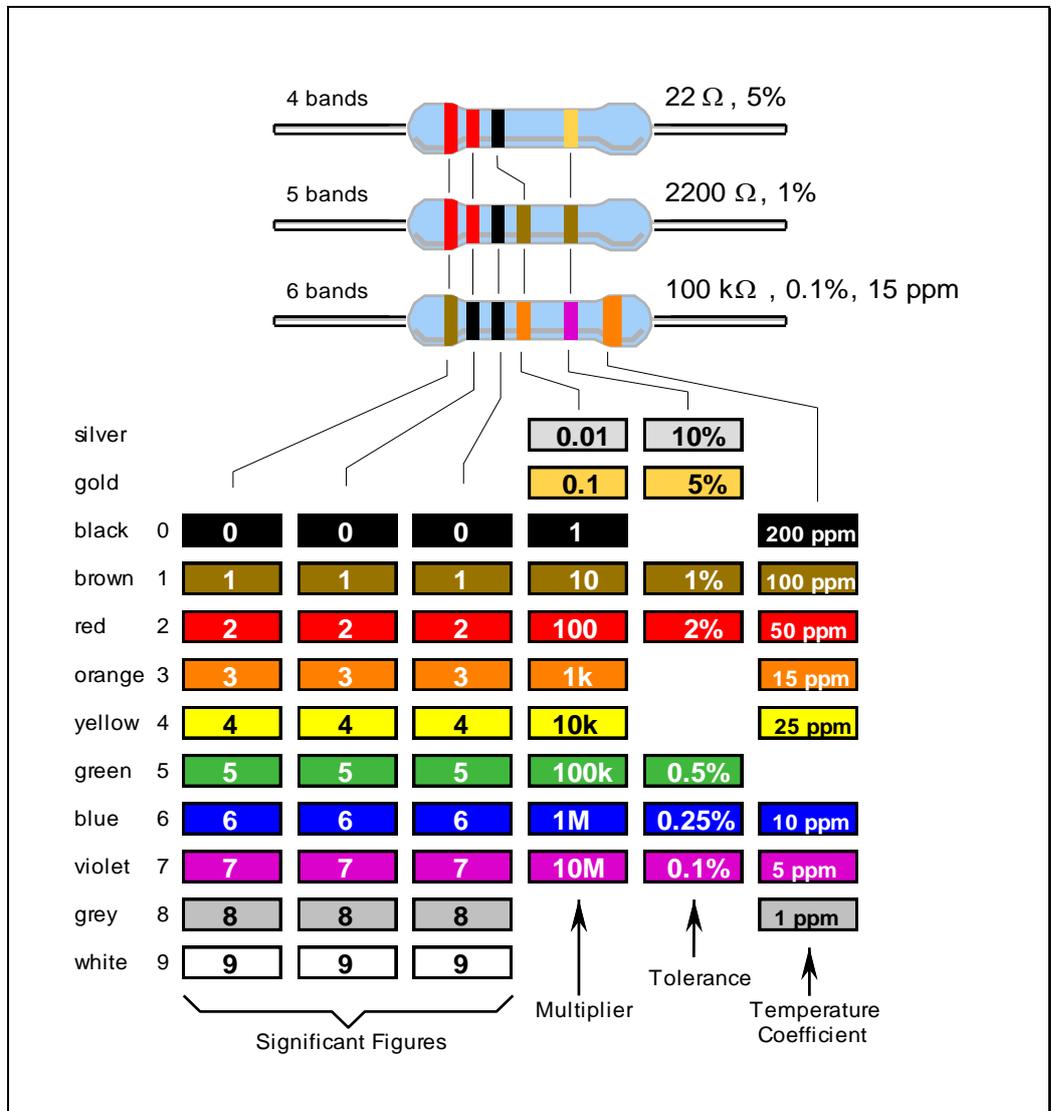


Figure 4.1 – Colour code marking of leaded resistors

The resistance *colour code* consists of three or four colour bands and is followed by a band representing the tolerance. The temperature coefficient band, if provided, is to the right of the tolerance band and is usually a wide band positioned on the end cap.

The resistance colour code includes the first two or three significant figures of the resistance value (in ohms), followed by a multiplier. This is a factor by which the significant-figure value must be multiplied to find the actual resistance value. (i.e. the number of zeros to be added after the significant figures).

Whether two or three significant figures are represented depends on the tolerance: $\pm 5\%$ and wider require two band; $\pm 2\%$ and tighter requires three bands. The significant figures refer to the first two or three digits of the resistance value of the standard series of values in a decade, in accordance with IEC 60063 as indicated in the relevant data sheets and shown in Table 4.1.

The colours used and their basic numerical meanings are recognized internationally for any colour coding used in electronics, not just resistors, but some capacitors, diodes, cabling and other items.

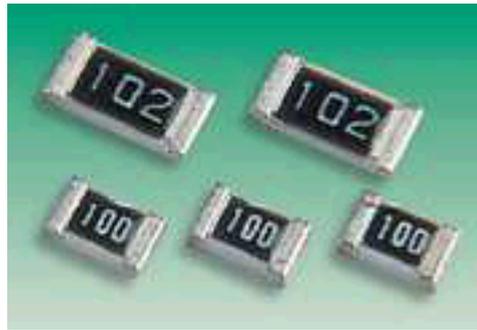
The colours are easy to remember: Black is the absence of any colour, and therefore represents the absence of any quantity, 0. White (light) is made up of all colours, and so represents the largest number, 9. In between, we have the colours of the rainbow: red, orange, yellow, green, blue and violet. These take up the numbers from 2 to 7. A colour in between black and red would be brown, which has the number 1. A colour intermediate to violet and white is grey, which represents the number 8.

The resistor colour code explained

4.10

Surface mount technology (SMT) chip resistors are frequently marked with a three-digit number, and some typical values are shown in the table below. The first two numbers are the significant digits of the value, and the last digit is the multiplier (the number of zeros to add to the first two digits). For example, a chip resistor labelled 102 has a value of 1 k Ω .

Resistor marking for chip resistors



Resistor Value	Resistor Marking
0.1 Ω	0R1
1 Ω	1R0
22 Ω	220
3.3 k Ω	332
100 k Ω	104
4.5 M Ω	155

When resistors are labelled with numbers, an additional letter is appended to indicate the tolerance. The table below lists the letters used for the tolerance.

We use a letter to indicate tolerance

Resistor Tolerance	IEC Labelling
1 %	F
2 %	G
5 %	J
10 %	K
20 %	M

The figure below shows the markings on a single in-line package (SIP) of networked resistors (several resistors are in the one package) and the markings on a chip resistor (with just a three-digit number for space reasons):

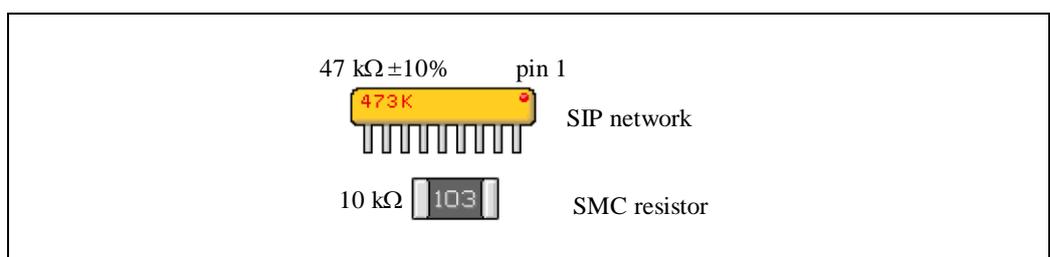


Figure 4.2 – Resistor markings

When resistors are labelled in diagrams, such as schematics, IEC 60062 calls for the significant figures to be printed as such, but the decimal point is replaced with the SI prefix of the multiplier. Examples of such labelling are shown below:

Resistor Value	IEC Labelling
0.1 Ω	0R1
1 Ω	1R0
22 Ω	22R
3.3 k Ω	3K3
100 k Ω	100K
4.5 M Ω	4M5

IEC labelling for diagrams

Note how the decimal point is expressed, that the ohm symbol is shown as an R, and that 1000 is shown as a capital K. The use of a letter instead of a decimal point solves a printing problem – the decimal point in a number may not always be printed clearly, and the alternative display method is intended to help misinterpretation of component values in circuit diagrams and parts lists.

We use a letter in place of a decimal point for labelling component values

In circuit diagrams and constructional charts, a resistor's numerical identity, or *designator*, is usually prefixed by 'R'. For example, R15 simply means resistor number 15.

A portion of a schematic diagram showing designators and IEC labelling is shown below:

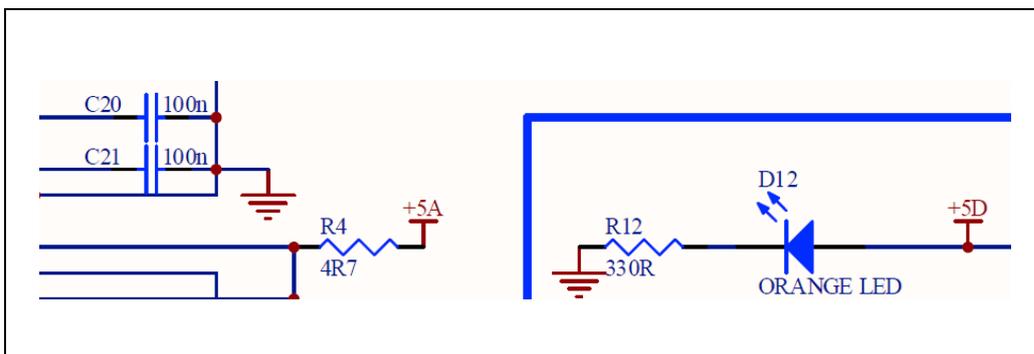


Figure 4.3 – Schematic portion showing IEC labelling

Note that resistor R4 has the value 4.7 Ω and resistor R12 has the value 330 Ω .

4.12

4.2.5 Temperature Coefficient (Tempco)

Temperature coefficient defined

The *tempco* is defined as $\Delta R / (R \cdot \Delta T)$, and is expressed in % / °C or ppm / °C. It is poorest for carbon composition resistors; for carbon film resistors it is typically negative, -200 to -500 ppm / °C. It is best for chip and metal film resistors (within ± 20 to 100 ppm / °C).

Wire wound resistors can have nearly zero ‘tempco’. They are wound from special alloys such as Constantan (55% copper, 45% nickel) which has a remarkably low tempco of 1-5 ppm / °C.

EXAMPLE 4.1 Effect of Temperature Changes on a Resistor

If the temperature coefficient of a resistor with $R_{\text{rated}} = 1 \text{ k}\Omega$ between -55 °C and +155 °C is $\pm 200 \times 10^{-6} / ^\circ\text{C}$, its resistance value will be:

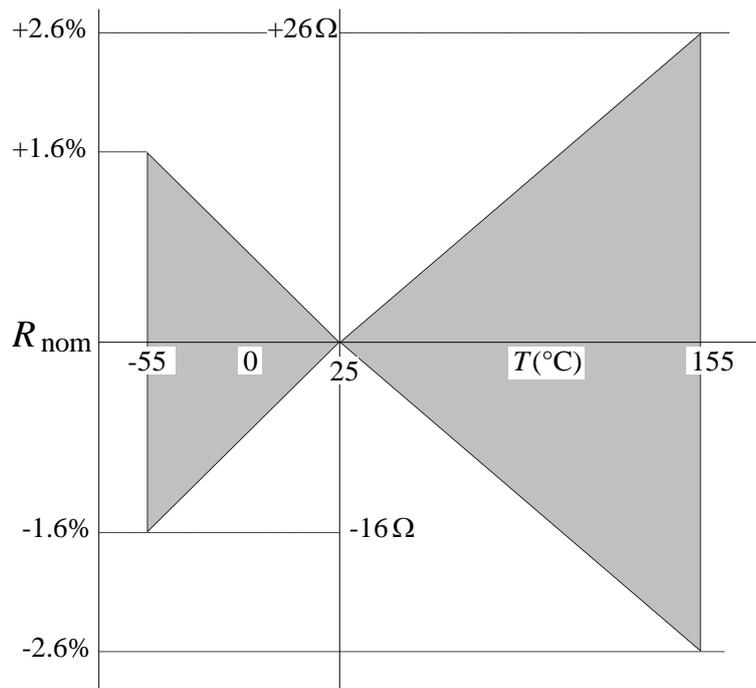
at +25 °C: 1000Ω (nominal = rated value)

at +155 °C: $1000 \Omega \pm (130 \times 200 \times 10^{-6}) \times 1000 \Omega = 1026 \Omega$ or 974Ω

at -55 °C: $1000 \Omega \pm (80 \times 200 \times 10^{-6}) \times 1000 \Omega = 1016 \Omega$ or 984Ω

If the temperature coefficient is specified as within $\pm 200 \times 10^{-6} / ^\circ\text{C}$, depending on temperature, the resistance will be within the shaded area shown below:

Tempco of resistors: effect of temperature changes



4.2.6 Voltage Coefficient (Vltco)

When a voltage is applied to a resistor there might be a slight decrease in resistance value (apart from temperature induced changes). The voltage coefficient is expressed in % / V or ppm / V. It can be large (-500 ppm / V) for carbon composition. The 'vltco' is about 5 to 30 ppm / V for carbon film, and 0.05 to 10 ppm / V for metal film or chip resistors.

Voltage coefficient defined

4.2.7 Stability

Stability refers to the change in resistance value, expressed in % or parts per million (ppm, i.e. 10^{-6}), following exposure to a specified environmental condition or stress: high or low temperature, time (ageing), full rated power, moisture, soldering heat, short-time overload, radiation. Wire-wound resistors are the most stable ones, followed, in order, by metal film, chip and carbon film.

Stability defined

4.2.8 Humidity Effects

On the surface of a high resistance value resistor, moisture can provide a low resistance leakage path. If moisture penetrates the coating, it can react chemically with the resistive film and alter it. The danger is mainly when the resistors are stored or the equipment is not used, because then the resistors are cold. The effect of prolonged humidity is also expressed in % or ppm change.

Humidity effects

4.2.9 Power Dissipation

Resistors must be operated within specified temperature limits to avoid permanent damage to the materials. The temperature limit is defined in terms of the maximum power, called the power rating, and a derating curve. The power rating of a resistor is the maximum power in watts which the resistor can dissipate. The maximum power rating is a function of resistor material, maximum voltage rating, resistor dimensions, and maximum allowable hot-spot temperature. The maximum hot-spot temperature is the temperature of the hottest part on the resistor when dissipating full-rated power at rated ambient temperature.

Power dissipation depends on the resistor construction, and depends on the ambient temperature

The maximum allowable power rating as a function of the ambient temperature is given by the derating curve. The figure below shows a typical power derating curve for a resistor.

Power derating curve for a resistor

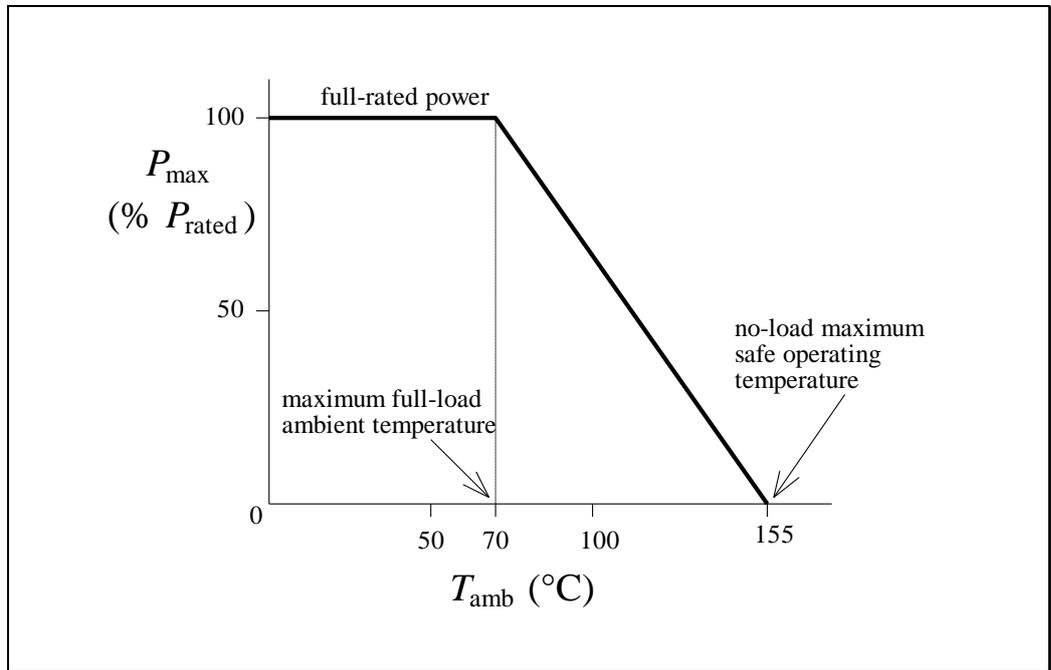


Figure 4.4 – Resistor derating curve

The derating curve is usually linearly drawn from the full-rated load temperature to the maximum allowable no-load temperature. A resistor may be operated at ambient temperatures above the maximum full-load ambient temperature if operating at lower than full-rated power capacity. The maximum allowable no-load temperature is also the maximum storage temperature for the resistor.

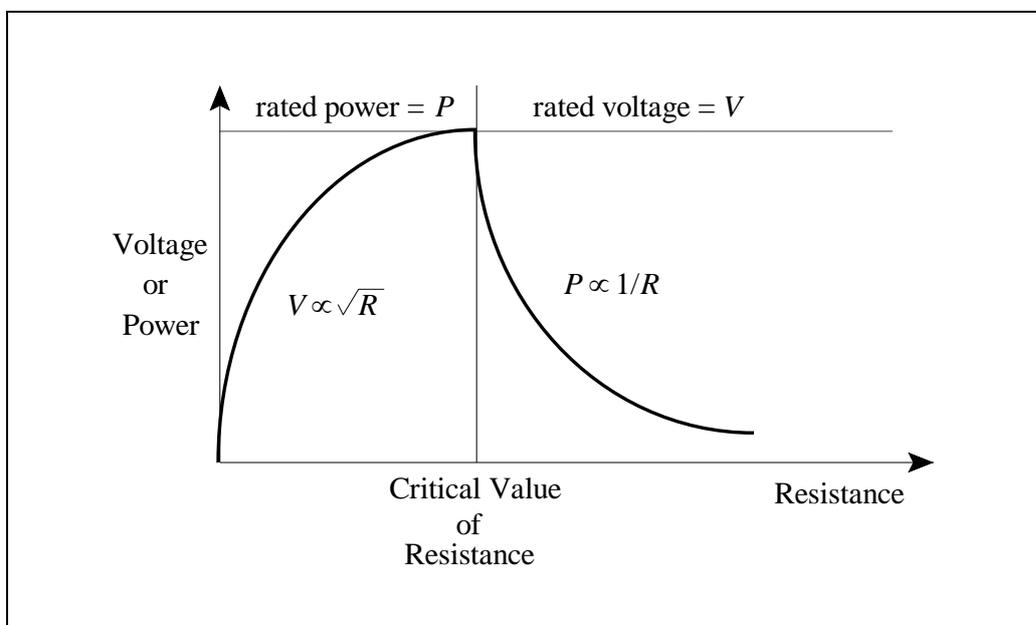
EXAMPLE 4.2 Derating a Resistor

According to Figure 4.4, a resistor rated 1 W can dissipate 1 W only if it is mounted so that air can circulate freely around it, and if $T_{\text{amb}} \leq 70^{\circ}\text{C}$. If the ambient temperature (i.e. *the temperature inside the equipment's enclosure*) is above 70°C , the power dissipation must be reduced to avoid the resistor temperature reaching a maximum temperature of 155°C . The thermal resistance can also be calculated from the derating curve as $\theta_R = (155^{\circ}\text{C} - 70^{\circ}\text{C})/P_{\text{rated}}$.

4.2.10 Voltage Rating

Discrete resistors are available in power dissipation ratings from 1/20 W to 2 W or more. However, for high values of resistance, *voltage*, rather than *power* is the limiting factor, ranging from 150 V for smaller packages to about 750 V for 2 W resistors. The critical resistance value, which is the boundary between the two limitations is $R = V^2/P$, where V and P are the rated voltage and power dissipation respectively.

For a given value of voltage and power rating, a critical value of resistance can be calculated. For values of resistance below the critical value, the maximum voltage is never reached; for values of resistance above the critical value, the power dissipated is lower than the rated power.



Sometimes voltage rating, rather than power rating, is a limiting factor of a resistor's operation

Figure 4.5 – Relationship of applied voltage and power above and below the critical value of resistance

4.2.11 Frequency Effects

The resistance of a resistor remains constant only at low frequencies. Frequency response is affected by distributed capacitance and inductance in the resistive path, lead inductance, capacitance from case to ground, skin effect, dielectric losses. The simplest approximate *equivalent circuit* of a resistor at high frequencies is a parallel resonant circuit, as shown in Figure 4.6, with capacitance and inductance figures shown for chip resistors of standard sizes.

Typical values of inductance and capacitance...

QUANTITY	CHIP PROPERTIES			
	THIN FILM	THICK FILM		
	1206 $R < 1\text{ k}\Omega$	1206	0805	0603
Capacitance	0.05 pF	0.05 pF	0.09 pF	0.05 pF
Inductance	2 nH	2 nH	1 nH	0.4 nH

... for a resistor's equivalent circuit

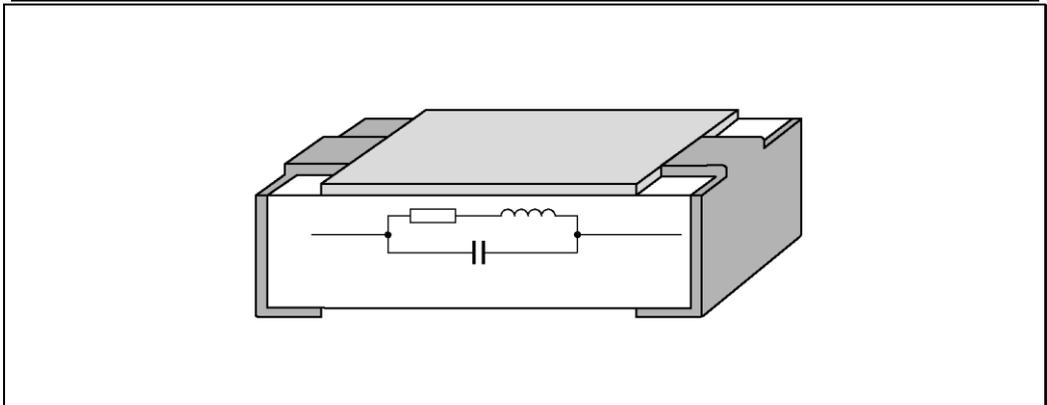


Figure 4.6 – Resistor equivalent circuit

The equivalent circuit parameters are not always supplied by the manufacturers. Instead, impedance versus frequency behaviour is given in plots of $|Z|/R$ versus frequency, and phase shift versus frequency as in the figure below:

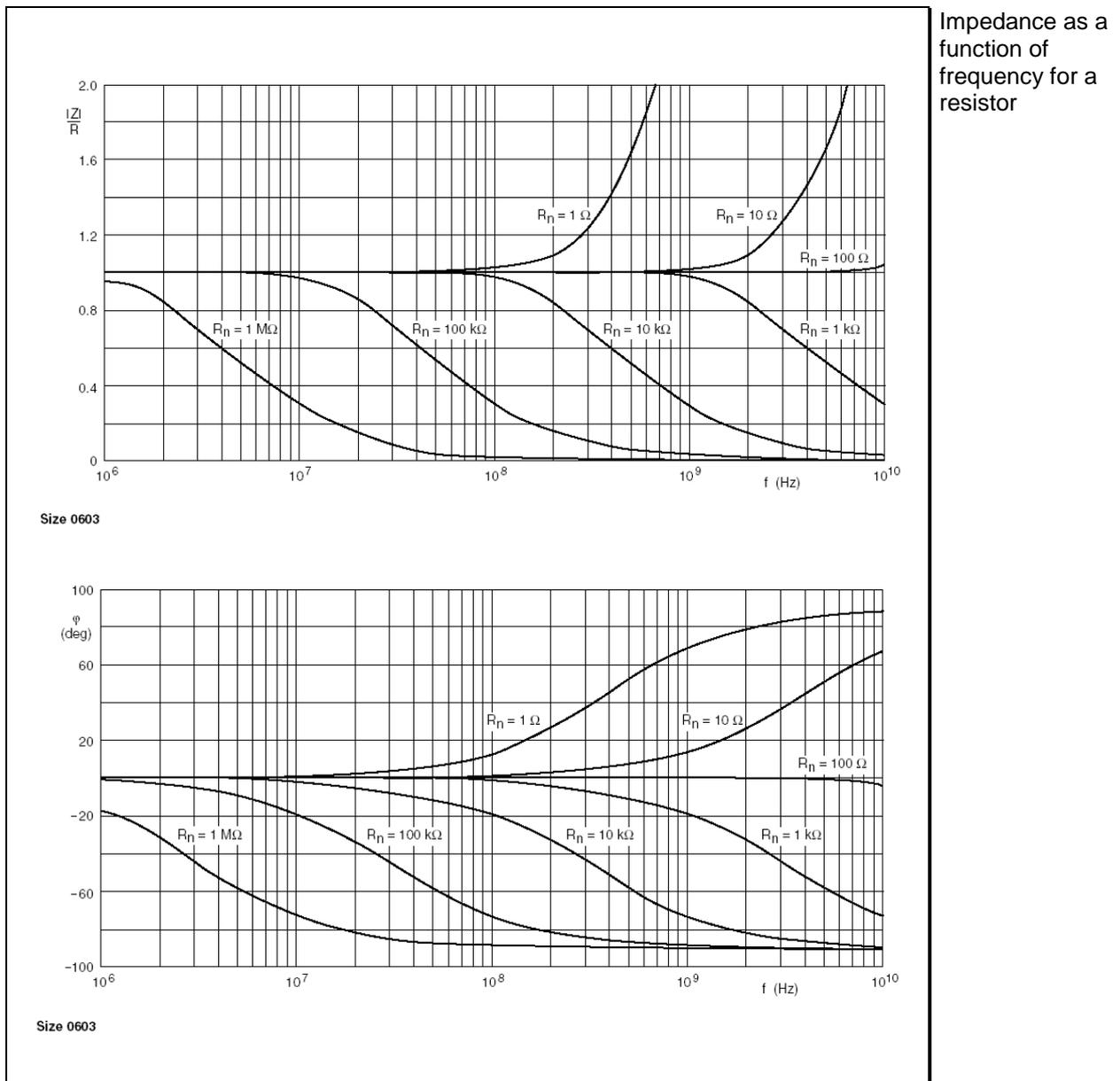


Figure 4.7 – Impedance as a function of frequency for a resistor

Generally the frequency response is best for low value, small size, not spiralled resistors (e.g. chip resistors). In Figure 4.7 one can see that a 10 kΩ chip resistor of size 0603 maintains its R value to about 70 MHz (a very high frequency). The response is poorest for high value, spiralled resistors. Wire-wound resistors are inductive even at audio frequencies.

4.2.12 Noise

All resistors generate a *thermal (Johnson) noise* voltage, caused by the random motion of the electrons. Since this source of noise is caused by the random motion of electrons, we expect that it is related to absolute temperature T . In fact thermal noise is *directly proportional* to T and, as T approaches zero, thermal noise also approaches zero. Since the noise is essentially random and follows a Gaussian distribution about 0 V, we are interested in the *RMS noise voltage*:

RMS thermal noise
voltage

$$V_n = \sqrt{4kTRB} \quad (4.4)$$

where:

V_n = RMS thermal noise voltage

k = Boltzmann's constant (1.38×10^{-23} J/K)

T = absolute temperature, in Kelvin (e.g. $298\text{K} = 25^\circ\text{C}$)

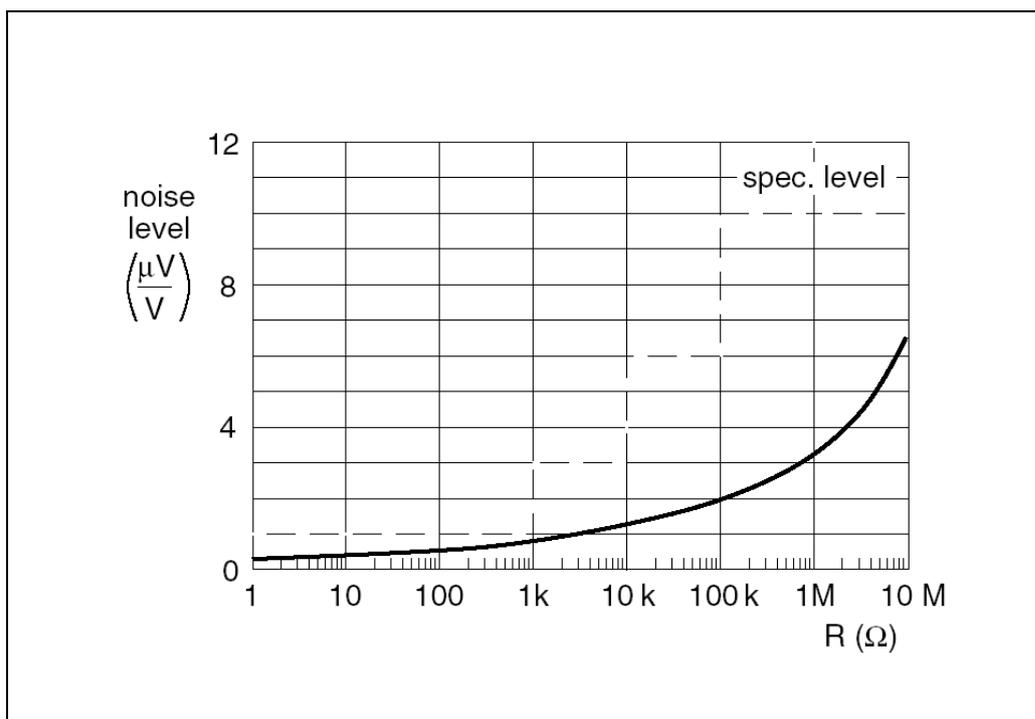
R = resistance value, Ω

B = noise bandwidth, Hz

It is useful to remember for thermal noise that a $1\text{k}\Omega$ resistor has a thermal noise power spectral density of $V_n^2/B = 16 \times 10^{-18} \text{ V}^2/\text{Hz}$. This can be written in RMS form as $V = 4 \text{ nV}/\sqrt{\text{Hz}}$ where the form $\text{nV}/\sqrt{\text{Hz}}$ is used to emphasize that the *RMS noise voltage* varies as the *square root* of the bandwidth. This noise is bothersome when the circuit handles low-level signals, e.g. at the input of a high-gain amplifier.

In addition to thermal noise, most resistors generate noise *due to the passage of current* through the resistor. This additional noise is dependent on the amount of current, the resistive material and the physical construction of the resistor. Laser trimming, which cuts a groove in the resistive material, affects the additional noise.

Typical noise levels due to current are shown below:



Typical noise levels in a resistor due to current

Figure 4.8 – Typical noise levels as a function of rated resistance

4.2.13 Reliability

Reliability refers to the probability that a resistor will still be within specifications after a given time under certain specified conditions. Reliability is expressed in terms of failure rate in percent per thousand hours of operation, or in units of Failure-In-Time (FIT) – a FIT being 1 failure in 1 billion device-hours of operation. A typical value, for good quality film resistors, operating at an ambient temperature of 25 °C, is 2 FITs. Resistors may fail catastrophically (open or short circuit) or may drift out of specification.

Reliability of resistors is excellent, and is mostly affected by the environment

Open circuits can be caused by cracked substrates, open welds, broken resistive elements and chemical corrosion. Make sure to *never stress* a resistor *mechanically*.

Short circuits may be caused by electrical breakdown (over-voltage), foreign objects and silver migration. Excessive drift is due to moisture penetration, corrosion, electrostatic discharge and transient overloading.

4.2.14 Derating

A policy of derating components leads to increased reliability and stability

Derating is a policy of *deliberately under-stressing components* so as to provide increased reliability and stability, e.g. using a 1/2 W resistor in circuit conditions demanding a 1/4 W dissipation is an effective and well established method of achieving a very reliable design. In most cases a component's reliability and stability improve when operating stresses like temperature, voltage and power dissipation are reduced. Unfortunately, this will usually require a larger size component.

Designers of military, medical and high-reliability equipment always derate component specifications. They account for extremes of power-supply fluctuation, ambient temperature, loss of active cooling systems, material variation, memory errors (in a digital system) due to high-energy particle radiation and the expected degree of "specsmanship" on the part of their component suppliers. Then, if they really want the product to work, designers test all components before putting them into the system.

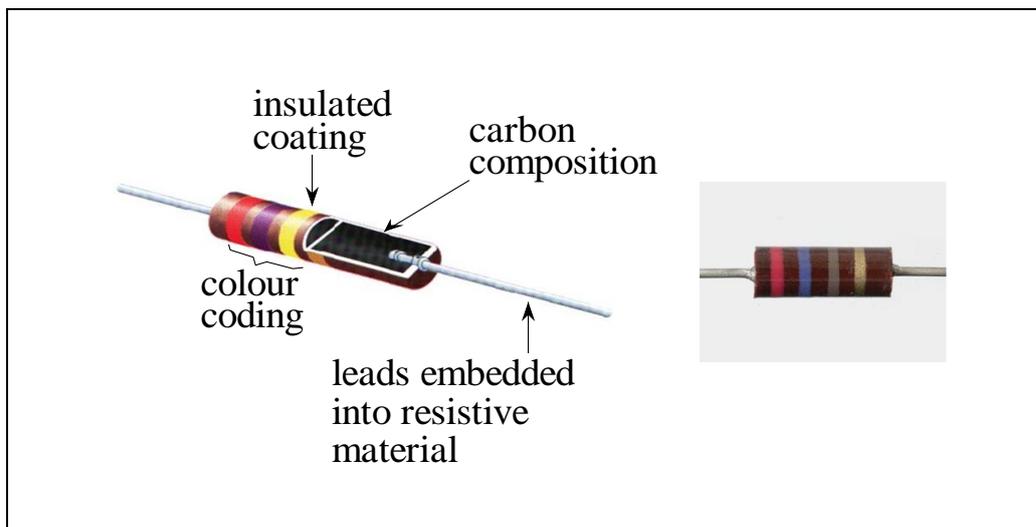
Not every project deserves such scrutiny, but it is standard practice to allocate a healthy margin of safety in an electronic design.

4.3 Resistor Types

This section briefly examines the construction method and materials used in the manufacture of different types of resistors, and therefore shows why the different types of resistors have such different characteristics.

4.3.1 Carbon Composition Resistors

Carbon composition resistors are made from a mixture of powdered carbon and a resin binder, and are pressed to form a rod. The resistor is then coated with epoxy. It is the oldest type of resistor, but is still used in some applications.



Carbon composition resistors...

Figure 4.9 – Carbon composition resistor

The main advantage of carbon composition resistors is their pulse handling capability. They are used as surge protection resistors in medical monitoring equipment and as output resistors in defibrillators. For example, in an emergency situation where monitoring equipment is attached to a patient (e.g. ECG), it may be necessary to apply a defibrillator pulse in the case of a cardiac arrest. In such circumstances there would not be time to disconnect all of the sensitive monitoring equipment and therefore this equipment must be protected.

...have the advantage of a high pulse handling capability

4.22

Carbon composition resistors, either mounted inside the equipment or moulded into the leads of the monitoring equipment, provide protection. These resistors are required to withstand all of the pulse energy – typically around 30 J.

Advantages: Low price, wide range, low inductance, good surge capability.

Disadvantages: Poor tolerance (5% at best), poor stability, moisture sensitivity, high noise figure. Seldom used except in special applications.

4.3.2 Carbon Film Resistors

A film (2 μm to 100 μm thick) of pure carbon is deposited (by pyrolytic decomposition at 1000 °C) on a small ceramic rod. The two ends are covered with metal caps and wire terminals soldered. To adjust the resistance value to within the required tolerance, a process called *spiralling* is used: a helical groove is cut in the resistive layer with an automatic grinding machine or a laser, under computer control, to trim (increase) the resistance to the rated value.

Carbon film resistors are general purpose resistors that are not used often anymore due to surface mount technology

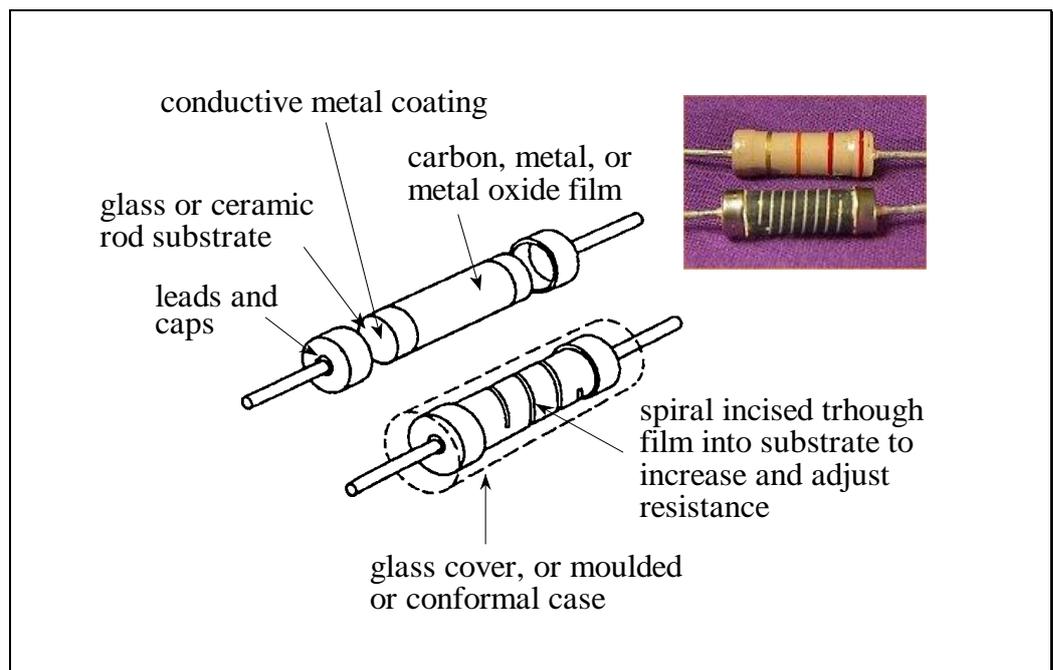


Figure 4.10 – Film resistor construction, spiralling and carbon film resistors

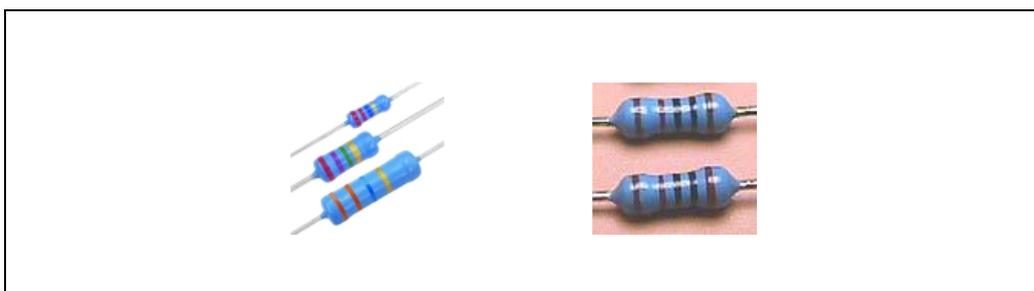
Unfortunately, spiralling also adds inductance to the resistor. Finally the resistor is coated with lacquer and marked. Carbon film resistors have replaced carbon composition resistors as general purpose, low cost resistors.

Advantages: Tighter tolerances (down to 0.5%), better stability, good high-frequency performance, lower noise. Power ratings 1/10 to 2 W. Distinctive feature: negative temperature coefficient of typically -200 to -300 ppm / °C.

Disadvantages: Large stray inductance, especially for high value resistance; poor surge capability. Carbon film resistors are *general purpose resistors*, used for less critical applications.

4.3.3 Metal Film Resistors

A film of low ‘tempco’ metal alloy (Ni, Cr, Au, Al) or of metal oxide is deposited on a ceramic body, then metal caps are added; a helical groove is cut to trim the value (spiralling, as in Figure 4.11), then the body is lacquered and marked.



Metal film resistors are used in precision applications that demand higher power ratings than surface mount components

Figure 4.11 – Metal film resistors

Advantages: Compared to carbon film: tolerances can be much tighter (typically 1% to 2% and can be accurate to 0.05% to 0.1%); can be of smaller size and lower power dissipation (down to 1/20 W). The ‘tempco’ is smaller: from ± 100 ppm / °C down to ± 25 ppm / °C. Stability, high frequency performance is superior.

Metal film are the resistors used in *precision* and *high precision applications*, such as gain setting in op-amp feedback networks and frequency setting in *RC* filters and oscillators.

4.3.4 Wire Wound Resistors

Resistive wire is wound on a ceramic rod or a glass core. Metal caps are pressed over the ends of the rod. The resistor is coated with cement or enamel (high power resistors).

Wire wound resistors are the most accurate and stable and can handle high powers, but they are bulky and expensive

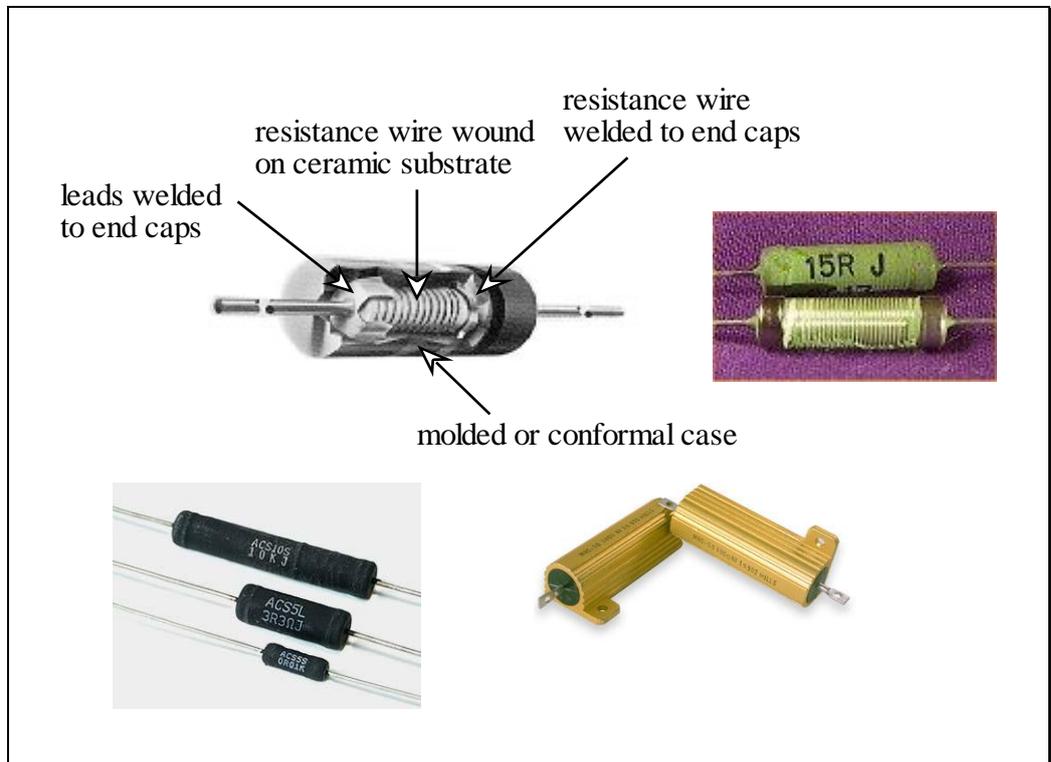


Figure 4.12 – Wire wound resistors

Advantages: High power, less than 20 W; highest precision: 0.01%; ‘tempco’ 0.2 to 2 ppm / °C.

Disadvantages: Large, expensive, very poor frequency response.

4.3.5 Chip Resistors

Chip resistors are surface mount components (SMC) and are made in either thick-film or thin-film.

In thick-film technology, a paste with metal oxides is screen-printed on a ceramic body, dried, then fired (this technology is also designated ‘*cermet*’). The composition of the metal oxides determines the approximate value of the resistor. Metallic end terminals are attached. The resistor is then laser trimmed to the final value and coated.

Thin-film resistors are made by sputtering (a method of vacuum deposition) the resistive material onto an insulating substrate, and then a pattern is etched into the metal layer.

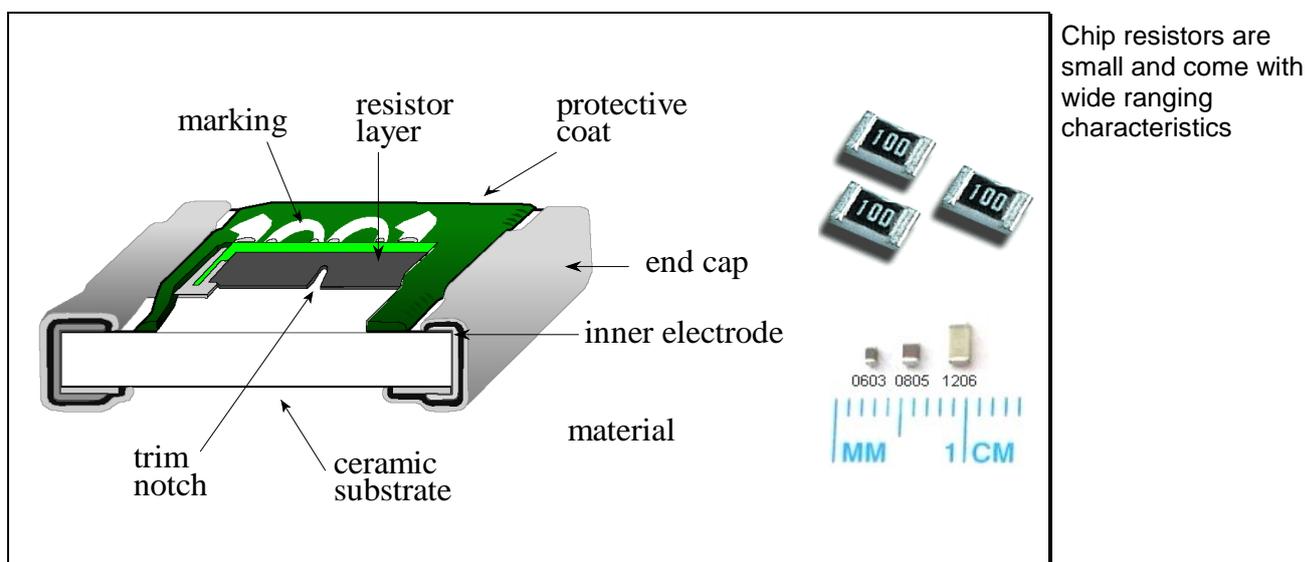


Figure 4.13 – Construction and sizes of chip resistors

Standard sizes and code designations are also shown in the table below.

SIZE CODE (metric)	Length (mm)	Width (mm)	Height (mm)	Power (W)	Mass (mg)
3216	3.2	1.6	0.55	0.25	10
2012	2.0	1.2	0.45	0.125	4
1608	1.55	0.85	0.45	0.100	2
1005	1.0	0.5	0.35	0.0625	0.8
0603	0.6	0.3	0.25	0.05	0.15

Sizes of chip resistors – note how light they are too!

4.26

Advantages: Very small size; range of values, tolerances, tempco and stability nearly as good as of metal film resistors.

Chip resistors are made both as general purpose and for precision applications. Usage has increased rapidly due to extensive adoption of surface mount assembly (SMA), where they are mandatory.

Chip resistors are by far the most common type of resistor in a modern circuit

All *modern circuits* now make use of chip resistors, with leaded resistors reserved for special applications. They are by far the most common type of resistor, and were a necessary development in achieving the miniaturization of everyday electronic devices, such as the mobile phone and MP3 player.

Thick film has the advantages of lower cost, of being able to handle more power, and of being able to service a higher range of ohmic values.

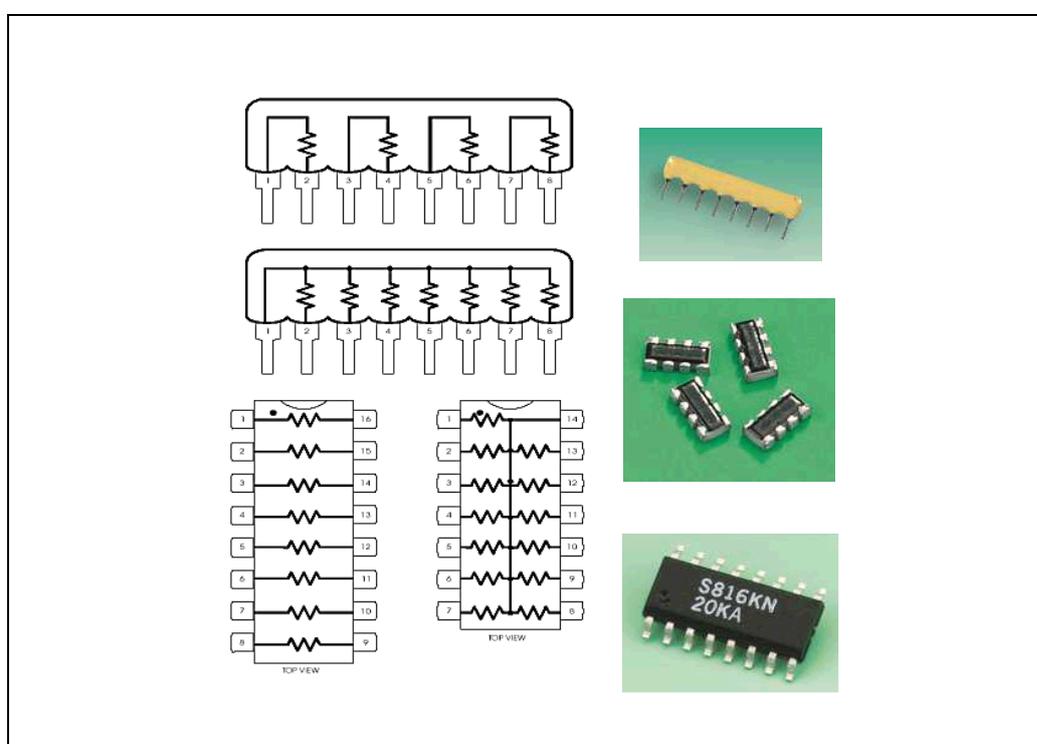
Thin film has the advantages of tighter absolute and ratio tolerances and more environmentally stable components with lower noise and tighter tempco than thick film. Thin film technology is used wherever precision resistors are needed.

Parameter	Thick-Film	Thin-Film
Resistance	3 Ω to 20 M Ω	10 Ω to 100 k Ω
Resistance Tolerance	0.5%, 1%, 2%, 5%	0.01% to 5%
Tempco	± 100 ppm/ $^{\circ}$ C	± 25 ppm/ $^{\circ}$ C
Tempco Tracking	100ppm/ $^{\circ}$ C	5ppm/ $^{\circ}$ C
Operating Temperature	-55 $^{\circ}$ C to +125 $^{\circ}$ C	-55 $^{\circ}$ C to +125 $^{\circ}$ C
Max. Operating Voltage	100 V	50 V
Power	0.125W to 0.5W	0.1W to 0.2W

Table 4.2 – Thick-Film vs. Thin-Film Resistors

4.3.6 Resistor Networks

Resistor networks can be discrete components in a single package, but typically are thick- or thin-film networks. Most leaded networks are in a Dual In-line Package (DIP) or a Single In-Line Package (SIP). Resistor networks for SMA are also readily available. Tolerances can be very tight, like for any metal-film or chip resistor. Particularly, the *tolerance on resistance ratios are extremely tight*, a few ppm / °C, which makes them extremely useful as accurate voltage dividers, attenuators, feedback networks for accurate gain op-amps and instrumentation amps, ladder networks for DACs and ADCs etc. other applications are digital pull-up / pull-down terminations in logic circuits.



Resistors networks save space but also provide matching component values

Figure 4.14 – Resistor networks

4.4 Choosing Resistors

The table below provides comparative data for different types of resistors.

Characteristic	Carbon Composition	Carbon Film	Metal Film	Wire Wound	Chip (thick- and thin-film)
Resistance	1 Ω to 100 M Ω	1 Ω to 10 M Ω	1 Ω to 10 M Ω	0.1 Ω to 10 M Ω	0.1 Ω to 10 M Ω
Tolerance at purchasing	5% to 20%	0.5% to 10%	0.1% to 10%	0.01% to 1%	0.5% to 10% Th F 0.01% to 10% Tn F
Power dissipation	1/8 W to 2W	1/10 W to 2 W	1/20 W to 2 W	0.1 W to 20 W	50 mW to 500 mW
Effect of soldering	2%	0.5%	0.05%	0.01%	0.25% Th F 0.05% Tn F
Frequency limit	1 MHz	100 MHz	400 MHz	50 kHz	100 MHz to 500 MHz
Tempco (ppm / °C)	-800 to +1600	-200 to -1000	± 20 to ± 200	± 20 to ± 200	± 25 to ± 100
Reliability	Good	Good	Good	Poor	Good
Stability	Poor	Good	Good	Very good	Good
Approx. cost	0.2 c	0.05c to 1c	1c to 4 c	50 c to \$1	0.1 c to \$10
Advantages	Low cost High reliability Surge capacity	Lowest cost	Good accuracy Good stability Small size	Highest accuracy and stability	Smallest size Typical for SMA
Disadvantages	Poor accuracy and stability	Relatively large tempco	Poor surge capacity	Very expensive Poor frequency response	

Table 4.3 – Features of Common Resistors

4.4.1 Summary of Resistor Characteristics According to Type

Carbon composition resistors have good pulse and surge response, but poor accuracy and stability. They tend to be replaced in most applications by carbon film resistors.

Carbon film resistors are used for general purpose applications, (e.g. consumer electronics or non-critical applications) where very high stability in time and small change with temperature are *not* required (the tempco is rather large, over -300 ppm / °C).

Metal film resistors are very accurate (readily available with 0.1% tolerance) and stable, with a very low temperature coefficient (from ± 10 to ± 100 ppm / °C). They are used as precision resistors in professional applications.

Chip resistors are either metal film or of the ‘cermet’ type, and have accuracy and stability specifications close to those of other film resistors. Chip resistors are very small in size and are surface mount components (SMC), intended for surface mount assembly (SMA). Usage of chip resistors is now almost universal in all large volume electronic products. The main limitation is the low power dissipation (to about 1/2 W).

Wire wound resistors are the most accurate and stable resistors, but are also the largest and most expensive. They also have a very poor frequency response. Usage is limited to very high precision or high power applications at power (50Hz) and low frequencies.

Resistor networks are used to replace sets of chip resistors in large-volume produced electronic products. Handling and mounting components on a PCB is often more expensive than the components themselves. The use of the smallest SMC is sometimes limited by the difficulty of picking and placing them correctly. The amount of handling and mounting of small SMC resistors can be reduced by combining a number of resistors in a network, produced in thick- or thin-film, on a single substrate, and mounted as a single multi-pin component. They are also used in circuit demanding extremely accurate and stable resistance ratios (op-amp gain settings, ADCs, DACs etc).

4.5 Capacitor Definitions and Basic Relations

Capacitors are the second-most-used passive component in electronic circuits (after the resistor). There is a wide variety of capacitor types, with substantial differences between their characteristics, depending on the dielectric and technology used. Each type has its own combination of features and drawbacks. We will examine, from a design engineer's viewpoint, the available choices among the most used types:

- *Film capacitors* with a number of different dielectric films
- *Ceramic capacitors* of *type I* (low ϵ_r) or *type II* (high ϵ_r) ceramic
- *Electrolytic capacitors* with aluminium or tantalum

Each type of capacitor is recommended for some applications and can be quite inadequate for some other applications. It is essential for any electrical engineer to be aware of:

- how the characteristics of capacitors are expressed
- the basic characteristics of various types of capacitors
- the main areas of application of each type of capacitor

Capacitance is defined as the *ratio of charge to voltage* between two conductors:

$$C = \frac{q}{v} \quad (4.5)$$

Of course:

$$v = \frac{q}{C} = \frac{1}{C} \int i dt \quad (4.6)$$

where i is the current charging or discharging the capacitor.

The instantaneous energy stored in a capacitor is:

$$E = \frac{1}{2} C v^2 \quad \text{or} \quad E = \frac{q^2}{2C} \quad (4.7)$$

The *reactance* of a capacitor is given by:

$$X_c = -\frac{1}{2\pi f C} \quad (4.8)$$

The capacitance of a parallel plate capacitor (neglecting fringing effects) is:

$$C = \frac{\epsilon_r \epsilon_0 A}{d} \quad (4.9)$$

where:

ϵ_r is the relative permittivity of the dielectric

ϵ_0 is the permittivity of vacuum: $\epsilon_0 = 8.85419 \times 10^{-12}$ [Fm⁻¹]

A is the area of overlap of the plates [m²];

d is the thickness of the dielectric [m]

Capacitors should be as small as possible. Since most needs are for low-voltage devices, much of the emphasis in capacitor manufacture is on thin and uniform dielectric, and on methods for obtaining a large overlap-area to volume ratio.

Film capacitors use flat-plate electrodes and dielectric films with a rather small relative permittivity, $\epsilon_r = 2 \dots 5$. They have excellent electrical characteristics, but are rather bulky and expensive at capacitance values above 0.1 to 1 μF .

Ceramic capacitors attain much larger capacitance / volume ratios by using dielectrics with a large relative permittivity, ϵ_r from about 100 to over 100,000.

4.32

Electrolytic capacitors have the highest capacitance / volume ratios, achieved by using chemically etched electrodes, with very large effective surface areas, and very thin dielectric films.

Ceramic class II capacitors and electrolytic capacitors show higher losses and generally poorer electrical characteristics than film capacitors. Their use is recommended for applications where large capacitance values are required, such as decoupling and rectifier filtering, because they are much smaller in size and their cost is lower.

4.6 Capacitor Characteristics

The characteristics and the features to consider when choosing a capacitor are: capacitor value, tolerance on value, rated voltage, surge voltage, leakage current, insulation (leakage) resistance, maximum current, rated pulse rise-time and ripple current. For insight into these factors, the materials and construction of the various capacitor types must be considered.

4.6.1 Rated Capacitance and Tolerance on Value

Like resistors, each capacitor has a rated (nominal) capacitance value and % tolerance on this value. The *preferred capacitance values*, depending on tolerance, are chosen based on the same IEC E-Series as used for the resistors. As with resistors, a precision capacitor with a tighter tolerance on value will also be more *stable in time* and will change less with temperature, i.e. will have a smaller *temperature coefficient of capacitance*.

Component tolerances are tolerances at time of purchase

4.6.2 Rated Voltage

The maximum working voltage of a capacitor is the sum of the DC voltage plus the AC peak voltage which may be applied continuously to its terminals. Operating a capacitor at a voltage lower than its maximum working value extends its life.

4.6.3 Surge Voltage

There is a maximum safe voltage to which a capacitor can be subjected under any combination of circumstances over a short period of time. This is the DC surge voltage rating. Above the surge voltage, the dielectric of the capacitor will break down. Normally, testing for surge voltage involves applying a signal several volts above the rated working voltage. It is applied via a 1 k Ω series resistor in repeated cycles of 0.5 minutes on and 5 minutes off.

4.6.4 Leakage Current

A relatively small direct current flows through a capacitor when a voltage is impressed across it. Electrolytic capacitors have the largest leakage currents.

4.6.5 Insulation (Leakage) Resistance

This is a measure of the ability of the charged capacitor to withstand leakage of DC current. For capacitors of less than 10 nF the insulation resistance should normally exceed 100 GΩ.

The leakage resistance, R_l depends on the resistivity ρ , area A , and thickness d of the capacitor's dielectric:

$$R_l = \frac{\rho d}{A} \quad (4.10)$$

Of course, d and A are the same as those used to calculate the capacitance. Hence, a capacitor (of a given type, construction and rated voltage) which has a larger capacitance, will have a lower value of leakage resistance, and vice-versa. Therefore, to provide a general specification for the leakage of capacitors of a given type and construction, usually the leakage time constant $\tau_l = R_l C$ is specified. The value of the leakage resistance for a particular capacitance value can then be calculated easily.

4.6.6 Maximum Current

Exceeding the maximum specified current through a capacitor can lead to fusing of the internal or external terminals, or of the capacitor's electrodes (sometimes very thin metal films).

Under *steady-state AC conditions* the values of current and voltage in an ideal capacitor are related by:

$$|\mathbf{I}| = 2\pi f C |\mathbf{V}| \quad (4.11)$$

where $|\mathbf{I}|$ and $|\mathbf{V}|$ are the RMS (or peak) values of current and voltage, respectively, and f is the frequency in Hz.

One can see that, besides a maximum allowed surge voltage across a capacitor, (set to avoid breakdown of the dielectric), in some cases there might be a *frequency dependent maximum allowed AC voltage across a capacitor, to avoid an excessive current*.

4.6.7 Rated Pulse Rise-Time

Since the current through a capacitor is the time derivative of charge, one can write:

$$i = \frac{dq}{dt} = C \frac{dv}{dt} \quad (4.12)$$

This equation points to a *limitation in maximum allowed rate of change of the voltage (pulse rise-time or voltage pulse slope)* across the capacitor, $dv/dt|_{\text{max}}$.

If the rate of change dv/dt exceeds the specified limit, the large resulting current might damage the terminals or the electrodes. The rated voltage pulse slope multiplied by the capacitance gives the peak allowed current through the capacitor.

4.6.8 Ripple Current

Electrolytic capacitors are frequently used in rectifier filters, where they are subjected to an AC voltage superposed on a DC voltage. Because of the ohmic and dielectric losses, the resulting AC current increases the temperature of the capacitor. The maximum value of the AC RMS current that may be applied to an electrolytic capacitor is termed *maximum ripple current*. The capacitor should be able to withstand this ripple current at 100 Hz up to 85 °C. The higher the ripple current the shorter the capacitor's life.

4.7 Types of Dielectrics

A real capacitor dissipates energy as well as stores it. This energy loss, which appears as heat, is a result of several effects: the *finite conductivity of the lead wires, electrode contacts and electrodes*; the *finite resistivity of the dielectric*, which results in the *DC leakage current*; and *AC dielectric losses*, which are determined by the polarisation mechanism within the dielectric and are frequency dependent.

Dielectric materials can be classified as *non-polar*, in which there are no dipoles before the electric field is applied; and *polar*, in which dipoles pre-exist the electric field.

4.7.1 Non-Polar Dielectrics

When an electric field is applied to a *non-polar dielectric*, its atoms or molecules are deformed, and an induced dipole moment appears. In non-polar dielectrics losses are very small up to very high frequencies. The two varieties of polarisation are:

- **electronic (optical)**: the cloud of electrons is displaced relative to the nucleus in the presence of an electric field. This mechanism shows no losses to $f = 10^{15}$ Hz and is typical for air, rare gases and polystyrene.
- **ionic**: in molecules having ionic bonds, the positive ions are displaced relative to the negative ions in the presence of an electric field. The upper frequency of ionic motion is about 10^{14} Hz. An example of an ionically bonded dielectric is polytetrafluoroethylene (PTFE), also known by DuPont's brand name as Teflon.

4.7.2 Polar Dielectrics

Polar dielectric materials contain permanent dipoles whose orientation, in the absence of a field, is random. An electric field aligns the dipoles, and electric charges are attracted to the surface of the material. When an AC electric field is applied, the dipoles are reoriented at each cycle. The internal friction of the dipoles inside the material leads to energy losses. Polar dielectrics produce generally the highest AC losses, that increase significantly with frequency. The upper frequency of this dipole motion is within 10 kHz to 1 GHz, depending on the material. There are three main types of polar dielectrics:

- **molecular:** a permanent dipole moment exists as a result of the molecular structure.
- **orientational:** the dipoles are larger than a single molecule. Examples: Transformer oils, certain ceramics and electrolytic dielectrics.
- **interfacial (dielectric absorption):** In some dielectrics, defects such as missing atoms, dislocations and impurity centres, can trap free electrons moving in the field near the electrodes of the capacitor. A local accumulation of charge in the dielectric (*dielectric absorption*) induces an image charge in the electrode. This charge is not released instantly when the field disappears. The mechanism operates from DC to about 100 Hz. It can cause the re-appearance of a charge on a capacitor immediately after it has been discharged, due to release of the trapped charge. Among capacitors showing dielectric absorption: aluminium electrolytic, paper, polyester film, mica.

4.8 Capacitor Models

The simplest model to account for capacitor AC losses is an *Equivalent Series Resistor* (ESR) R_s in series with an ideal capacitor C_s as shown in Figure 4.15a below:

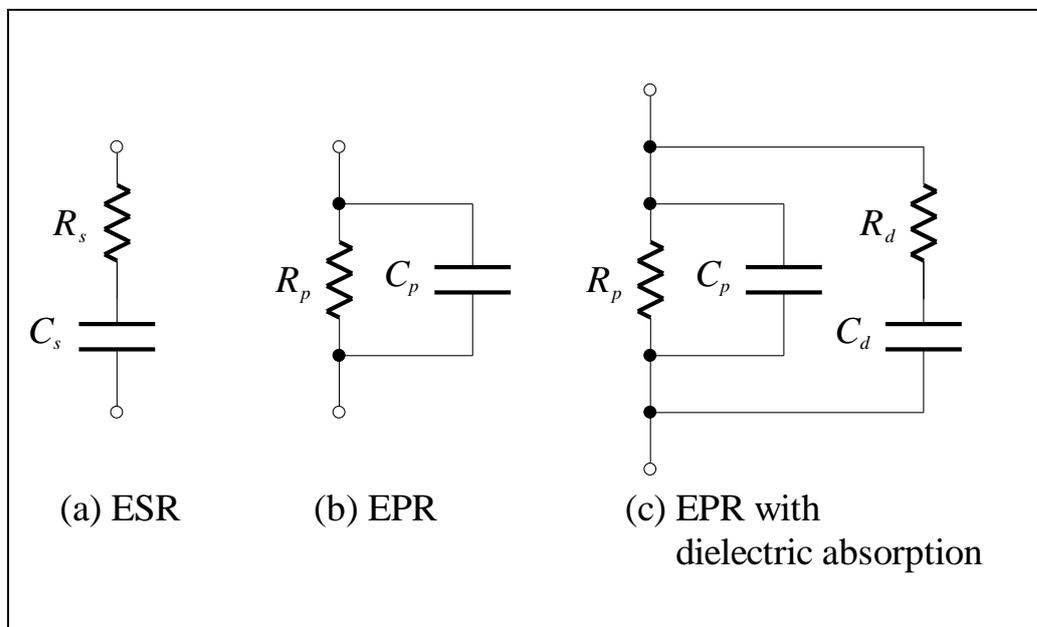


Figure 4.15 – Simplified Models of a Capacitor

In this model, all internal series resistors of a capacitor are lumped into the ESR to represent the losses in the terminals and dissipation in the dielectric.

Another model, useful to represent both AC dielectric losses and DC leakage, is the *Equivalent Parallel Resistor* (EPR) equivalent circuit, with R_p parallel to an ideal capacitor C_p (Figure 4.15b).

Note that C_s is not equal to C_p , and the components in the model generally are frequency dependent.

The model of Figure 4.15c represents *a capacitor with dielectric absorption*. The [C_d in series with R_d] combination, in parallel to [C_p and R_p], models this effect. C_d is the capacitance corresponding to the charge absorbed in the dielectric; R_d in series with C_d accounts for the (relatively long – a few seconds) time constant of the charge release from the dielectric.

The conversion from parallel to series and from series to parallel model parameters can be made using the following relations:

$$R_s = \frac{R_p}{1 + R_p^2 \omega^2 C_p^2} \quad C_s = \frac{1 + R_p^2 \omega^2 C_p^2}{R_p^2 \omega^2 C_p} \quad (4.13)$$

and:

$$R_p = R_s + \frac{1}{R_s \omega^2 C_s^2} \quad C_p = \frac{C_s}{1 + R_s^2 \omega^2 C_s^2} \quad (4.14)$$

4.8.1 Quality of a Capacitor

There are different ways to express the quality of a capacitor, i.e. to show how small its energy losses are relative to the energy stored. These can be understood by referring to Figure 4.16:

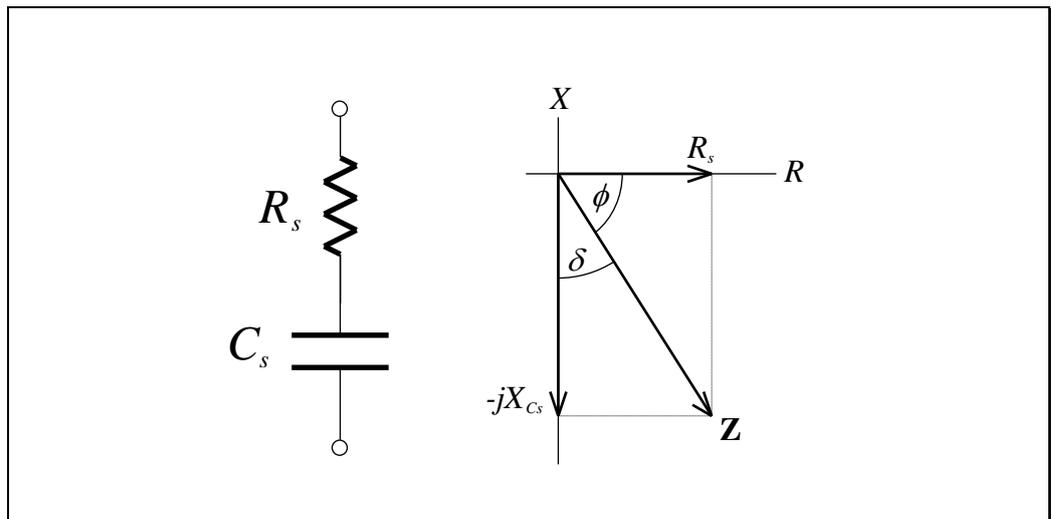


Figure 4.16 – Losses of a Capacitor

The *quality factor*, or Q , of a network is defined as 2π times the ratio of the maximum stored energy to the energy dissipated per period at a given frequency. The higher the value of Q , the closer to ideal a capacitor is.

Quality factor for a capacitor is therefore defined as:

$$Q = \frac{X_{C_s}}{R_s} = \frac{1}{2\pi f R_s C_s} = 2\pi f R_p C_p \quad (4.15) \quad \text{Quality factor defined}$$

Bridges often measure the *Dissipation Factor DF* of a capacitor, which is the reciprocal of Q , and is often expressed in %. We can write:

$$DF = \frac{1}{Q} = \tan \delta = \cot \phi = 2\pi f R_s C_s \quad (4.16) \quad \text{Dissipation factor defined}$$

The *loss angle* δ is the deviation of the capacitor's impedance phasor angle from 90° , i.e., from the phase of an ideal capacitance (see Figure 4.16).

The *Power Factor PF* can also be used to specify the losses. The Power Factor is defined as the cosine of the phase angle between the voltage and current vector:

$$PF = \cos \phi = \sin \delta = \frac{R_s}{|Z|} \quad (4.17) \quad \text{Power factor defined}$$

For low-loss dielectrics, $\tan \delta$ and $\cos \phi$ are approximately equal, and can be used to express dielectric loss. In a “low-loss” capacitor, the dissipation factor is small, typically less than 0.1%.

4.8.2 Equivalent Series Resistance (ESR)

The Equivalent Series Resistance is an important parameter when the capacitor is used for decoupling purposes. The ESR can be obtained from:

$$\text{ESR} = \frac{\tan \delta}{2\pi f C_s} \quad (4.18)$$

4.8.3 Series Resonant Frequency (SRF)

The simplified models shown in Figure 4.15 describe well the capacitor's behaviour at relatively low (at most audio) frequencies. A very accurate model over a wide frequency range is difficult to produce, because capacitor parameters are distributed. A reasonably accurate high frequency model is shown below:

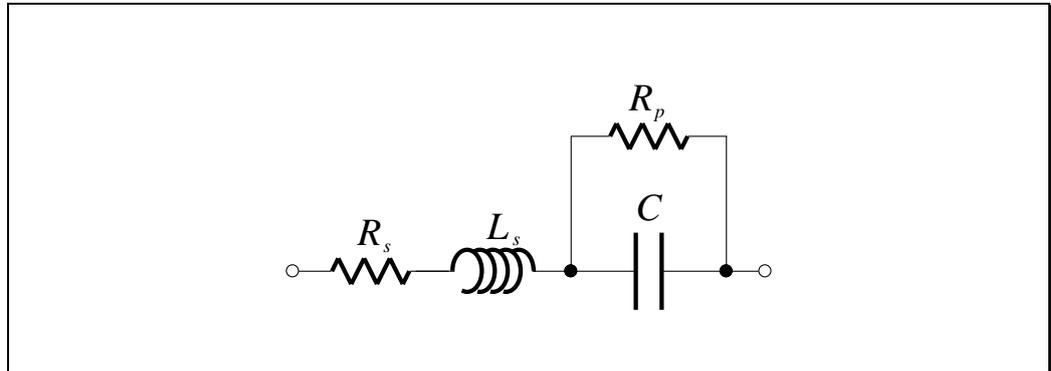


Figure 4.17 – High-Frequency Model of a Capacitor

The dielectric losses are represented by R_p and the conductor (series) losses by R_s . The series inductance L_s models the *inductance of the leads and of the capacitor structure itself*. Leadless chip capacitors and ceramic capacitors generally show smaller inductance.

If R_p is neglected, the capacitor behaves as a typical *series RLC resonant circuit*, whose impedance versus frequency curve is shown below:

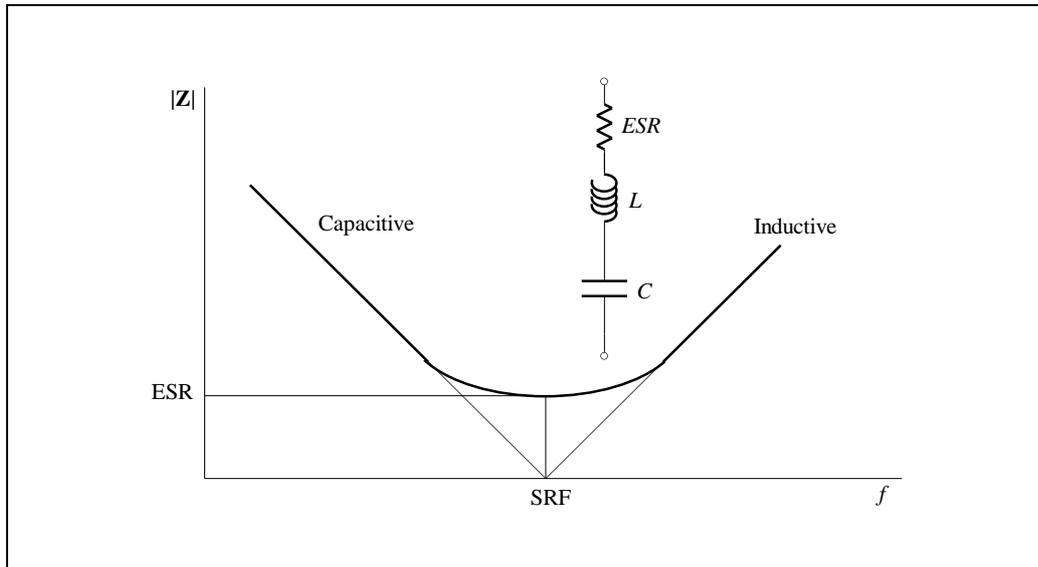


Figure 4.18 – Simplified High-Frequency Model of a Capacitor

The minimum impedance of the capacitor, reached at the Series Resonant Frequency (SRF), is the Equivalent Series Resistance (ESR). Above the SRF, $|Z|$ increases with f , i.e. *the capacitor behaves as an inductor!*

For a particular family of ceramic capacitors, the impedance curves vary:

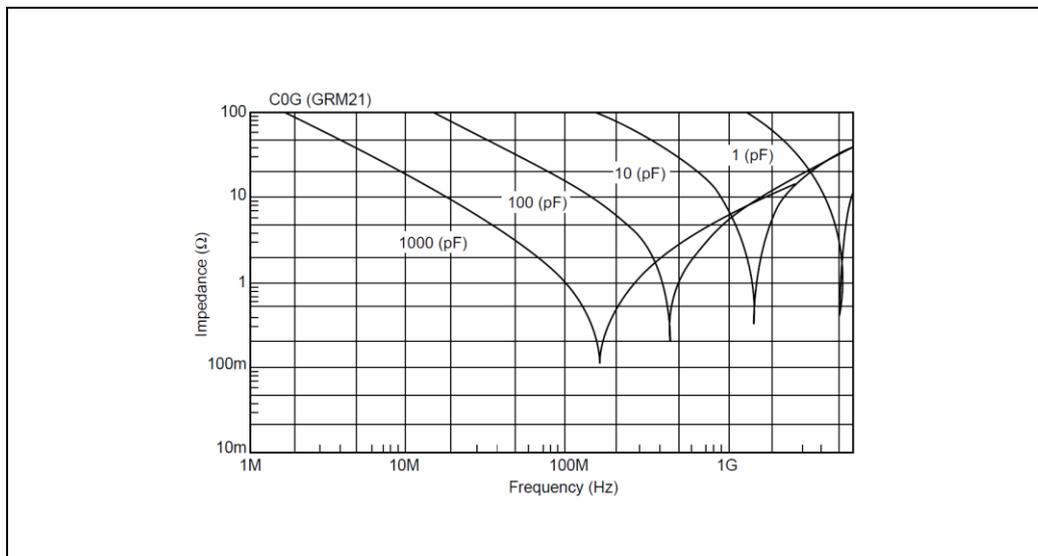


Figure 4.19 – Impedance versus Frequency Variation

shows that in a capacitor family (same type and construction), *the larger the capacitance, the smaller the Series Resonant Frequency (SRF)*.

4.9 Film Capacitors

Film capacitors are very widely used. The availability of extremely thin films and the wide variety of materials provides the versatility for a variety of applications including filtering, coupling, bypassing, timing, and noise suppression.

Film capacitors can operate at relatively high temperature, have high insulation resistance, have good stability and are available in tolerances as tight as 0.5%. The self-healing property of metallised films is useful in some applications, where surge voltages might happen occasionally. Three main construction techniques are used: wound foil, metallised film, stacked film.

4.9.1 Wound Foil Capacitors

A wound foil capacitor is made of two aluminium foils separated by sheets of dielectric and rolled into a compact cylinder. Contacts to foil are made by welding or inserting tabs during winding (Figure 4.20a) or, in the extended foil type, by allowing the foils to extend beyond the dielectric on opposite sides (Figure 4.20b). After winding, leads are attached to the tabs or exposed foil edges. After the leads are attached the assembly is moulded, potted, dipped in a protective resin coating or sealed in a metal can.

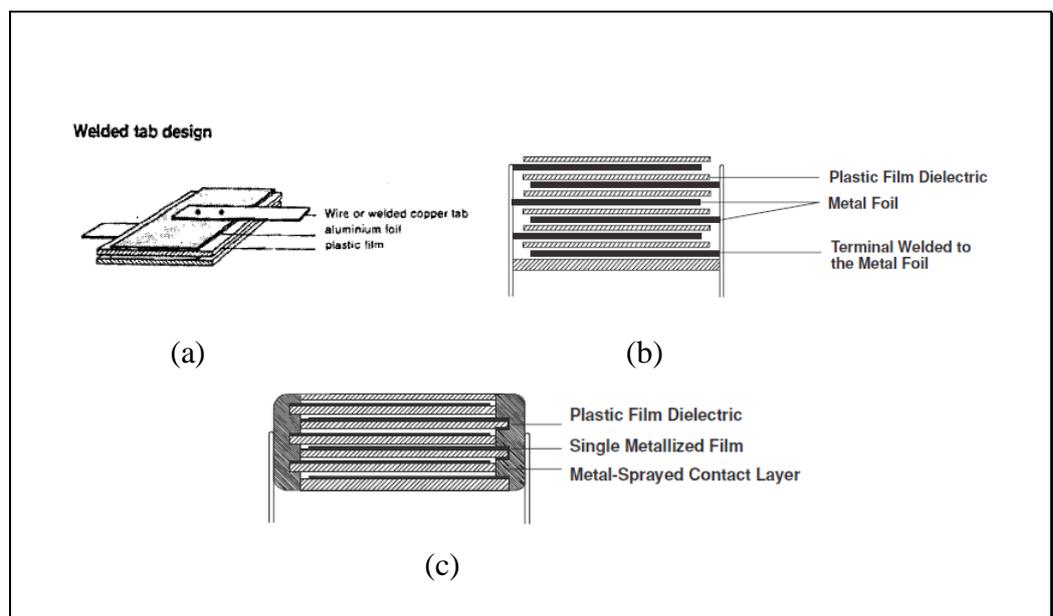


Figure 4.20 – Film Capacitors

4.9.2 Metallised Film Capacitors

Metallised film capacitors are made by vacuum deposition of aluminium 0.1 μm thick directly onto the dielectric film. The pattern is basically that of an extended foil configuration.

After winding, contact to each end of the roll is made with a fine spray of molten metal, to which leads are finally soldered (Figure 4.20c). This construction reduces the volume of low-voltage large-value capacitors and provides a voltage breakdown property known as 'self-healing' or 'self-clearing'. During the 'healing stage' of the manufacturing process, the rated voltage is applied to the capacitor. If a defect should occur in the dielectric, the discharge current through the defect generates enough heat to vaporise the thin metal electrodes. This isolates the defect site and permits restoration of insulation. Self-healing also happens under moderate surge conditions in normal operation of metallised film capacitors (in the wound foil construction, the electrodes are much thicker, and a breakdown results in a permanent short). Self-clearing requires a minimum energy of 10 to 50 pJ. Low values of metallised film capacitors used in low-voltage high impedance applications may be shorted, and fail instead of clearing.

4.9.3 Stacked Film Capacitors

Stacked film capacitors are of a newer construction. Metallised films are wound onto a large cylinder, then cut into rectangular sections. Connections are made to alternate electrodes on opposite ends, resulting in a stack of metallised film connected in parallel. The structure is similar to that of multilayer ceramic capacitors, except the dielectric is much thinner. This compensates for the low permittivity of film dielectrics and produces a much better capacitance/volume ratio. Stacked film chip capacitors are an alternative to multilayer ceramic capacitors in some applications.

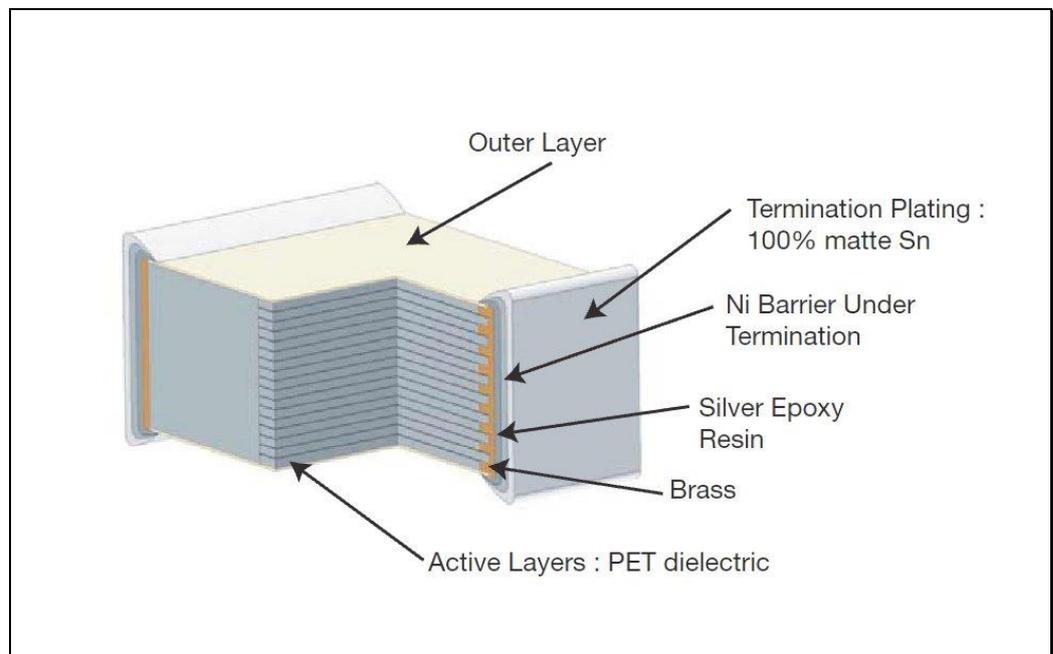


Figure 4.21 – A Stacked Film Chip Capacitor

4.9.4 Basic Properties of Film Capacitor Dielectrics

Film capacitors are available in values of 100 pF to 1 μ F (seldom to 10 μ F) and in voltage ratings from 50 V to several thousand volts. Tolerances are 20% down to 0.5% for some types. Dissipation Factors (DF) are less than 1% at 25 °C. Films with low DF at room temperature have generally low DFs over the entire temperature range. Insulation resistance generally decreases with temperature, sometimes by up to two or three orders of magnitude at 125 °C. Film capacitors can be used at high frequencies, depending on the size and length of the leads. The basic properties of film dielectric materials are listed in Table 4.4 and Figure 4.22 below.

Dielectric	Code	Permit- tivity ϵ_r	DF 1 kHz 25 °C %	DF 1 MHz 25 °C %	Max. Temp °C	$\Delta C/C$ @ -40 °C %	$\Delta C/C$ @ 100 °C %	Leakage Time Const. (25 °C) $M\Omega \times \mu F$
Paper	P	3	0.5	3	100	-8	4	40,000
Polyester	KT	3.3	0.5	2	125	-4	3	100,000
Polycarbonate	KC	2.9	0.12	1.1	125	-0.7	1	300,000
Polystyrene	KS	2.4	0.02	0.04	85	0.8	-0.4 (70 °C)	500,000
Polypropylene	KP	2.2	0.02	0.04	100	1	-1.5	100,000
Polysulfone (Polyphenylene sulfide)	KPS	3	0.1	0.18	150	0.6	-0.7	100,000
PTFE or Teflon (polytetra- fluoroethylene)		2.1	0.02		150	1	-2	5,000,000

Table 4.4 – Basic Properties of Film Capacitor Dielectrics

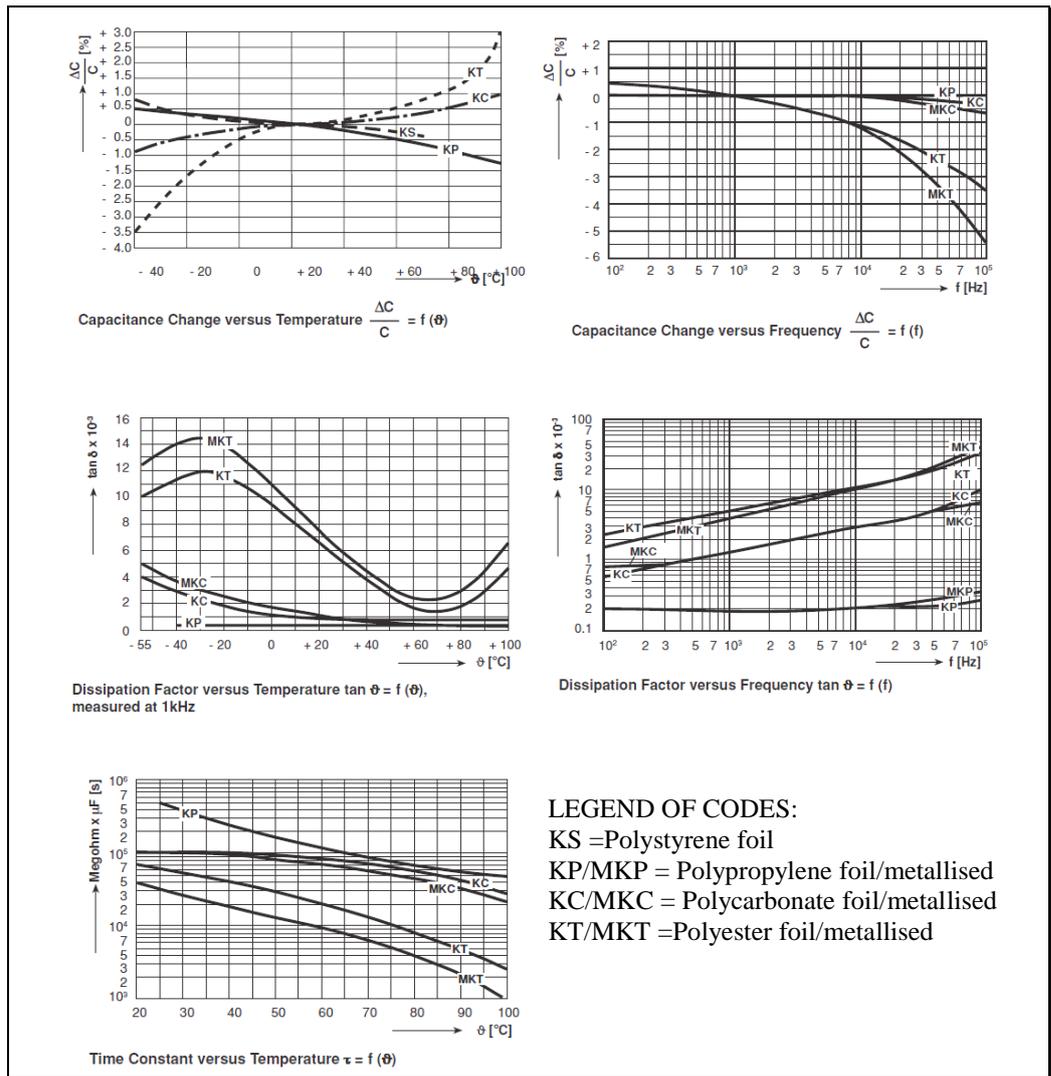


Figure 4.22 – Changes in Capacitance and Loss angle with Temperature and Frequency of Film Capacitors

4.9.5 Tolerance

The tolerance of film capacitors is given by a letter – some of the more common are shown in the table below:

	Code	B	C	D	F	G	J	K	M	Z
Tolerance	$C < 10 \text{ pF} \pm \text{pF}$	0.1	0.25	0.5	1	2				
	$C > 10 \text{ pF} \pm \%$			0.5	1	2	5	10	20	+80 -20

Table 4.5 – Common Tolerance Codes of Film Capacitors

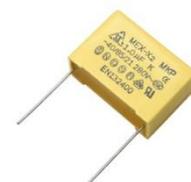
4.9.6 Recommended Applications for Film Capacitors

From Table 4.4 and Figure 4.22 one can see that the films showing the lowest losses at high frequencies and the best temperature stability are polystyrene, polypropylene, polycarbonate and PTFE (Teflon).

Polystyrene film (KS) has excellent electrical characteristics. The loss angle of polystyrene film capacitors is low over a wide frequency range but the electrodes degrade performance with higher capacitance values as frequency increases. A device under 1 nF will typically rise from $\tan \delta < 1 \times 10^{-4}$ at 1 kHz to less than 1×10^{-3} at 1 MHz. For values between 10 and 100 nF, in a typical example, the power factor ($\tan \delta$) will be as low as 1×10^{-4} at 1 kHz, but at 1 MHz, the power factor might increase to 5×10^{-3} . As a major drawback, the maximum ambient temperature of KS capacitors is only 85 °C. Also, capacitors over 10 nF tend to be rather bulky. Solvents affect the film, so sealed encapsulation is sometimes needed.



Polypropylene film (KP) is another excellent choice for precision and HF applications. The dissipation factor can stay below 1×10^{-3} to over 10 MHz. Polypropylene has a higher operating temperature (125 °C) and capacitors can be made smaller.



Polystyrene and polypropylene films are the first choice for high frequency, high precision, or high voltage applications due to their very low dissipation factor, low temperature coefficient and high dielectric strength. KS and KP capacitors are recommended for circuits where capacitance stability and low losses are critical, e.g. frequency dependent applications such as accurately tuned filters, oscillators, timers, integrators, ADCs and VFCs. For circuits of the same category, operating at higher frequencies of, say, above 1 to 10 MHz, ceramic class 1 capacitors would probably be preferable.



Polycarbonate (KC) film has a relatively low loss angle, but the loss angle increases slightly more with frequency than the films mentioned above. Polycarbonate shows a small change of capacitance with temperature and can be made as a thinner film, so devices are generally smaller.



PTFE (Teflon) film has an extremely high insulation resistance, up to a high temperature, and is therefore excellent when very low leakage is required. PTFE also maintains low dielectric losses up to microwave frequencies. PTFE capacitors are of higher cost than the other film capacitors. They are used mainly in dedicated applications, e.g. microwave, high temperature, or where extremely low leakage is required, where the other films cannot compete.



Polyester film (KT) capacitors have higher losses and are less stable than the other film capacitors, but they are of lower cost and smaller size, because of a slightly higher relative permittivity.

Polyester and polycarbonate dielectrics are used in general purpose applications where a small DC bias voltage and small AC voltages at low frequencies are usual. Polyester film capacitors (so-called 'green caps') are recommended mainly for audio- or video-frequency coupling, bypassing or simple lowpass or highpass filtering, where tolerance and stability are not critical. The most important advantages are the high capacitance per volume for polyester and the capacitance stability over a wide temperature range for polycarbonate.



Polyphenylenesulfide (PPS) (or Polysulfone) is a newer dielectric. Its high melting point allows it to be used in non-encapsulated SMD capacitors, because it can withstand the soldering heat unprotected by a case. PPS is about as stable, and has losses as low as, polycarbonate.



Paper capacitors (P) have the disadvantages of a larger variation in capacitance with temperature change and a shorter service life relative to most other types. Paper capacitors are still used for medium capacitance values of approximately 1 nF to 1 μ F, mainly at power line frequency, as for example in interference suppression capacitors.

4.10 Ceramic Capacitors

Ceramic dielectrics and ceramic capacitors using these dielectrics are divided into three classes.

Class 1 ceramic dielectrics are materials with relatively low permittivity ($\epsilon_r = 6 \dots 600$), good control of tolerances, excellent stability and ageing characteristics, low dissipation and very good Q up to very high frequencies. The capacitance versus temperature characteristics are well-controlled and basically linear, with *specified temperature coefficients*. They are used in such applications as oscillators and filters where low losses, capacitance drift compensation and high stability are required.

Class 2 ceramic dielectrics are materials of higher permittivity ($\epsilon_r = 250 \dots 100,000$), which allow much higher capacitance/volume efficiencies, but show higher losses and have non-linear capacitance-temperature characteristics; the capacitance is also voltage dependent and subject to ageing. They are used for coupling and decoupling, notably in pulse and high-frequency circuits where their small series inductance is valuable.

Class 3 ceramic dielectrics offer a still higher volumetric efficiency, but again this is at the expense of poor accuracy and stability and a low dissipation factor. They are also not normally able to withstand high voltages. The dielectric used is often barium titanate that has a dielectric constant of up to about 1250. A typical class 3 capacitor will change its capacitance by -22% to +50% over a temperature range of +10°C to +55°C. It may also have a dissipation factor of around 3 to 5%. It will have a fairly poor accuracy (commonly 20%, or -20%/+80%). As a result, class 3 ceramic capacitors are typically used for decoupling or in other power supply applications where accuracy is not an issue. However they must not be used in applications where spikes are present as these may damage the capacitor if they exceed the rated voltage.

4.10.1 EIA temperature coefficient codes

In order that the performance of ceramic capacitors can be standardized and easily defined, a set of codes has been defined by the Electronic Components Industry Association (ECIA) which are called the Electrical Industries Association (EIA) standards. These codes enable ceramic capacitor performance to be defined in an easily managed way. The codes are different though for class 1 and class 2 ceramic capacitors.

4.10.2 Class 1 capacitor codes

Class 1 capacitors are comprised of a three character EIA code:

1. The first character is a letter which gives the significant figure of the change in capacitance over temperature in ppm/°C.
2. The second character is numeric and gives the multiplier.
3. The third character is a letter and gives the maximum error in ppm/°C.

The table below details what each of the EIA codes means.

FIRST CHARACTER (LETTER) SIGNIFICANT FIGURES		SECOND CHARACTER (DIGIT) MULTIPLIER		THIRD CHARACTER (LETTER) TOLERANCE	
C	0.0	0	-1	G	±30
B	0.3	1	-10	H	±60
L	0.8	2	-100	J	±120
A	0.9	3	-1000	K	±250
M	1.0	4	+1	L	±500
P	1.5	6	+10	M	±1000
R	2.2	7	+100	N	±2500
S	3.3	8	+1000		
T	4.7				
V	5.6				
U	7.5				

Table 4.6 – EIA Temperature Codes for Class 1 Ceramic Capacitors

As an example, one common type of class 1 capacitor is a C0G and this will have 0 drift, with an error of ±30 ppm/°C.

Industry commonly uses an "N/P" designation system that is obsolete, but is more intuitive, and more reflective of what is actually in production. The "N" is used for capacitors with a negative temperature coefficient, and a "P" for those

with a positive coefficient. Manufacturers usually use both in their catalogs, while distributors' catalogs often use only the N/P codes.

Industry:	P100	NP0	N030	N075	N150	N220	N330	N470	N750	N1500	N2200
EIA:	M7G	C0G	B2G	U1G	P2G	R2G	S2H	T2H	U2J	P3K	R3L

Table 4.7 – Some Commonly Available Class 1 EIA and Industry Codes

For example, a P100 class 1 capacitor has a temperature coefficient of +100 ppm/°C, whilst an N470 has -470 ppm/°C.

4.10.3 Class 2 capacitor codes

Class 2 capacitors are comprised of a three character EIA code:

1. The first character is a letter. This gives the low-end operating temperature.
2. The second is numeric and this provides the high-end operating temperature.
3. The third character is a letter which gives capacitance change over that temperature range.

The table below details what each of the EIA codes means.

FIRST CHARACTER (LETTER) LOW TEMPERATURE	SECOND CHARACTER (DIGIT) HIGH TEMPERATURE	THIRD CHARACTER (LETTER) CHANGE			
X	-55 °C	2	+45 °C	D	±3.3%
Y	-30 °C	4	+65 °C	E	±4.7%
Z	+10 °C	5	+85 °C	F	±7.5%
		6	+105 °C	P	±10%
		7	+125 °C	R	±15%
				S	±22%
				T	+22% / - 33%
				U	+22% / - 56%
				V	+22% / - 82%

Table 4.8 – EIA Temperature Codes for Class 2 Ceramic Capacitors

Two very common examples of class 2 ceramic capacitors are the X7R capacitor which will operate from -55 °C to +125 °C with a capacitance change of up to ±15%, and the Z5U capacitor which will operate from +10 °C to +85 °C with a capacitance change of up to +22% to -56%.

4.10.4 Construction of Ceramic Capacitors

Ceramic capacitors are manufactured in disk or square plate form, in tubular form, or as multilayer or 'monolithic' capacitors. The dielectric material is mainly barium titanate, calcium titanate or titanium dioxide with small amounts of other additives for specific characteristics.

Disk capacitors are made from carefully formulated powders, milled to produce a small particle size. The powder is compressed into a thin disk or square plate which is then fired at a high temperature (1200... 1400 °C) to fuse the material. Electrodes are screen printed on each side of the disk and fired (at about 800 °C). Leads are soldered with a high melting temperature solder, then the capacitors are lacquered or immersed in epoxy resin to provide a protective coating. The capacitance value is marked on the body in clear text or in colour code. The temperature coefficient or temperature dependence are also indicated by colour coding. Disk capacitors have a limited range of electrode area and thickness. The dielectric formulation is varied to achieve a wide range of capacitance values.

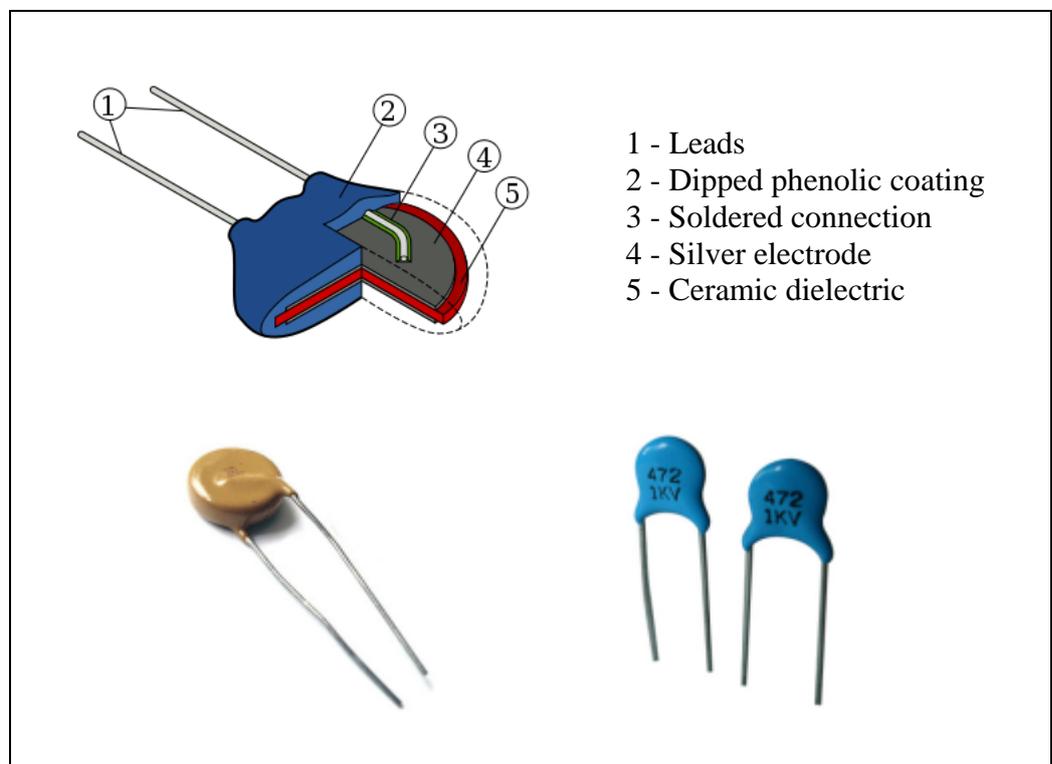


Figure 4.23 – Ceramic Disk Capacitors

Multi-layer ceramic capacitors (MLCC) First, a slurry consisting of dielectric powder, a binder and a solvent is cast into a thin sheet on a stainless-steel or plastic belt. After drying, the electrodes are printed on the sheets, which are then stacked and compressed. The stacks are cut into individual capacitors, heat treated and fired at a high temperature. Finally terminals are attached on both ends and the device is encapsulated. The range of capacitance values, for a given dielectric, is realized by changes in electrode area and the number of layers in the stack.

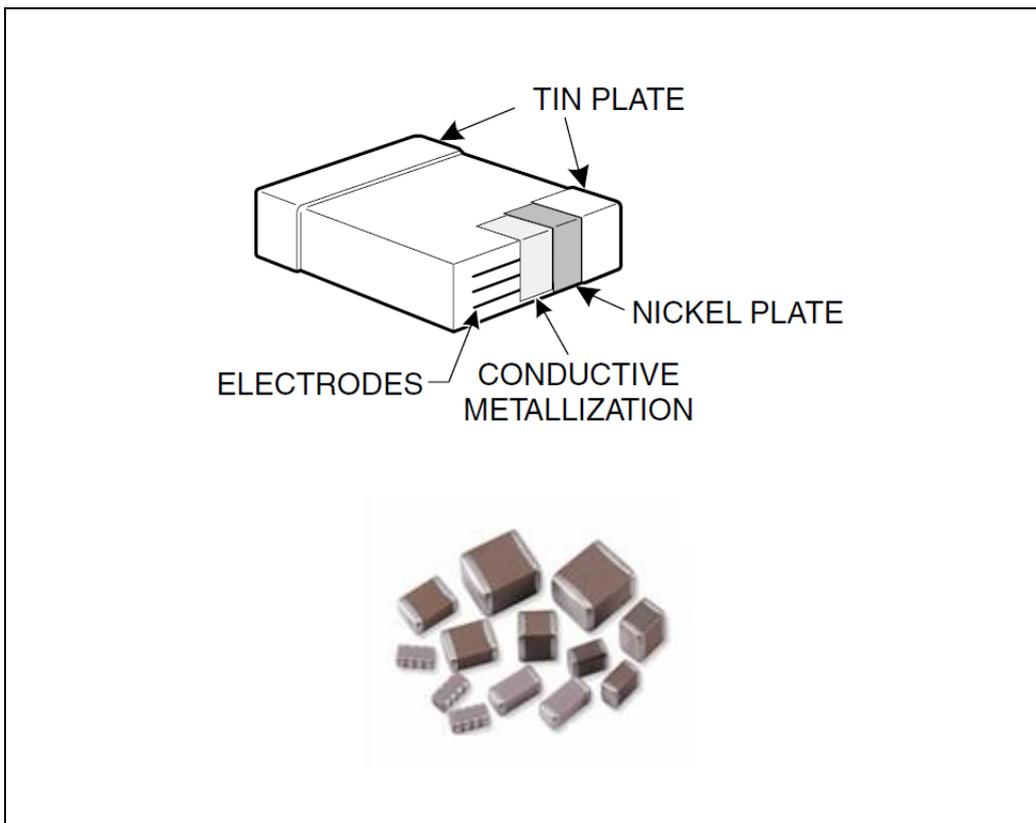


Figure 4.24 – Multi-layer Ceramic Capacitors

4.10.5 Characteristics of Ceramic Capacitors

The characteristics and applications of ceramic capacitors are quite different for class 1 and class 2 dielectrics.

Class 1 dielectrics can be used for capacitance values of 1 pF to 10 nF (in chip form, to 100 nF) with tolerances of $\pm 20\%$ to $\pm 1\%$ (or ± 0.1 pF for $C < 10$ pF), and voltage ratings to kilovolts. The specified temperature coefficient of capacitance extends from P100 ($+100$ ppm/ $^{\circ}\text{C}$) to N470 (-470 ppm/ $^{\circ}\text{C}$), with additional ranges from N750 to N4700 (see Figure 4.25). The most frequently used dielectric is C0G of nominally 0 ppm/ $^{\circ}\text{C}$, but actually guaranteed to be within ± 30 ppm/ $^{\circ}\text{C}$. The capacitance of class 1 ceramic capacitors is very stable in storage or in operation. The dissipation factor for C0G, for example, is less than 0.1 % over the full temperature and frequency range, up to several tens of MHz. The voltage coefficient is essentially zero.

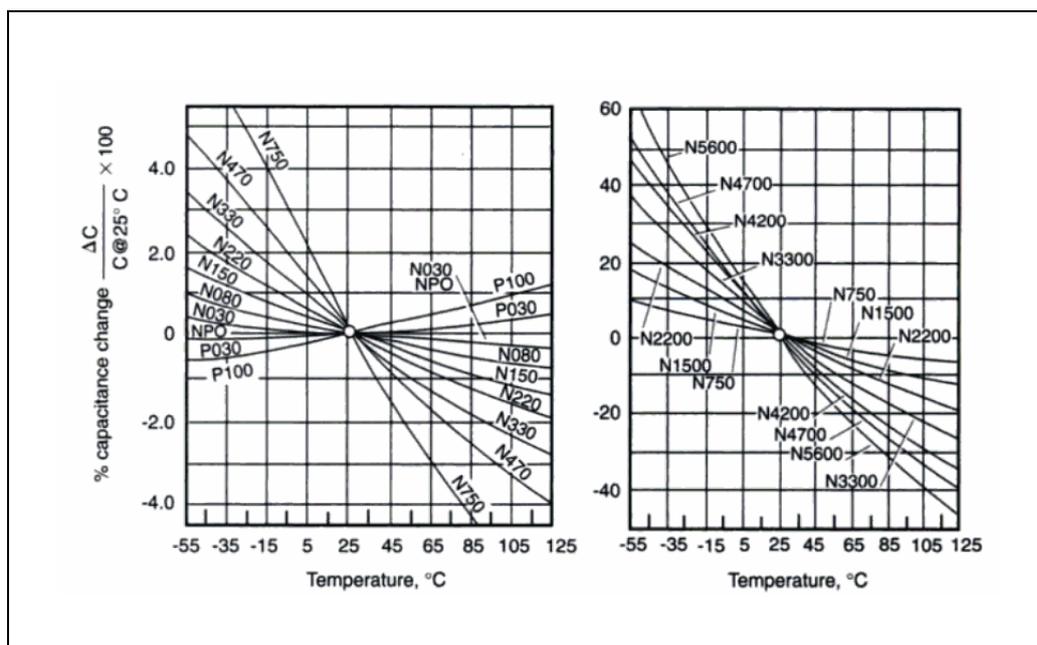


Figure 4.25 – Capacitance Variation with Temperature for Class 1 Ceramic Dielectrics

Class 2 ceramic dielectrics are used for general purpose applications, where small size is important, and stability and tolerance are less important (the designation of the most frequently used material for Class 2 are X7R and Z5U). Capacitance values range from 1 pF to 1 μ F in disk capacitors, and to 10 μ F in multi-layer capacitors. Typical voltage ratings are up to 1000 V in disk, and up to 200 V in MLCC. Temperature, frequency and voltage dependence of capacitance (up to -50% for some types), as well as variations due to ageing are large in class 2 capacitors. The dissipation factor is typically 1% to 10% and is very dependent on temperature (decreases at higher temperatures) and on frequency (increases to over 30% at 1 MHz).

4.10.6 Applications of Ceramic Capacitors

The main advantages of *class 1 ceramic* capacitors over film type capacitors are:

- generally lower cost
- better control and wide range of temperature coefficients
- lower inductance and good high-frequency characteristics

The main disadvantages over film capacitors:

- higher losses than polystyrene, polypropylene or polycarbonate film
- lower leakage resistance than most film capacitors

Class 1 capacitors are used in circuits requiring stability and low loss (high Q), particularly at high frequencies, say over 0.1 to 1 MHz, over the full temperature range.

The main advantages of *class 2 ceramic* capacitors for general purpose applications are:

- lower cost for values to 10 nF
- comparable in cost between 10 nF and 1 μ F
- lower inductance and better high-frequency performance, especially compared to electrolytic capacitors.

Applications of class 2 capacitors are filtering ripple, DC blockage, coupling and decoupling components in circuitry where stability is not an important criterion but low inductance and a high self-resonant frequency are important.

4.11 Electrolytic Capacitors

Electrolytic capacitors fall into two main categories – aluminium and tantalum.

4.11.1 Aluminium Electrolytic Capacitors

Aluminium electrolytics are of the foil type, with an electrolyte that can be aqueous, paste or dry. The anode is made of high purity aluminium foil, 25 to 100 μm thick. The foil is usually electrochemically etched to increase its surface by a factor of 8 to 30 or more. The foil is then "anodised" to produce electrochemically a layer of aluminium oxide (Al_2O_3 with $\epsilon_r = 8.4$) which is the dielectric. The voltage used for the final stage of anodising (the "forming voltage") determines the oxide thickness and voltage rating (about 2/3 of the forming voltage, which can be as high as 600 V).

Non-solid polarised electrolytics. In this, the most common structure, (Figure 4.26) a second, non-oxidised aluminium foil is the cathode connection. The *cathode* aluminium foil serves only as an electrical contact to the *electrolyte*, (e.g. glycol-borate) which is the actual cathode. (In nonpolarised electrolytics, the cathode is also anodised, but the capacitance is halved). The anode and cathode foils are welded to lead wires and then wound into a tight capacitor. In newer constructions, the foils are arranged in stacks, with a tab extension on each foil, to reduce ESR and inductance.

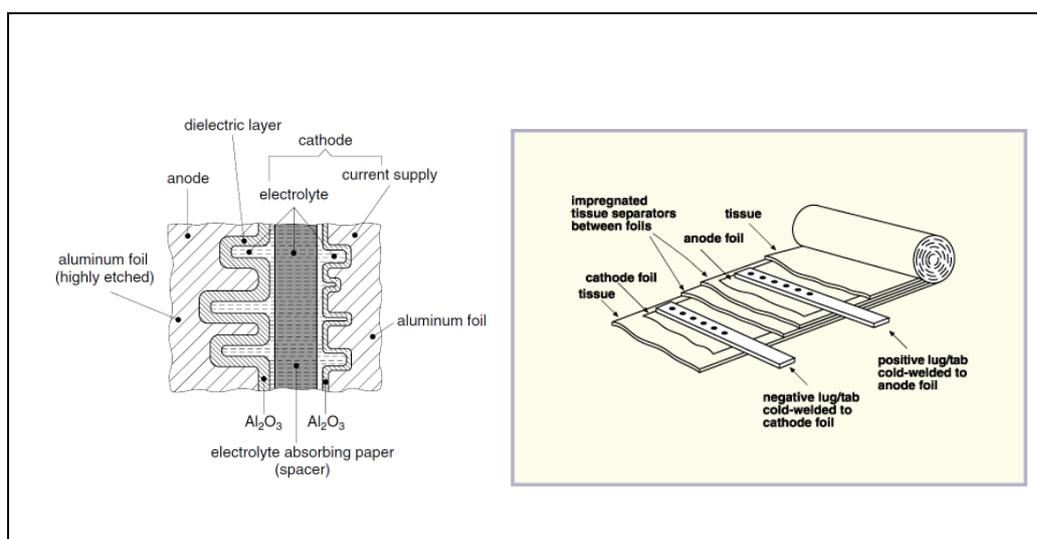


Figure 4.26 – Structure of a Non-Solid Electrolytic Capacitor

4.60



Solid Electrolyte Aluminium is a newer type of aluminium electrolytic, using technologies similar to solid electrolyte tantalum capacitors. Solid electrolyte capacitors have better frequency characteristics, (to > 100 kHz) and lower leakage than non-solid aluminium electrolytics. SMD implementations of solid electrolyte capacitors are increasingly being used.

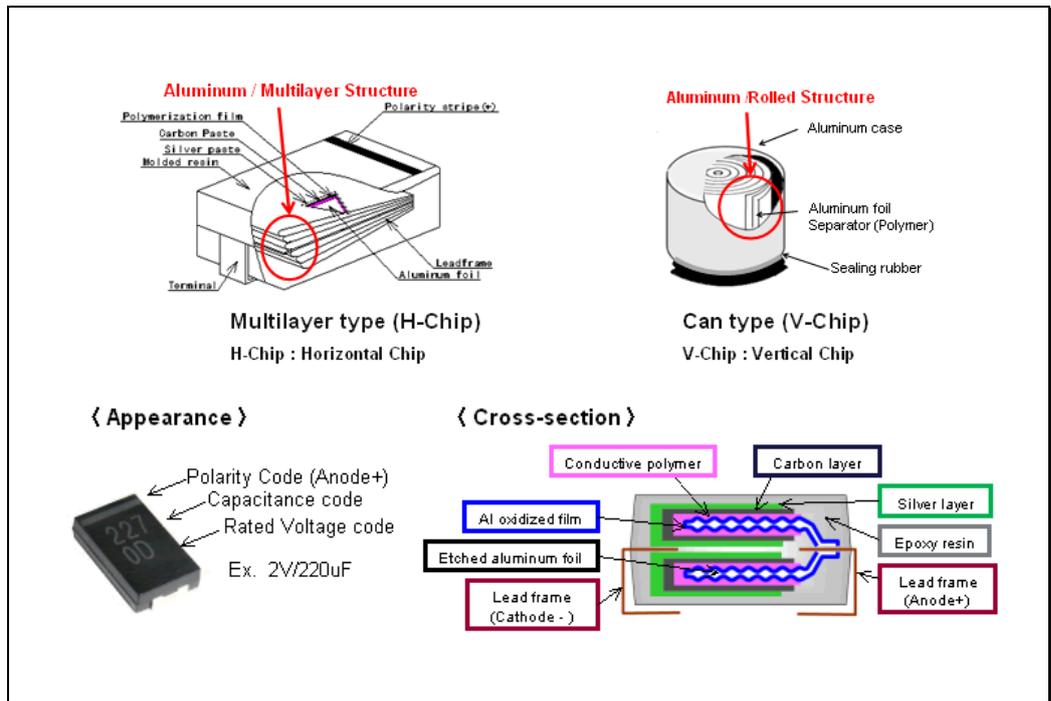


Figure 4.27 – Structure of a Non-Solid Electrolytic Capacitor

Very recently, new types of Sintered Aluminium Solid Electrolytic Capacitors, e.g. "Alsicon" and "OS-CON" (the latter one, using an organic solid electrolyte) have been developed, which compete favourably in characteristics with solid tantalum electrolytics, especially in high-frequency applications, up to 1 MHz.

Aluminium electrolytics are available in capacitor values of 1 μ F to 1 F, with voltage ratings of 3 to 475 V. Higher voltages imply lower capacitance per unit volume, because the aluminium oxide film must be thicker. Tolerances are typically -20% to +150% (for applications in coupling or decoupling, a minimum capacitance value must be guaranteed, but a larger capacitance is usually not harmful).

Polarised aluminium electrolytic capacitors can withstand a reverse voltage of up to 1.5 V without noticeable effect on their operating characteristics. Excess voltage applied for short periods will cause some change in capacitance but will not lead to failure. On the other hand, exposure to reverse or excess forward voltage for a longer time leads to rapid heating of the capacitor and to breakdown. The specified maximum *AC ripple current* (when filtering AC rectifier based power supplies) must also not be exceeded to avoid overheating.

Newer aluminium electrolytic capacitors are made with a built-in pressure relief mechanism. This is designed to open and slowly release gas pressure that may build up if the device overheats during operation.

The *capacitance* of electrolytics is very dependent on temperature, (Figure 4.28). It decreases significantly at lower temperatures. Over extended periods of time (years) aluminium electrolytics show a gradual decrease in capacitance, due to loss of electrolyte (drying) through the seals.

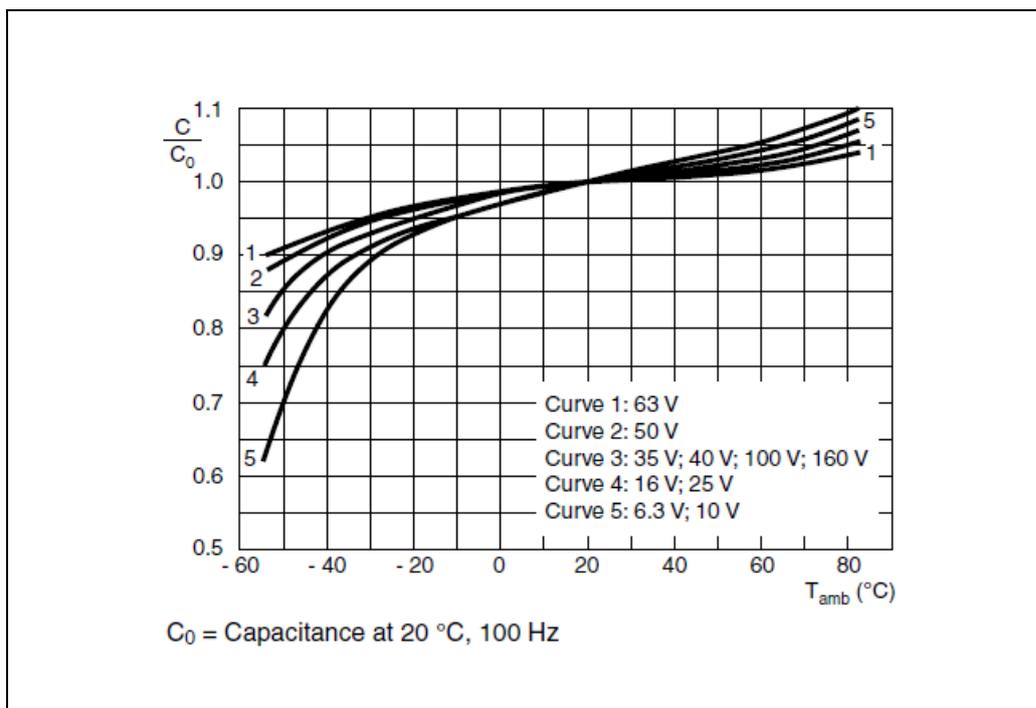


Figure 4.28 – Dependence on Temperature of an Electrolytic Capacitor

The capacitance also decreases quite significantly with increasing frequency, from about 1...10 kHz (Figure 4.29). On the other hand, the dissipation factor increases rapidly with frequency above 10 kHz. All these frequency effects limit the use of these capacitors, e.g. as coupling capacitors to ≤ 20 kHz. The losses, expressed by the ESR, increase at low temperatures.

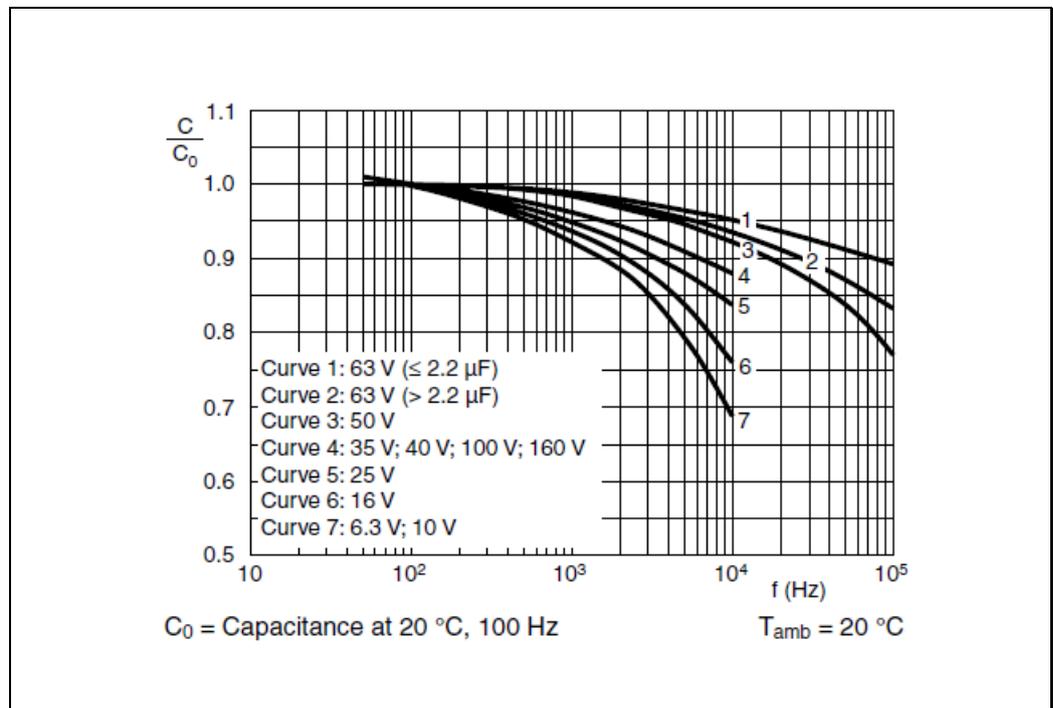


Figure 4.29 – Dependence on Frequency of an Electrolytic Capacitor

The leakage current is quite large (caused by the oxide film not being a perfect insulator), at 25°C , typically one to a few μA , and depends on time from application of a DC voltage. The leakage current increases significantly with applied voltage and with temperature. After long periods of storage, the oxide layer may partially dissolve in the electrolyte, and leakage may be catastrophic. To restore the oxide, the capacitor must be "reformed", by increasing very gradually the applied DC voltage, until the rated values are reached.

4.11.2 Applications of Aluminium Electrolytic Capacitors

These devices provide high capacitance in a small space at a low cost.

Electrolytic capacitors are polarised. If connected incorrectly, the insulating oxide film is not formed and there is no capacitance. Reverse connection eventually causes overheating and then failure.

Main advantages

- low cost
- large capacitance per unit volume
- very large capacitance values are available – to 1 F
- impedance and ESR can be small up to a few MHz for capacitances $< 100\mu\text{F}$

Main disadvantages

- poor tolerance on values (typically -20% ,+50%)
- capacitance very dependent on temperature, frequency, storage/usage time
- high losses, especially at frequencies ≥ 20 kHz
- large leakage current (μA), strongly dependent on temperature, voltage, time
- electrolytics are basically polarised capacitors (the peak of the AC voltage component must be less than the DC bias voltage to *make sure that the anode is at any instant positive relative to the cathode*), although nonpolarised versions (with half the volume efficiency), are produced

Main applications

- filtering in power supplies
- coupling, effective to about 20kHz
- decoupling, bypass, effective to a few MHz for $C < 100\mu\text{F}$

4.11.3 Tantalum Electrolytic Capacitors

Tantalum capacitors offer a form of capacitor that provides a very high capacity density – as much as three times better capacitance / volume efficiency than aluminium electrolytic capacitors. Tantalum capacitors are widely used in many mass produced items of electronics equipment where there is a need for a small size and a high level of capacitance.

There are three types of tantalum electrolytic capacitors: foil, liquid electrolyte and solid electrolyte.

Tantalum foil capacitors are made the same way as aluminium electrolytic capacitors. The dielectric is tantalum pentoxide (Ta_2O_5 with $\epsilon_r = 26$). They are available in sizes of 0.1 to 3000 μF , at voltages up to 450 V. The capacitance is less dependent on temperature than in Al electrolytics, but losses and leakage current, although smaller by one to two orders of magnitude than in Al electrolytics, are similarly temperature dependent. This type of tantalum capacitor is used only in the higher voltage ranges, as a better quality, but more expensive alternative to Al electrolytics. For lower voltage applications it compares unfavourably with the other two types of tantalum electrolytics, which use sintered anodes. As high voltage (above 100V) usage in electronics has decreased since the advent of transistors, this type of tantalum capacitor is seldom used.

Liquid electrolyte (wet slug) tantalum capacitors have the highest volumetric efficiency of any capacitor. This is due to the very large surface area possible in the porous tantalum pellet anode. The anode pellet is produced by sintering powdered tantalum around a tantalum lead wire with a technology ensuring a light, porous assembly. The anode pellet is then anodised to produce the thin Ta oxide layer. The anode is enclosed in a case, plated with silver and platinum and filled with electrolyte, which serves as a cathode. Due to the low reactivity of the tantalum oxide, electrolytes of high conductivity can be used, and the ESR is low.

The main application of liquid electrolyte tantalum capacitors is in power supply filters. The maximum voltage is typically 125 V and the capacitance ranges from 1 to 2000 μF . Although much more volume efficient, the liquid electrolyte tantalum capacitor has two main disadvantages over solid tantalum capacitors: possibility of leakage of the corrosive electrolyte; and the necessity to prevent the application of even short duration reverse voltage (it might lead to a catastrophic short-circuit, and sometimes explosion).

Solid tantalum capacitors are the variety that are most commonly used. The anode is produced the same way as that of the liquid electrolyte capacitor. After the pellet is anodised, several films of solid cathode material are produced by pyrolytic conversion of manganous nitrate solution into manganese dioxide, MnO_2 , which is a reasonably good conductor, serves as the first layer of the cathode, which is extended with colloidal graphite, then silver paint, to provide a good connection to the metal cathode.

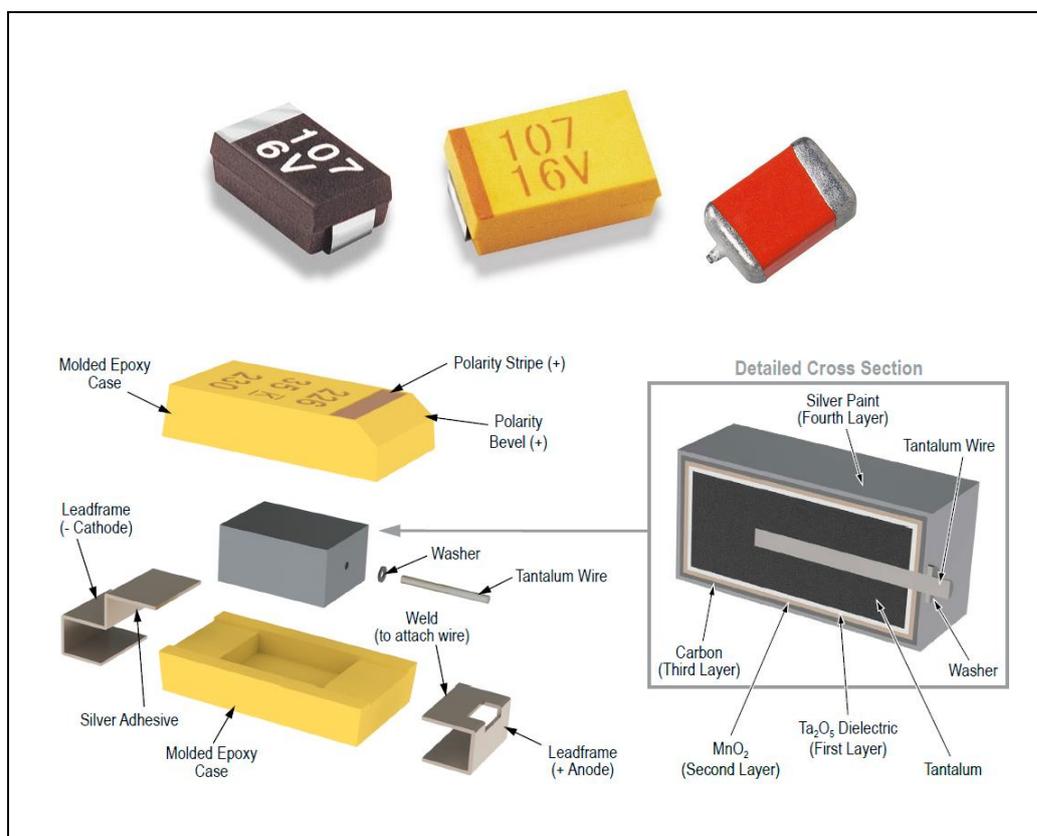


Figure 4.30 – Solid Tantalum Capacitors

4.11.4 Applications of Tantalum Capacitors

Tantalum capacitors offer many advantages over other types of capacitor. This has meant that their use has risen considerably over the years, and now they are widely used in all forms of electronics equipment. The advantages are:

- ***Volumetric efficiency:*** Tantalum capacitors offer a very high level of volumetric efficiency – much greater than many other types. In particular they are better than Al electrolytic capacitors which are their main rival.
- ***Good characteristics:*** The frequency response of tantalum capacitors is superior to that of Al electrolytic capacitors. They also have a lower series resistance and lower leakage current. This means that they are more suitable for use in a number of applications where Al electrolytics could not be used.
- ***High reliability:*** Tantalum capacitors are more reliable than many other forms of capacitor. Provided they are operated within their ratings they are able to provide an almost unlimited life. Their use is not time limited as in the case of Al electrolytic capacitors.
- ***Wide operating temperature range:*** Tantalum capacitors are able to operate over a very wide temperature range. They are often specified for operating over the range -55°C to $+125^{\circ}\text{C}$, with a variation as little as 10%. This makes them an ideal choice for use in equipment used in harsh environmental conditions.
- ***Compatibility with modern production methods:*** Modern production techniques often expose components to high temperatures during soldering as the whole assembly is heated by infra-red heat. Using conventional leaded components only the board surface was heated and the amount of heat conducted by the leads was usually insufficient to damage the components. Tantalum capacitors are able to withstand the temperatures of SMT production and are therefore ideal for use in many new electronics designs.

Tantalum capacitors have a number of disadvantages, and these need to be considered when using them in new designs.

- ***Low ripple current ratings:*** It is hardly surprising in view of their size that tantalum capacitors do not have a high ripple current rating. They should not normally be used in areas that require high levels of current to be passed.
- ***Not tolerant to reverse or excess voltage:*** Tantalum capacitors do not like reverse or excess voltage. Even spikes can destroy them. If they are exposed to excess or reverse voltages, then they can explode.
- ***More expensive than other types:*** Tantalum capacitors are more expensive than many other forms of capacitor. As a result their cost should be considered during the design phase as the other benefits may outweigh any increased costs.

4.12 Mica Capacitors

Silver mica capacitors are made by plating silver electrodes directly on to a mica dielectric. Several layers are used to achieve the required capacitance. Wires for the connections are added and then the whole silver mica capacitor assembly is encapsulated to provide protection.

Silver mica capacitors are able to provide very high levels of accuracy, stability and low loss. As a result, silver mica capacitors have found many uses in radio frequency applications, particularly for oscillator and filter circuits where their stability, accuracy and low loss (leading to high Q) were needed. Although not as widely used these days, they can still be obtained and are used where stability of value is of the utmost importance and where low loss is required.



The reason for the continued use of silver mica capacitors is the fact that they can offer very high levels of performance, better in many areas than any other type of capacitor. However in many applications, other more modern technologies provide levels of performance that meet the requirements.

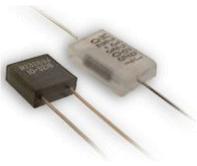
The particular properties of the silver mica capacitor are summarised below:

- **High accuracy:** Silver mica capacitors can be obtained with tolerance figures of $\pm 1\%$. This is much better than virtually every other form of capacitor available today.
- **Temperature co-efficient:** The temperature co-efficient of silver mica capacitors is much better than most other types of capacitor. The temperature coefficient is positive and is normally in the region 35 to 75 ppm / $^{\circ}\text{C}$, with +50 ppm / $^{\circ}\text{C}$ being an average value.
- **Value range:** Values for silver mica capacitors are normally in the range between a few picofarads up to two or possibly three nanofarads.
- **Low capacitance variation with voltage :** Silver mica capacitors exhibit very little voltage dependence.
- **High Q :** Silver mica capacitors have very high levels of Q that are almost independent of frequency.

4.13 Glass Capacitors

Glass capacitors are made from potash lead glass, drawn into a thin ribbon which is then stacked with alternate layers of aluminium foil. Alternate foils are then welded to lead wires, cover glass is added and the assembly sealed at high temperature.

Glass dielectric capacitors offer very high levels of performance, although their cost is high when compared to many other forms of capacitor. Typically a glass capacitor will have a relatively low capacitance value. The values of glass capacitors may range between a fraction of a picofarad up to two to three nanofarads. As such these capacitors are used mainly in radio frequency (RF) circuit design. The supply of glass capacitors is limited to a small number of manufacturers and suppliers. Glass capacitors offer several advantages over other types of capacitor:



- **Low temperature coefficient:** Glass capacitors have a low temperature coefficient. Figures of just over 100 ppm / °C are often obtained for these capacitors.
- **No hysteresis:** Some forms of capacitor exhibit hysteresis in their temperature characteristic. This is not the case for glass capacitors which follow the same temperature / capacitance curve when the temperature is rising and falling.
- **Zero ageing rate:** Many electronics components change their value with age as chemical reactions take place within the component. Glass capacitors do not exhibit this effect and retain their original value over long periods of time.
- **No piezoelectric noise:** Some capacitors exhibit the piezoelectric effect to a small degree. This can result in effects such as microphony (voltages caused by vibration) on oscillators. Where this could be a problem, the use of glass capacitors could help solve the problem.
- **Extremely low loss / high Q:** Glass capacitors are very low loss as there is virtually no dielectric loss. This enables very high Q circuits to be built using them provided the other components (e.g. inductors) are not lossy.
- **Large RF current capability:** Some capacitors are not able to withstand large values of current. This is not the case for glass capacitors which are suitable for use in RF high power amplifiers, etc.
- **High operating temperature capability:** Glass dielectric capacitors are able to operate at very high temperatures. Many are able to operate at temperatures up to about 200 °C without fear of damage or performance shortfall.

4.14 Choosing Capacitors

The following diagrams facilitate selection of capacitor types for specific applications. These diagrams should point to the types of capacitors that could be used for a given capacitance range, tolerance on value and frequency range. Of course, price and availability must also be considered. There is a wide overlap in specifications among the various families of capacitors.

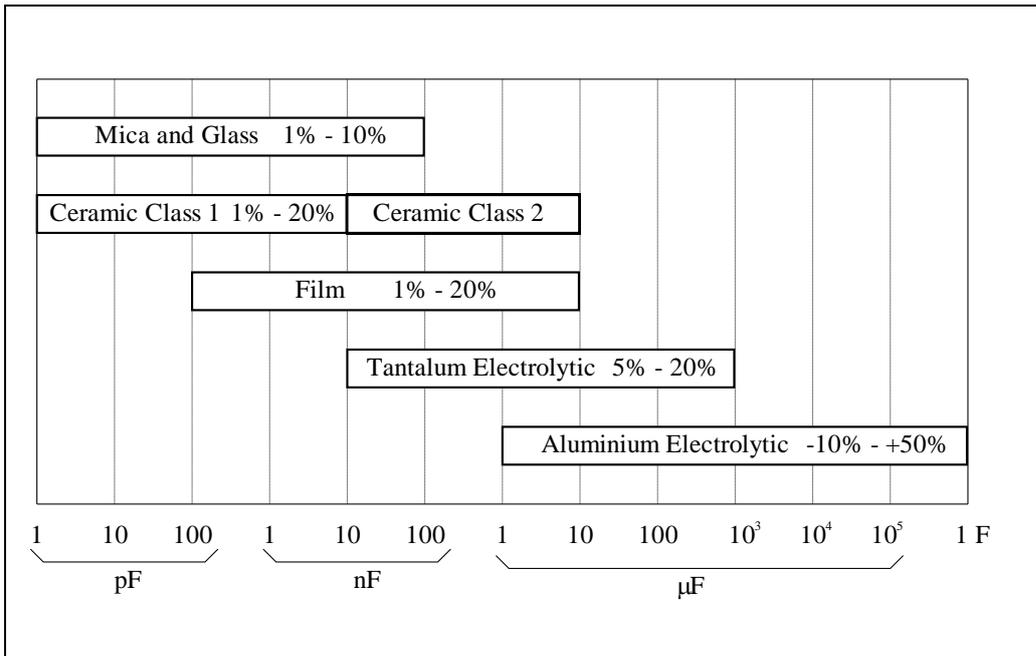


Figure 4.31 – Range of Capacitance Values and Tolerances for Different Capacitor Types

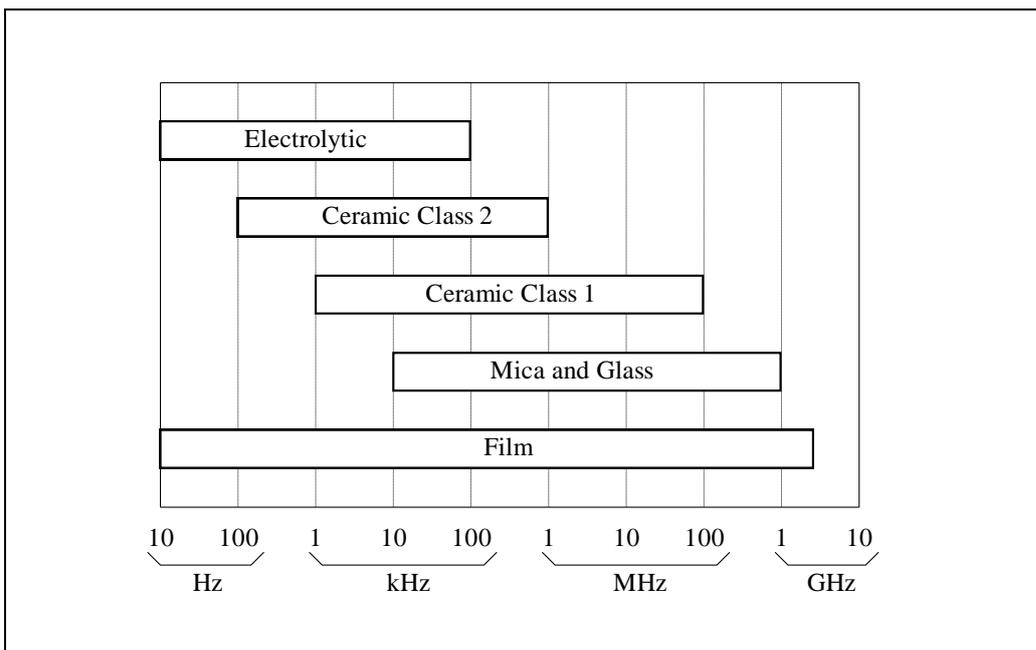


Figure 4.32 – Useful Frequency Ranges for Different Capacitor Types

Type	Advantages	Disadvantages
Polystyrene	Low cost Low DF Wide range of values Good stability Tolerance to 0.5%	Temperature < +85 °C Large case size for C > 10 nF Relatively high inductance
Polypropylene	Low cost Low DF Wide range of values Good stability Tolerance to 1%	Temperature < +105 °C Relatively large Relatively high inductance
Polycarbonate	Low cost Good stability Wide temp. range	Large High inductance
Polysulfone (Polyphenylene- sulfide)	Low cost Good stability Wide temp. range	Large High inductance
Polyethylene- terephthalate (Teflon)	Good stability Temperature > +125 °C Wide range of values Tolerance to 1%	Large High inductance High cost
Polyester	Low cost Small size	Moderate stability Temp. and frequency dependent Largest DF of film capacitors
Ceramic Class 1	Small size Low cost Good stability Wide range of values Tolerance to 1% Low inductance	DF larger than film capacitors at frequencies < 10 kHz
Ceramic Class 2	Wide range of values Low inductance	Poor stability Poor DF
Mica	Low loss at HF Low inductance Very stable Tolerance to 1%	Quite large Low values (< 10 nF) High cost
Aluminium Electrolytic	Large values Small size High voltages	High leakage Usually polarised Poor accuracy Poor stability Inductive
Tantalum Electrolytic	Large values Very small size Medium inductance Reliable	Quite high leakage Usually polarised High cost Poor stability Poor accuracy

Table 4.9 – Features of Common Capacitors

4.15 Decoupling Capacitors

One of the wider uses of capacitors, in both analog and digital circuits, is for '*decoupling*' or '*bypassing*' a power supply, an IC, or a resistor, by *shunting them with a negligibly small impedance at AC or pulse signal frequencies*.

High-speed or high-level circuits can inadvertently interact with low-level circuits on the same printed circuit board (PCB) by way of an impedance common to both circuits. Such a common impedance might be:

- the internal AC impedance of the DC power supply
- the impedance of the leads from the power supply to the positive supply or negative supply connecting pads on the PCB
- the ground connection from the PCB to the power supply
- the 'invisible resistance' of the ground track on the PCB, if it is common to high-level and low-level signals, or to digital and analog sub-circuits
- a voltage dropping resistor or divider, common to both sub-circuits
- the 'invisible resistance' of a common track on the PCB

To avoid as much as possible this type of parasitic coupling, one must take care to:

- carefully layout the PCB
- properly bypass the likely offending common impedances

A most likely offending impedance, that usually is common to many different subcircuits on a PCB, is the AC internal resistance of the DC power supply.

Voltage regulator IC's ensure that the power supply has a very small internal resistance, typically a few milliohms, but only to about 100 to 1000 Hz. Above this frequency, the AC internal impedance of the voltage regulator IC increases rapidly, because the inner control loop gain drops at higher frequencies. A small AC impedance of a power supply can be ensured above 1 kHz only by using suitable bypass capacitors.

For the decoupling to be effective, the bypass impedance must be maintained at a small value over a much wider frequency range than the actual useful signal frequency range. Parasitic coupling may happen not only at signal frequencies, but also at a very low or a very high frequency, well outside the useful signal band. And, as long as *there still is sufficient parasitic loop gain available in the circuit*, parasitic oscillations may occur. This can happen at a very low frequency (so called 'motor-boating') or at a high frequency (say MHz), where only an oscilloscope might help in detecting them.

The capacitors most used for decoupling are electrolytic capacitors, which, depending on value, can ensure a small impedance from very low frequencies, say a few Hz or tens of Hz up to, say, 0.1 MHz...10 MHz.

The minimum value of bypass impedance that a capacitor can provide is set by its equivalent series resistance, ESR. To assess the suitability of a capacitor for decoupling, particularly for decoupling switching power supplies, the ESR is as important as the capacitance value.

Of course, if the impedance of the capacitor, due to series resonance, instead of decreasing continuously with frequency ($X_C = 1/2\pi fC$) starts increasing above the *Series Resonant Frequency (SRF)*, the decoupling effect gradually disappears.

Of the different capacitor types, ceramic capacitors have usually the smallest inductances, and the highest *SRF* (typically to well over 10 MHz). Modern film capacitors are now also manufactured in low-inductance versions. Lead-less capacitors, e.g. SMC chip capacitors, have generally higher *SRFs* than the equivalent wire terminal capacitors. Electrolytic capacitors on the other hand, because of large capacitance values, have generally lower *SRF*.

The best practice, especially for decoupling the supply terminals of fast pulse circuits or high-frequency circuits, is to place a large capacitor – say an electrolytic of 10 μF or 100 μF in parallel with a smaller capacitor, say a 10 nF or 100 nF ceramic capacitor, across the positive supply to common and negative supply to common power supply input pads of the printed circuit board, as shown below:

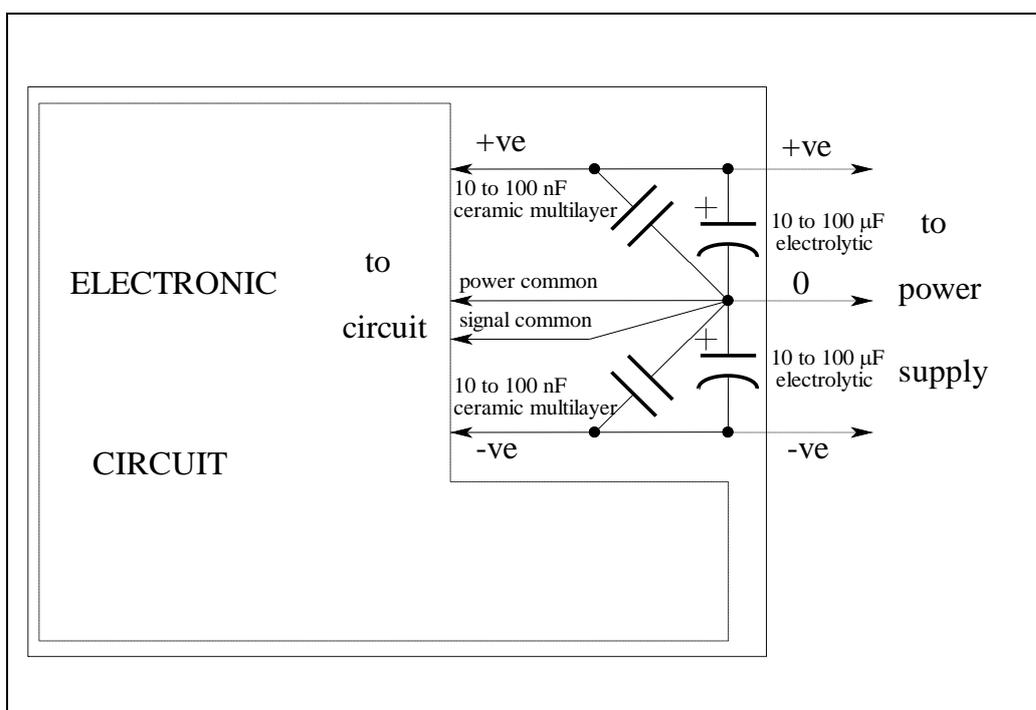


Figure 4.33 – Decoupling Power Supply Entry to a Printed Circuit Board

Note the single point star connection to the '0 V' of the power supply, the power common, and the signal common.

The two different capacitor types used in the decoupling would normally have quite different Series Resonant Frequencies, and would make sure that a small bypass impedance is maintained over a wider range of frequencies.

The large electrolytic capacitor will take care of decoupling, i.e. ensure a small impedance of a few $\text{m}\Omega$ to a few Ω , from about, say, 10 Hz, to about a few tens or hundreds of kHz. The ceramic capacitor will continue maintaining a small bypass impedance into the tens of MHz.

Besides decoupling the supply rails, for fast pulse/digital ICs it is strongly recommended that a small 10 nF to 100 nF ceramic capacitor should be placed directly across the supply and common pads of each IC on the PCB.

In any IC decoupling circuit, the capacitor is essentially a local energy source that supplies current to the chip during switching. Without bypassing, the impedance of the PCB tracks causes a voltage drop on the supply line. Depending on the frequency, the typical unbypassed dynamic impedance of the positive supply track can be about 50 to 100 Ω . This is enough to produce a considerable drop during short current pulses unless a bypass capacitor is used.

As an example, assume that a current swing of 300 mA with a duration of 3 ns is produced by a digital IC, i.e. it requires briefly a charge:

$$\Delta Q = I\Delta t = 0.3 \times 3 \times 10^{-9} = 0.9 \times 10^{-9} \text{ C}$$

If the voltage drop is to be limited to $\Delta V = 0.1 \text{ V}$, the bypass capacitor required is:

$$C = \Delta Q / \Delta V \approx 10 \text{ nF}$$

The recommended capacitor type would be a multilayer ceramic (typically class 2) capacitor.

Of course, if the pulse duration were to be longer, say about 30 ns in the pulse conditions of our example, the required capacitor would be about 100 nF, again multilayer ceramic.

To ensure bypassing as far as possible towards higher frequencies, as required for fast pulse circuits, the capacitor should be a leadless ceramic multi-layer (chip) capacitor, and the tracks on the PCB to the nodes to be bypassed should be as short as feasible. This bypass capacitor will also ensure that there is no electromagnetic radiation caused by large pulse currents through the PCB tracks.

4.16 References

Analog Devices: *Hardware Design Techniques*; Section 9 of Practical Analog Design Techniques, Analog Devices, 1996.

Becker, J.: *Understanding Passive Components*; viewed 6 March 2007, <www.epemag.com>.

Brokaw, P. & Barrow, J.: *Grounding for Low- and High-Frequency Circuits*, Analog Dialogue, 23-3 1989.

Greb, V. & Grasso, C.: *Don't let rules of thumb set decoupling capacitor values*; in EDN, 1 Sep 1995, pp 141-147.

Hyslop, M.S.: *Use Power Bypassing and Bussing for High-Performance Circuits*; in Electronic Design, 27 Sep 1990, pp 110-115.

Pecht, M., Lall, P., Ballou, G., Sankaran, C., Angelopoulos, N.: *Passive Components*; in The Electrical Engineering Handbook, CRC Press LLC, 2000

Piotech: *Data Sheet: Chip Resistors with Au-Terminations*; Rev. 1, viewed 6 March 2007 <www.piotech.com.tw>

Sinclair, I.R.: *Passive Components – a User's Guide*, B&H Newnes, Oxford, 1994.

Stere, R.: *Project A: Passive Components*, UTS, 1997.

Whitaker, J.C., editor-in-chief: *The Electronics handbook, Section II, Properties of Materials and Components*, CRC Press & IEEE Press, 1996, pp 135-182.

Wong, T.: *Choosing Capacitors*; Electronics World & Wireless World, April 1994, pp 327-329.

Yageo: *Components Data Books*: PA08 Fixed Resistors; PA05 Film Capacitors; PA06 Ceramic Capacitors; PA01 Electrolytic Capacitors. Yageo, Taiwan, 1994-1996.

5 Sensitivity, Various Responses

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5.1 Sensitivity

The need for sensitivity analysis

Our designs so far have been constructed with ideal elements. With this assumption, circuits that produce identical responses are interchangeable. In practice element values deviate from their nominal values due to tolerances related to their manufacture. Deviations will also be caused by chemical changes because of aging and changes caused by the environment such as temperature and humidity. A change in an element's value may affect specifications such as ω_0 , Q_0 and gain. Such effects will be important in choosing one circuit over another when implementing a filter.

We are interested in how a characteristic $y(x)$ depends on an element x . If the nominal value of x is x_0 , then variations in $y(x)$ produced by changes in x can be expressed as a Taylor series:

The change in a function expressed as a Taylor series

$$y(x_0 + \Delta x) = y(x_0) + \frac{\partial y(x_0)}{\partial x} \Delta x + \frac{\partial^2 y(x_0)}{\partial x^2} \frac{\Delta x^2}{2} + \dots \quad (5.1)$$

If Δx is small, then we may truncate the Taylor series so that we have the first order Taylor series approximation:

$$y(x_0 + \Delta x) \approx y(x_0) + \frac{\partial y(x_0)}{\partial x} \Delta x \quad (5.2)$$

or, equivalently:

The change in a function expressed as a first-order Taylor series approximation

$$\Delta y(x_0) \approx \frac{\partial y(x_0)}{\partial x} \Delta x \quad (5.3)$$

We are interested in relative, not absolute, changes in y and x , so we normalise Eq. (5.3) to get:

$$\frac{\Delta y(x_0)}{y(x_0)} \approx \left[\frac{\partial y(x_0)}{\partial x} \frac{x_0}{y(x_0)} \right] \frac{\Delta x}{x_0} \quad (5.4)$$

We now define the *relative sensitivity* as:

$$S_x^y = \frac{\partial y}{\partial x} \frac{x}{y} = \frac{\partial y/y}{\partial x/x} = \frac{\partial(\ln y)}{\partial(\ln x)} \quad (5.5) \quad \text{Relative sensitivity defined}$$

5.4

5.1.1 Properties of the Sensitivity Function

Relative sensitivity of independent variable products

Assume that y is a function of several variables, such as x_1 , x_2 and x_3 . One of the forms we will encounter most often is:

$$y = x_1^a x_2^b x_3^c \quad (5.6)$$

The natural logarithm of this function is:

$$\ln y = a \ln x_1 + b \ln x_2 + c \ln x_3 \quad (5.7)$$

Using Eq. (5.5), we then have:

$$S_{x_1}^y = a, \quad S_{x_2}^y = b, \quad S_{x_3}^y = c \quad (5.8)$$

If y is expressed as the product of two other functions, then:

$$S_x^y = \frac{\partial(\ln y_1 y_2)}{\partial(\ln x)} = \frac{\partial(\ln y_1)}{\partial(\ln x)} + \frac{\partial(\ln y_2)}{\partial(\ln x)} \quad (5.9)$$

That is, we have:

$$S_x^y = S_x^{y_1} + S_x^{y_2} \quad (5.10)$$

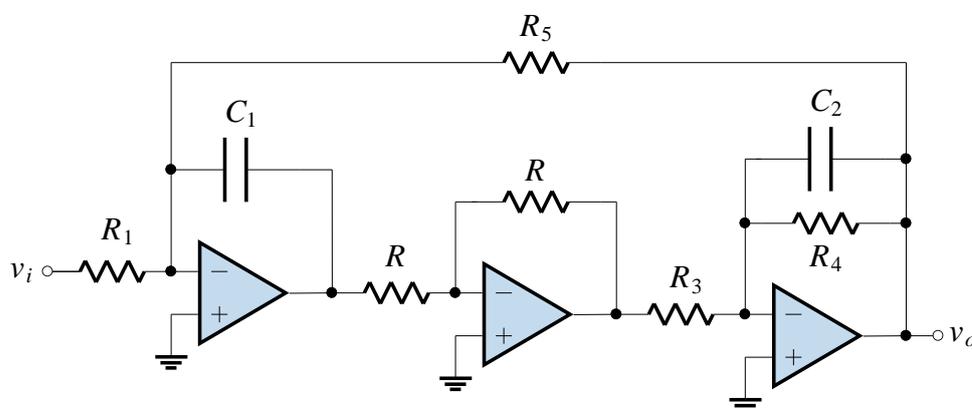
Relative sensitivity of dependent variable products and quotients

Similarly, if $y = y_1/y_2$, then:

$$S_x^y = S_x^{y_1} - S_x^{y_2} \quad (5.11)$$

EXAMPLE 5.1 Sensitivity Analysis of the Tow-Thomas Biquad

Consider the Tow-Thomas biquad circuit below:



We rewrite the original Tow-Thomas design equations in the form of Eq. (5.6):

$$\omega_0 = \frac{1}{\sqrt{R_3 R_5 C_1 C_2}} = R_3^{-1/2} R_5^{-1/2} C_1^{-1/2} C_2^{-1/2}$$

$$Q_0 = \sqrt{\frac{R_4^2 C_2}{R_3 R_5 C_1}} = R_4^1 C_2^{1/2} R_3^{-1/2} R_5^{-1/2} C_1^{-1/2}$$

$$H = \frac{R_5}{R_1} = R_5^1 R_1^{-1}$$

We can then write the sensitivities directly (using Eq. (5.6)), for example:

$$S_{R_3}^{\omega_0} = -1/2, \quad S_{R_5}^{\omega_0} = -1/2, \quad S_{C_1}^{\omega_0} = -1/2, \quad S_{C_2}^{\omega_0} = -1/2$$

ω_0 sensitivities of the lowpass biquad

This means that a change of 1% in any of the four circuit parameters would cause a decrease in ω_0 of 1/2%. A change in R_4 does not change ω_0 at all.

The Q_0 sensitivities have a range of values which are the exponents in

$$Q_0 = R_4^1 C_2^{1/2} R_3^{-1/2} R_5^{-1/2} C_1^{-1/2} :$$

$$S_{R_4}^{Q_0} = 1, \quad S_{C_2}^{Q_0} = 1/2, \quad S_{R_3}^{Q_0} = -1/2, \quad S_{R_5}^{Q_0} = -1/2, \quad S_{C_1}^{Q_0} = -1/2$$

Q sensitivities of the lowpass biquad

5.6

From $H = R_5^1 R_1^{-1}$, the sensitivities of the gain are:

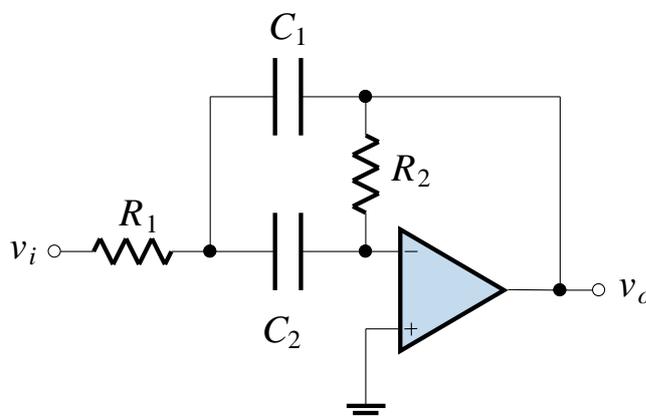
Gain sensitivities of
the lowpass biquad

$$S_{R_5}^H = 1, \quad S_{R_1}^H = -1$$

Values of $\frac{1}{2}$ and 1 for the sensitivity are very low. This analysis gives one of the reasons why the Tow-Thomas biquad circuit is popular: the sensitivity values are much lower than other circuits which have the same response.

EXAMPLE 5.2 Sensitivity Analysis of the Friend Circuit

Consider now the Friend circuit:



With $C_1 = C_2 = C$, the design equations are:

$$\omega_0 = \frac{1}{C\sqrt{R_1R_2}}, \quad Q_0 = \frac{1}{2}\sqrt{\frac{R_2}{R_1}}$$

or:

$$\omega_0 = C^{-1}R_1^{-1/2}R_2^{-1/2}, \quad Q_0 = 1/2R_2^{1/2}R_1^{-1/2}$$

From these equations, the sensitivities are:

$$S_C^{\omega_0} = -1, \quad S_{R_1}^{\omega_0} = -1/2, \quad S_{R_2}^{\omega_0} = -1/2$$

ω_0 sensitivities of
the Friend circuit

and:

$$S_{R_1}^{Q_0} = -1/2, \quad S_{R_2}^{Q_0} = 1/2$$

Q_0 sensitivities of
the Friend circuit

Again, all values are low, meaning that the Friend circuit has a very stable response.

5.1.2 Bode Sensitivity

Another sensitivity figure we use relates $\mathbf{T}(j\omega)$ to some circuit parameter, such as an element, Q_0 or ω_0 .

We begin with the definition of *Bode sensitivity*:

Bode sensitivity defined

$$S_x^{\mathbf{T}(\mathbf{s})} = \frac{x}{\mathbf{T}(\mathbf{s})} \frac{\partial \mathbf{T}(\mathbf{s})}{\partial x} \quad (5.12)$$

Under sinusoidal, steady-state conditions, we let $\mathbf{s} = j\omega$ so that:

$$\mathbf{T}(j\omega) = |\mathbf{T}(j\omega)| e^{j\theta(\omega)} \quad (5.13)$$

The Bode sensitivity becomes:

$$S_x^{\mathbf{T}(j\omega)} = \frac{x}{|\mathbf{T}(j\omega)| e^{j\theta(\omega)}} \frac{\partial}{\partial x} [|\mathbf{T}(j\omega)| e^{j\theta(\omega)}] \quad (5.14)$$

Show that this simplifies to:

$$S_x^{\mathbf{T}(j\omega)} = \frac{x}{|\mathbf{T}(j\omega)|} \frac{\partial}{\partial x} |\mathbf{T}(j\omega)| + jx \frac{\partial \theta(\omega)}{\partial x} \quad (5.15)$$

From this equation we see that:

Magnitude and phase sensitivity defined

$$S_x^{|\mathbf{T}(j\omega)|} = \operatorname{Re} S_x^{\mathbf{T}(j\omega)}, \quad S_x^{\theta(\omega)} = \frac{1}{\theta(\omega)} \operatorname{Im} S_x^{\mathbf{T}(j\omega)} \quad (5.16)$$

EXAMPLE 5.3 Sensitivity of a Second-Order Lowpass Transfer Function

We will calculate the sensitivity when $x = Q_0$ for a second-order lowpass transfer function:

$$\mathbf{T}(s) = \frac{\omega_0^2}{s^2 + (\omega_0/Q_0)s + \omega_0^2}$$

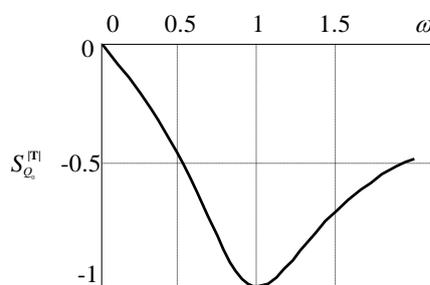
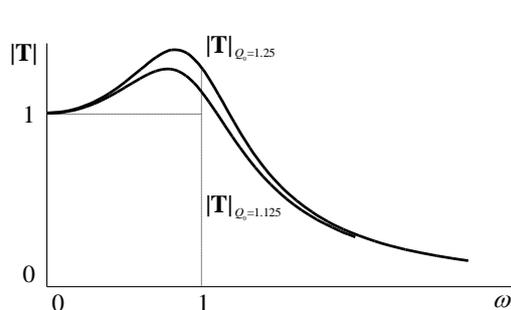
We compute the Bode sensitivity function:

$$S_{Q_0}^{\mathbf{T}(s)} = \frac{Q_0}{\mathbf{T}} \frac{\partial \mathbf{T}}{\partial Q_0} = \frac{-(\omega_0/Q_0)s}{s^2 + (\omega_0/Q_0)s + \omega_0^2}$$

We next let $s = j\omega$ and determine the real part of the resulting function:

$$S_{Q_0}^{|\mathbf{T}(j\omega)|} = \operatorname{Re} S_{Q_0}^{\mathbf{T}(j\omega)} = \frac{-\omega_0\omega/Q_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\omega_0\omega/Q_0)^2}}$$

This sensitivity can be graphed versus ω on a computer (Matlab, Excel) to show how Q_0 changes the magnitude response. For $Q_0 = 1.25$, frequency normalised plots of the magnitude and the sensitivity are shown below:



Magnitude sensitivity for changes in Q for a normalised lowpass second-order transfer function

We can always confirm our theoretical sensitivities by actually simulating the circuit with real components using PSpice and performing a Monte Carlo analysis.

5.2 Chebyshev Response

The idea behind the Chebyshev response

The Butterworth response meets (or exceeds) the passband specifications at a single point only (when $\omega = \omega_p$). At all other frequencies in the passband, the Butterworth response easily meets the specifications. One result of this is that the transition band is not as narrow as is needed in many applications. We now describe a different type of response, one in which the passband specification is “spread” across the passband frequencies.

The response that achieves this goal is called the Chebyshev response. The magnitude function for a Chebyshev response is given below:

Chebyshev magnitude response defined

$$|\mathbf{T}_n(j\omega)| = \frac{1}{\sqrt{1 + \varepsilon^2 C_n^2(\omega)}} \quad (5.17)$$

where:

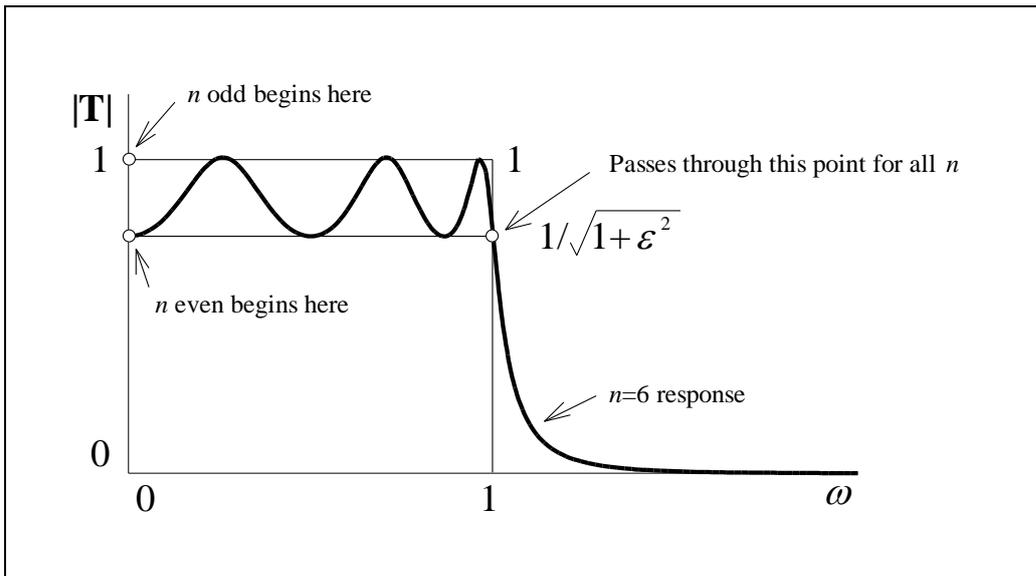
Chebyshev polynomial defined

$$C_n(\omega) = \cos(n \cos^{-1} \omega), \quad \omega \leq 1 \quad (5.18)$$

Show that the inverse cosine function becomes imaginary for $\omega > 1$. Hence show that:

$$C_n(\omega) = \cosh(n \cosh^{-1} \omega), \quad \omega > 1 \quad (5.19)$$

A plot of the Chebyshev response for $n = 6$, for one value of ϵ , is shown below:

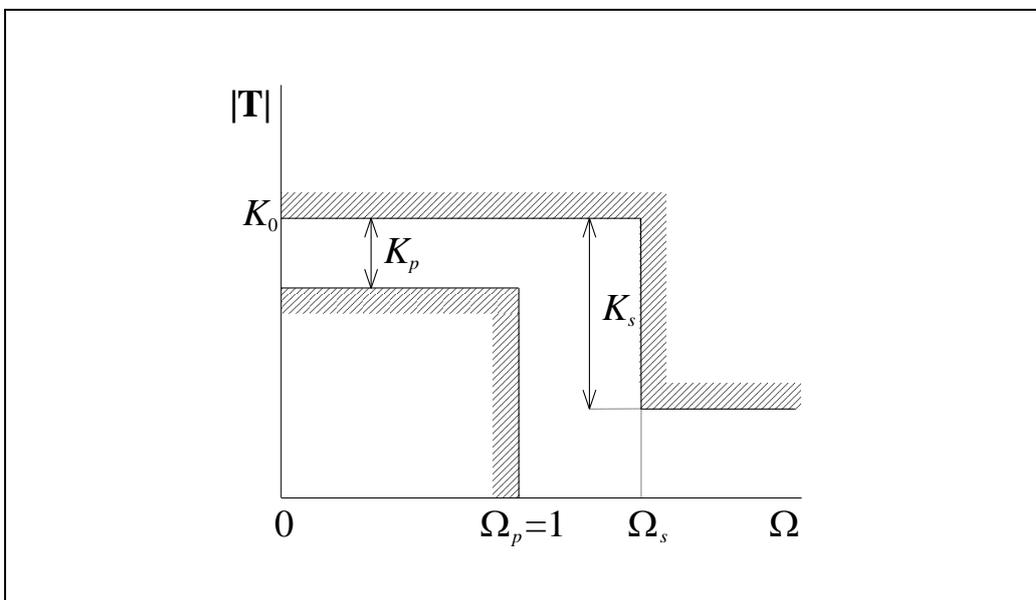


A typical Chebyshev magnitude response

Figure 5.1

Due to the rippling nature of the response in the passband, the Chebyshev response is said to have an *equal-ripple* characteristic. Unlike the Butterworth response, the end of the passband (or ripple band) is always $\omega = 1$. Therefore, we will discuss the Chebyshev filter design equations in terms of the following lowpass prototype specifications:

The Chebyshev passband is *equal-ripple*



Lowpass specifications for the Chebyshev response

Figure 5.2

5.12

Note that this specification allows for a maximum gain of K_0 dB in the passband. We seek the order n and the constant ε of the Chebyshev filter that will satisfy these specifications.

The maximum ripple in the passband, K_p dB, occurs when $C_n^2(\Omega) = 1$. We then have:

$$\begin{aligned} A &= 20 \log |\mathbf{T}(j\Omega)| \\ K_0 - K_p &= 20 \log H - 10 \log (1 + \varepsilon^2) \end{aligned} \quad (5.20)$$

and since:

$$K_0 = 20 \log H \quad (5.21)$$

then:

$$\varepsilon = \sqrt{10^{0.1K_p} - 1} \quad (5.22)$$

The Chebyshev parameter ε in terms of specifications

The minimum attenuation in the stopband must satisfy the following equation:

$$K_s = 10 \log [1 + \varepsilon^2 C_n^2(\Omega_s)] \quad (5.23)$$

Rearranging, we have:

$$\varepsilon^2 \cosh^2(n \cosh^{-1} \Omega_s) = 10^{0.1K_s} - 1 \quad (5.24)$$

Substituting Eq. (5.22) and taking the square root, we get:

$$\cosh(n \cosh^{-1} \Omega_s) = \sqrt{\frac{10^{0.1K_s} - 1}{10^{0.1K_p} - 1}} \quad (5.25)$$

If we define (as for the Butterworth response):

$$M = \sqrt{\frac{10^{0.1K_s} - 1}{10^{0.1K_p} - 1}} \quad (5.26)$$

M for the Chebyshev response (same as Butterworth)

then the order n for a Chebyshev response is:

$$n = \frac{\cosh^{-1} M}{\cosh^{-1} \Omega_s} \quad (5.27)$$

n defined for the Chebyshev response (different to Butterworth)

The pole locations for the Chebyshev response are located on an ellipse. The pole locations are given by $\mathbf{S}_k = \Sigma_k \pm j\Omega_k$, where:

$$v = \frac{1}{n} \sinh^{-1} \left(\frac{1}{\epsilon} \right) \quad (5.28)$$

$$\Sigma_k = -\sinh(v) \cos(\psi_k) \quad (5.29)$$

The pole locations for the Chebyshev response

$$\Omega_k = \cosh(v) \sin(\psi_k) \quad (5.30)$$

and ψ_k are the familiar Butterworth angles obtained by treating the filter as an n th order Butterworth filter.

For complex poles, we then convert to the standard second-order form by:

$$\Omega_{0k} = \sqrt{\Sigma_k^2 + \Omega_k^2} \quad (5.31)$$

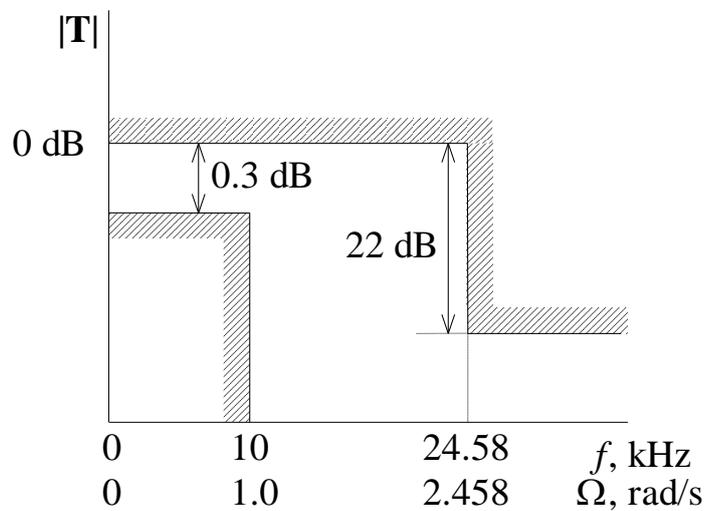
$$Q_{0k} = \frac{\Omega_0}{-2\Sigma_k} \quad (5.32)$$

Complex pole locations in terms of standard second-order quantities

5.14

EXAMPLE 5.4 Design of a Chebyshev Lowpass Filter

We require a lowpass filter to satisfy the specifications given below:



The normalised frequencies are also shown in the figure above with the end of the ripple band indicated as $\Omega = 1$. We first compute M :

$$M = \sqrt{\frac{10^{0.1 \times 22} - 1}{10^{0.1 \times 0.3} - 1}} = 46.93$$

We then find the order of the filter:

$$n = 2.993, \text{ round up to } 3$$

Compare this with $n = 5$ if we had used a Butterworth response. Next we use the Butterworth angles ψ_k for $n = 3$ and compute ε and the pole locations:

$$\varepsilon = 0.2674, \quad v = \frac{1}{3} \sinh^{-1} \left(\frac{1}{0.2674} \right) = 0.6765$$

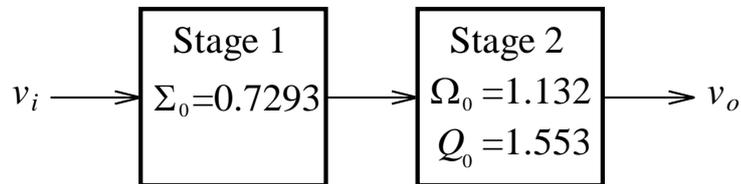
$$\mathbf{p}_1, \mathbf{p}_2 = -0.3646 \pm j1.072$$

$$\mathbf{p}_3 = -0.7293$$

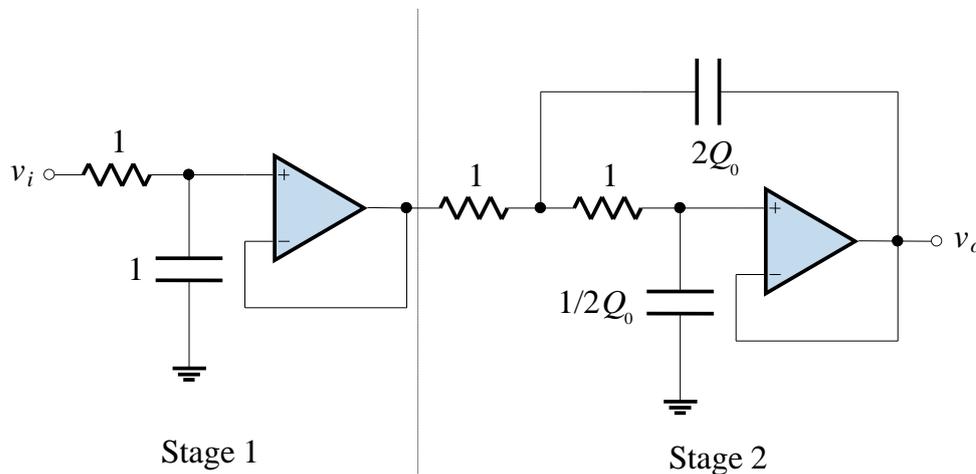
For the complex conjugate poles, we have:

$$\Omega_0 = 1.132 \quad \text{and} \quad Q_0 = 1.553$$

In block diagram form, we have the following implementation:



We can use a simple *RC* first-order circuit for the first stage, and a Sallen-Key circuit to realise the second stage:



Two applications of frequency scaling are required. The first will undo the normalising that made the end of the ripple band $\Omega = 1$. This applies to both stages. We then have to denormalise each stage separately since their pole locations did not lie on the unit circle.

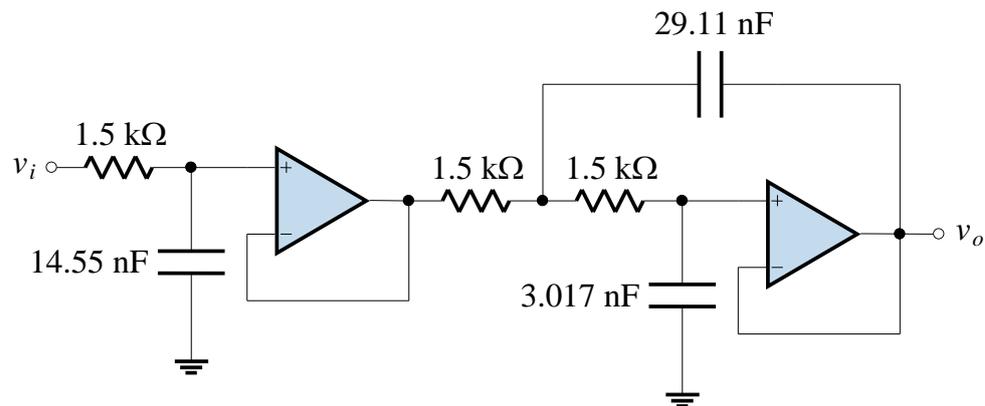
Chebyshev filter stages require two frequency scaling factors

5.16

Since there was no requirement on capacitor or resistor values, we arbitrarily choose $k_m = 1500$ to obtain reasonable element values. The following table shows the various scaling factors:

Stage	Scaling Factors		
	k_{f1}	k_{f2}	k_m
1	$2\pi \times 10^4$	0.7293	1500
2	$2\pi \times 10^4$	1.1323	1500

Using these scaling factors, we obtain the circuit below, and the design is complete.



Chebyshev and Butterworth filters compared

The Chebyshev filter's advantages over the Butterworth filter are that it has higher attenuation in the stop band and steeper rolloff near the cutoff frequency. It's disadvantages are that it requires higher values of Q_0 than the Butterworth filter which leads to difficulties in achieving circuit realisations (see the Sensitivity section) and it's phase characteristic is more nonlinear.

5.3 Inverse Chebyshev Response - Overview

A popular filter response is the inverse Chebyshev response. The inverse Chebyshev response can be generated in three steps. Starting with the magnitude of the lowpass Chebyshev response squared:

$$|\mathbf{T}_{C_n}(j\omega)|^2 = \frac{1}{1 + \varepsilon^2 C_n^2(\omega)} \quad (5.33)$$

we subtract this from 1, giving:

$$1 - |\mathbf{T}_{C_n}(j\omega)|^2 = \frac{\varepsilon^2 C_n^2(\omega)}{1 + \varepsilon^2 C_n^2(\omega)} \quad (5.34)$$

We then apply a highpass to lowpass transformation by replacing ω with $1/\omega$.

The inverse Chebyshev response is then the square root of this expression:

$$|\mathbf{T}_{IC_n}(j\omega)| = \frac{\varepsilon C_n(1/\omega)}{\sqrt{1 + \varepsilon^2 C_n^2(1/\omega)}} \quad (5.35)$$

Inverse Chebyshev
magnitude response
defined

These three steps are shown graphically in the figure below:

The steps leading from the Chebyshev magnitude response to the inverse Chebyshev magnitude response

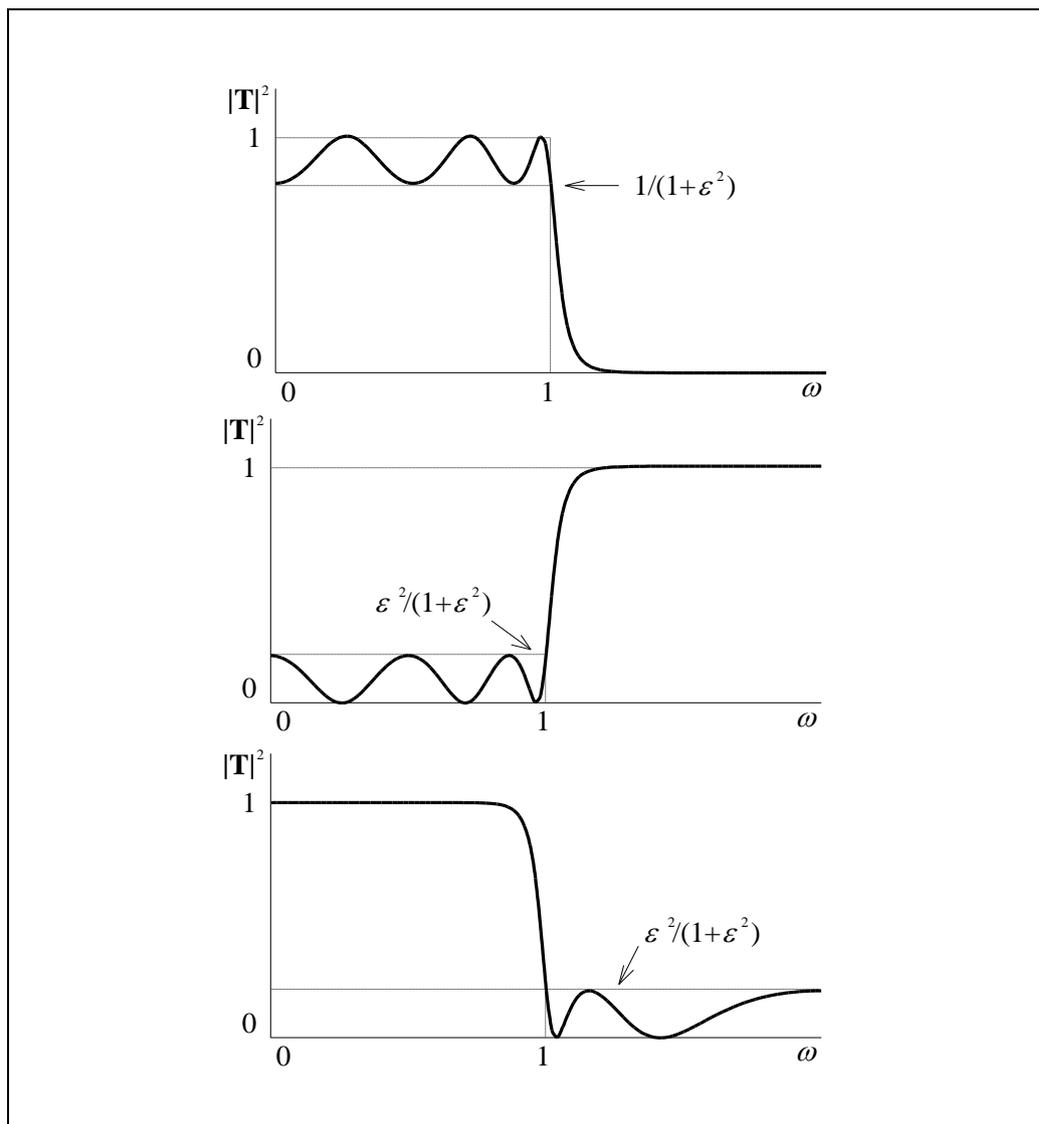


Figure 5.3 – Steps leading to the Inverse Chebyshev Response

The inverse Chebyshev response has zeros as well as poles

The poles of the inverse Chebyshev response are just the inverse of the poles for the Chebyshev response. Notice also that there are points in the stopband where the response is zero. This means that the inverse Chebyshev response also has zeros on the imaginary axis.

From Figure 5.3, we can see that we should always frequency scale a specification so that the stopband has a frequency of $\Omega_s = 1$. The order of an inverse Chebyshev filter is given by an equation similar to Eq. (5.27).

It can be shown that under certain conditions, the inverse Chebyshev response is “flatter” than the Butterworth response in the passband. The inverse Chebyshev response is equal-ripple in the stopband.

The inverse Chebyshev response can be “flatter” than the Butterworth response

5.3.1 Comparison of Butterworth, Chebyshev and Inverse Chebyshev Responses

Now that we have seen a few responses, it is interesting to examine the advantages and disadvantages of each one, and so determine the factors leading to the “best” filter for a given application.

We will use the following specification as an example:

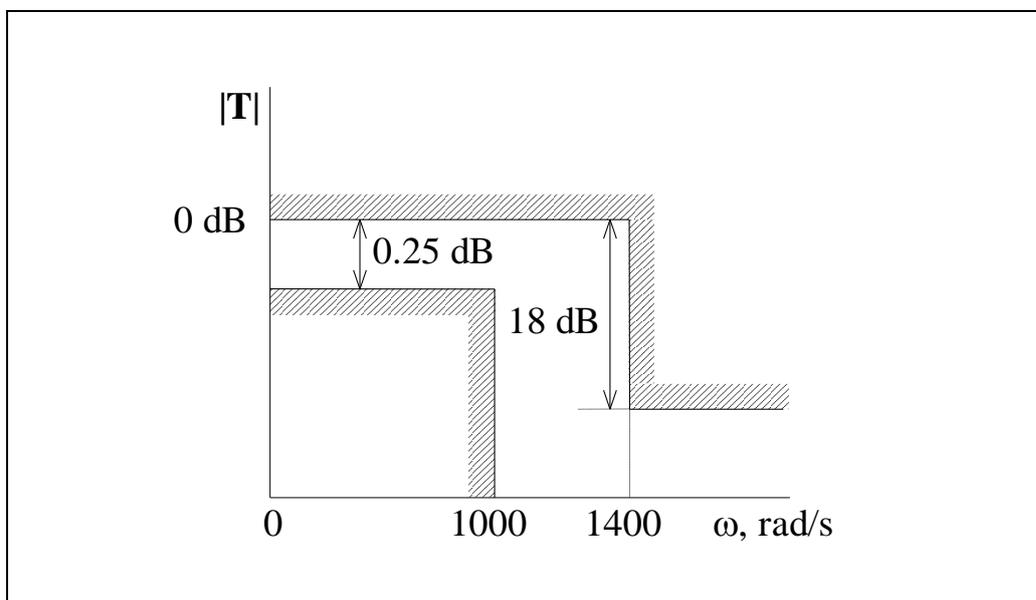


Figure 5.4

Since $\Omega = 1$ is the 3 dB frequency for a Butterworth response, the edge of the passband for a Chebyshev response, and the edge of the stopband for an inverse Chebyshev response, we will frequency scale so that for all responses we have $\Omega = 1$ as the 3 dB frequency (for response comparison purposes only).

To compare filter responses, frequency scaling is required

5.20

A plot of the three responses for this special case is shown below:

Butterworth, Chebyshev and inverse Chebyshev magnitude response stopbands compared

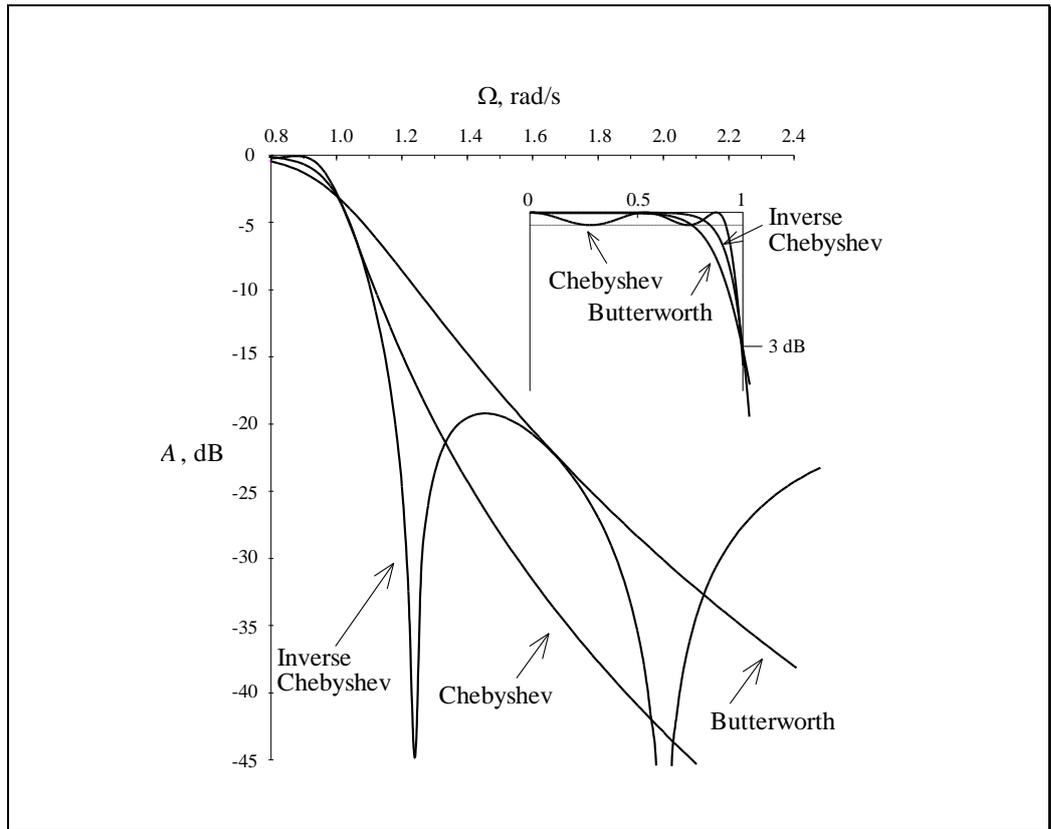


Figure 5.5

A close view of the responses near $\Omega = 1$ is shown below:

Butterworth, Chebyshev and inverse Chebyshev magnitude response pass bands compared

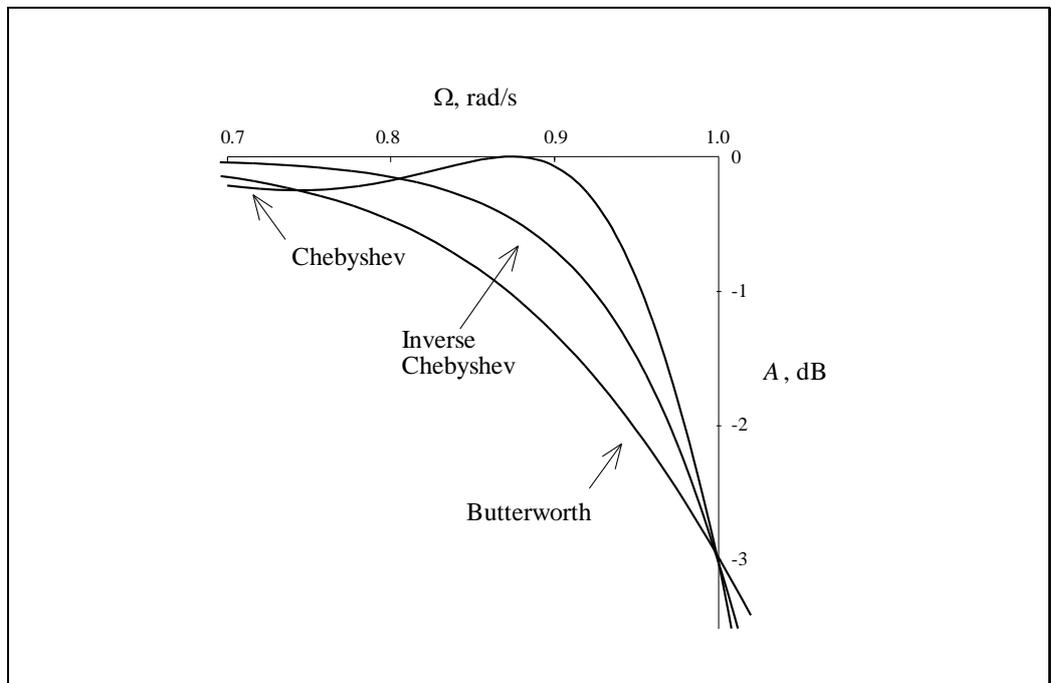


Figure 5.6

From the previous two figures, we will make some generalisations about the responses.

Generalising the characteristics of the Butterworth, Chebyshev and inverse Chebyshev magnitude responses

1. Passband Response

The inverse Chebyshev is always better than the Butterworth, and it is better than the Chebyshev except near $\Omega = 1$. For low frequencies the inverse Chebyshev is always the best of the three.

2. Stopband Response

The inverse Chebyshev response meets the K_s specification at the lowest frequency. The Butterworth and Chebyshev responses have greater attenuation than the inverse Chebyshev response for higher values of frequency. This is generally not significant if K_s is the minimum value of attenuation required for the entire stopband.

3. Transition Band

The inverse Chebyshev response has the most rapid change from K_s to K_p , or the sharpest cutoff. The Butterworth response is the worst, with the Chebyshev in between.

4. The Q_0 Required

The Q_0 for the inverse Chebyshev response is the same as the Q_0 required for the Chebyshev response if ε is the same, and both are greater than the Q_0 required for the Butterworth response.

5. Circuit Realisations

The Butterworth and Chebyshev responses can be realised with simple first- and second-order circuits (such as the biquad or Sallen-Key circuit). The inverse Chebyshev, due to its zeros on the imaginary axis, requires the use of lowpass notch circuits which are complicated and difficult to design (by hand).

5.4 Cauer (Elliptic) Response - Overview

The Cauer response has the smallest transition band

The response due to Cauer is equal ripple in both the pass and stop bands. The equal ripple property achieves the fastest transition from pass to stop bands for a given order n . Conversely, the Cauer filter requires a smaller n than any of the other three responses to meet the transition band requirements.

The design of a Cauer filter is quite complicated and involves elliptic functions.

A typical Cauer response is shown below:

A typical Cauer response showing equal ripple pass and stopbands

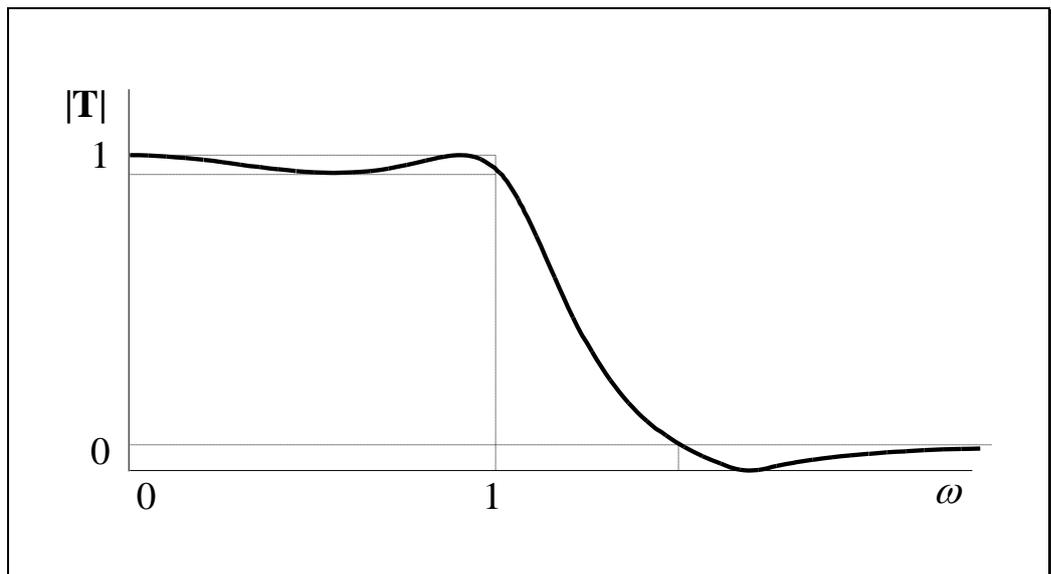


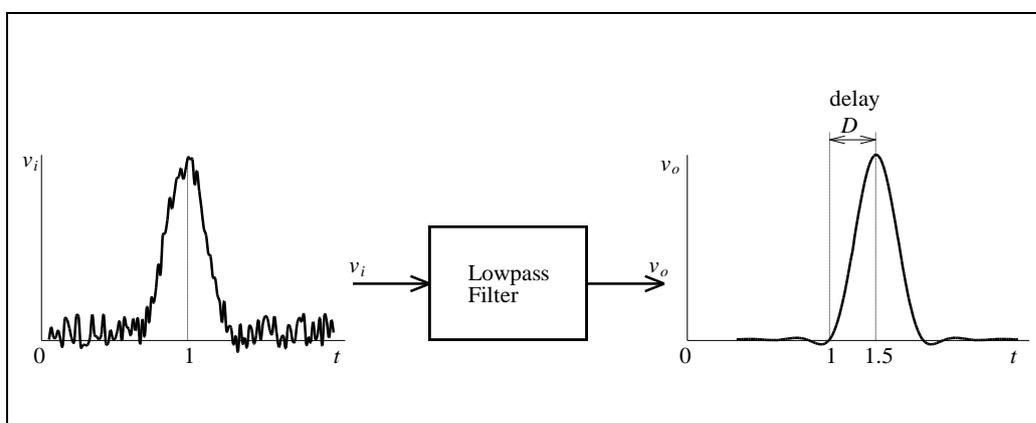
Figure 5.7

5.5 Phase Responses

Most filter specifications deal with the magnitude response. In systems where the filter is designed to pass a particular “waveshape”, phase response is extremely important. For example, in a digital system we may be sending 1’s and 0’s using a specially shaped pulse that has “nice” properties. At the receiver it is passed through a lowpass filter to remove high frequency noise. The filter introduces a delay of D seconds, but the output of the filter is as close as possible to the desired pulse shape.

A particular phase response is crucial for retaining a signal’s “shape”

This is illustrated below:



A filter introducing delay, but retaining signal “shape”

Figure 5.8

To the pulse, the filter just looks like a delay. We can see that distortionless transmission through a filter is characterised by a constant delay of the input signal:

$$v_o(t) = K v_i(t - D)$$

(5.36) Distortionless transmission defined

In Eq. (5.36) we have also included the fact that all frequencies in the passband of the filter can have their amplitudes multiplied by a constant without affecting the waveshape. Also note that Eq. (5.36) applies only to frequencies in the passband of a filter - we do not care about any distortion in a stopband.

5.24

We will now relate these distortionless transmission requirements to the phase response of the filter. From Fourier analysis we know that any periodic signal can be decomposed into an infinite summation of sinusoidal signals. Let one of these be:

$$v_i = A \cos(\omega t + \phi) \quad (5.37)$$

From Eq. (5.36), the output of the filter will be:

$$\begin{aligned} v_o &= KA \cos[\omega(t - D) + \phi] \\ &= KA \cos(\omega t - \omega D + \phi) \end{aligned} \quad (5.38)$$

The input and output signals differ only by the gain K and the phase angle which is:

Distortionless transmission requires a linear phase

$$\theta = -D\omega \quad (5.39)$$

That is, the phase response must be a straight line with negative slope that passes through the origin (or $\pm n\pi$).

In general, the requirement for the phase response in the passband to achieve distortionless transmission through the filter is:

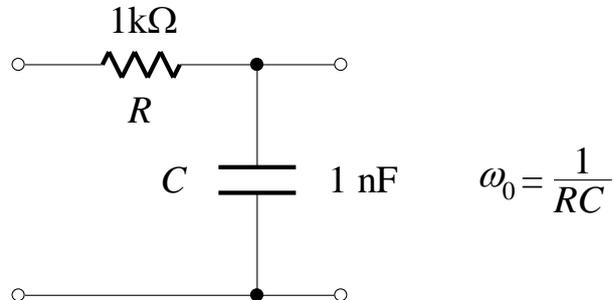
Group delay defined

$$\frac{d\theta}{d\omega} = -D \quad (5.40)$$

The delay D in this case is referred to as the *group delay*. (This means the group of sinusoids that make up the waveshape have a delay of D).

EXAMPLE 5.5 Distortionless Transmission through an RC Lowpass Filter

We would like to determine the maximum frequency for which transmission is practically distortionless in the following simple filter:



We would also like to know the group delay caused by this filter.

We know the magnitude and phase response:

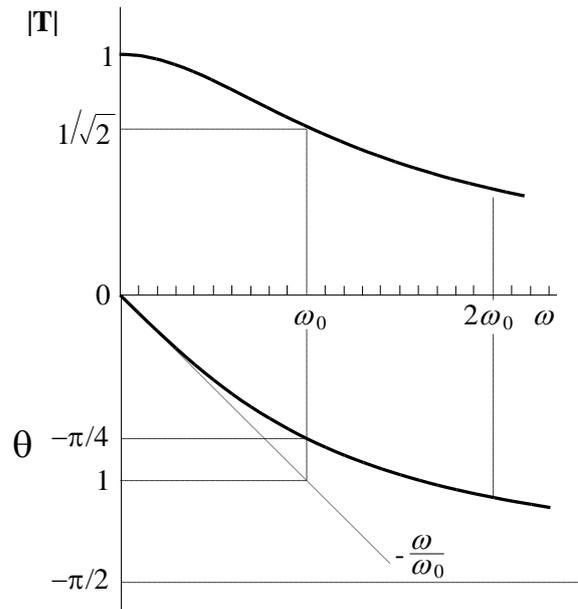
$$|\mathbf{T}(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}$$

$$\theta(\omega) = -\tan^{-1}(\omega/\omega_0)$$

5.26

These responses are shown below:

The deviation from linear phase and constant magnitude for a simple first-order filter



Suppose we can tolerate a deviation in the magnitude response of 1% in the “passband”. We then have:

$$\frac{1}{\sqrt{1+(\omega/\omega_0)^2}} \geq 0.99$$

$$\omega \leq 0.1425\omega_0$$

Also, suppose we can tolerate a deviation in the delay of 1% in the “passband”. We first find an expression for the delay:

$$D = -\frac{d\theta}{d\omega} = \frac{1}{1+(\omega/\omega_0)^2} \frac{1}{\omega_0}$$

and then impose the condition that the delay be within 1% of the delay at DC:

$$\frac{1}{1+(\omega/\omega_0)^2} \geq 0.99$$

$$\omega \leq 0.1005\omega_0$$

Considering the restrictions on ω imposed the constraints given above, we must have $\omega \leq 0.1\omega_0$ for practically distortionless transmission. The delay for $\omega \leq 0.1\omega_0$, according to the equation for D above, is approximately given by:

$$D \approx \frac{1}{\omega_0}$$

Approximate group delay for a first-order lowpass circuit

For the values shown in the original RC circuit, the group delay is approximately $1 \mu\text{s}$. In practice, variations in the magnitude transfer function up to the half-power frequency are considered tolerable (this is the bandwidth B of the filter). Over this range of frequencies, the phase deviates from the ideal linear characteristic by at most $1 - \pi/4 = 0.2146$ radians (see the graph of θ above). Frequencies well below ω_0 are transmitted practically without distortion, but frequencies in the vicinity of ω_0 will suffer some distortion.

There is a special class of filters, called *Bessel-Thomson* filters, which achieve a maximally flat delay in their passband. The magnitude response is also flat, but it starts rolling off at an earlier frequency than the Butterworth response.

5.6 Delay Equalisation

A way to achieve distortionless transmission through a filter is to use delay equalisation. The technique is as follows:

The idea behind delay equalisation

1. A filter is designed to provide the desired magnitude response.
2. The phase of the resulting filter is then supplemented over a frequency range of importance by a cascade of first or second-order allpass filters.

5.6.1 Allpass Filters

Allpass filters change the phase only

Allpass filters, as their name suggests, do not affect the magnitude response. They only contribute to the overall phase response. Consider the following circuit:

A first-order allpass circuit

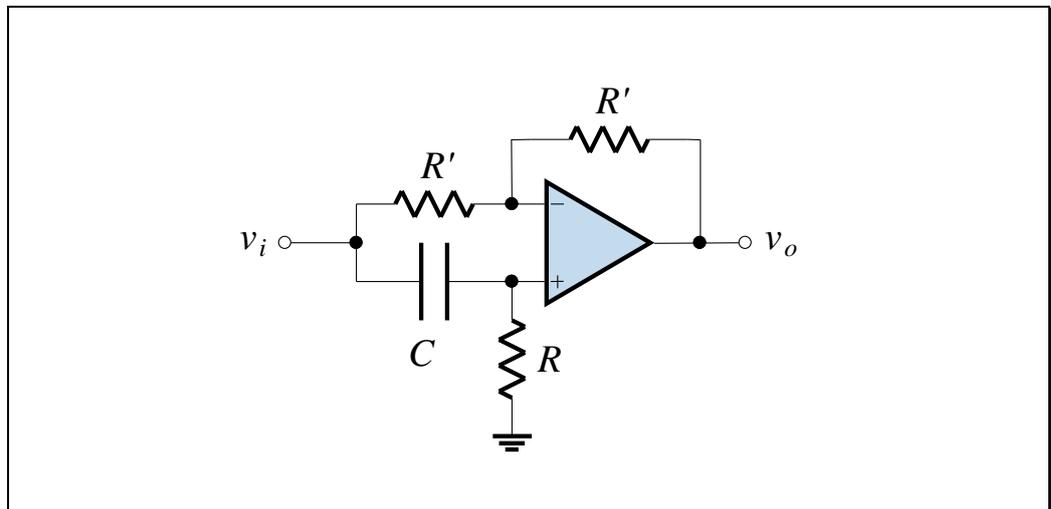


Figure 5.9

Show that the transfer function is given by:

$$\mathbf{T}_1(\mathbf{s}) = \frac{\mathbf{s} - 1/RC}{\mathbf{s} + 1/RC} \quad (5.41)$$

If we let:

$$\sigma_1 = 1/RC \quad (5.42)$$

then the transfer function is:

$$\mathbf{T}_1(\mathbf{s}) = \frac{\mathbf{s} - \sigma_1}{\mathbf{s} + \sigma_1}$$

(5.43) Standard form for a first-order allpass transfer function

For this function, we have:

$$|\mathbf{T}_1(j\omega)| = 1$$

(5.44)

and:

$$\theta(\omega) = 180^\circ - 2 \tan^{-1}(\omega/\sigma_1)$$

(5.45)

The delay is:

$$D_1(\omega) = -\frac{d\theta}{d\omega} = \frac{2/\sigma_1}{1 + (\omega/\sigma_1)^2}$$

(5.46) The delay introduced by a first-order allpass filter

EXAMPLE 5.6 Delay Equalisation

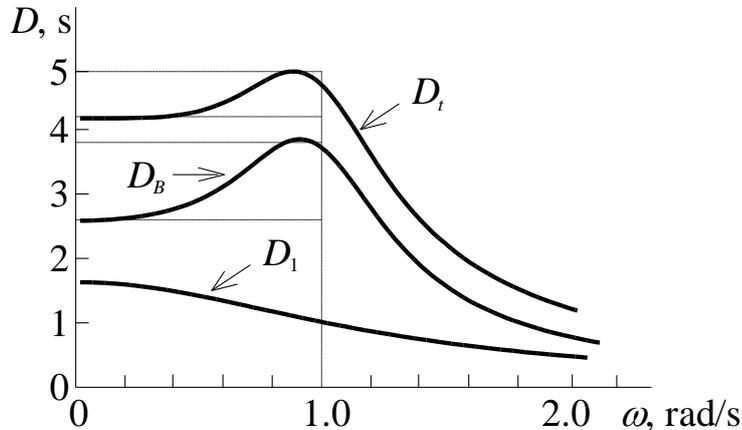
Suppose we have a fourth-order Butterworth lowpass filter which needs equalisation. If we use a first-order allpass circuit to equalise the delay, then the total delay will be:

$$D_t(\omega) = D_B(\omega) + D_1(\omega)$$

Our objective is to select D_1 so that D_t is maximally flat at $\omega = 0$. Accomplishing this objective is complicated, but the constraint on σ_1 turns out to be, for a Butterworth filter of order n :

$$\sigma_1 = \left(2 \sin \frac{3\pi}{2n} \right)^{1/3}$$

Therefore, for $n = 4$, we find that $\sigma_1 = 1.227$. We use the circuit in Figure 5.9, which is designed using Eq. (5.42), and frequency scaled with the same value that the Butterworth filter is scaled. The plots of the three delays are given below:



Delay equalisation of a normalised fourth-order Butterworth filter using a first-order allpass filter in cascade

Delay equalisation introduces additional group delay

From the curves we see that the maximum change in D_B is 1.3 for the Butterworth delay response, but the addition of the allpass section results in $\Delta D_t = 0.75$, a significant improvement. We can also see that a flattening of the delay curve came at the expense of increased delay through the system.

Delay equalisation is normally performed by higher order allpass filters

We can perform “better” equalisation with higher-order allpass filters. Such filters can be made up as a cascade of first- and second-order allpass filters.

5.7 References

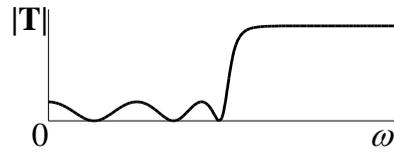
Lathi, B. P.: *Modern Digital and Analog Communication Systems*, Holt-Saunders, Tokyo, 1983.

Van Valkenburg, M. E.: *Analog Filter Design*, Holt-Saunders, Tokyo, 1982.

Quiz

Encircle the correct answer, cross out the wrong answers. [one or none correct]

1.



The response shown is:

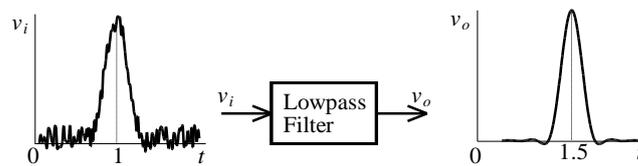
- (a) Inverse Chebyshev (b) Cauer (c) Chebyshev

2.

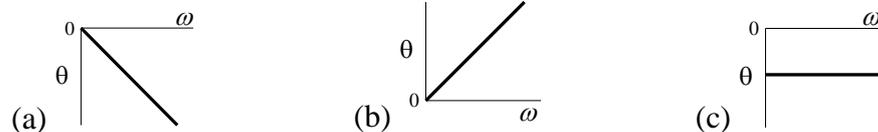
For a normalised Chebyshev lowpass filter with $0 < \varepsilon < 1$, the half-power frequency is:

- (a) $\cos\left[\frac{1}{n} \cos^{-1}(\varepsilon)\right]$ (b) $\cosh\left[\frac{1}{n} \cosh^{-1}\left(\frac{1}{\varepsilon}\right)\right]$ (c) $\cosh\left[n \cosh^{-1} \frac{1}{\varepsilon}\right]$

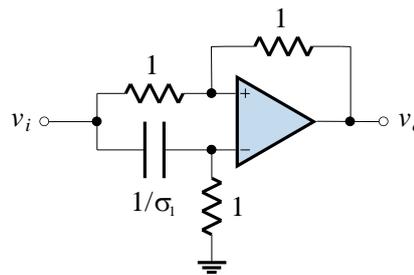
3.



For the filter shown, the phase response in the passband would resemble:



4.



The group delay $D(\omega)$ of the circuit shown is:

- (a) $\frac{1/\sigma_1}{1+(\omega/\sigma_1)^2}$ (b) $\frac{2/\sigma_1}{\sqrt{1+(\omega/\sigma_1)^2}}$ (c) $\frac{1}{\sigma_1} \tan^{-1}(\omega/\sigma_1)$

5.

For a second-order lowpass transfer function, $S_{\omega_0}^{|T(j\omega)|}$ at $\omega = \omega_0$ is:

- (a) 1 (b) $-2Q_0$ (c) 0

Answers: 1. a 2. b 3. a 4. b 5. a

Exercises

1.

For the Sallen-Key circuit shown in Exercise 4 of the Lecture 2 notes, derive the following sensitivities: $S_x^{\omega_0}$, $S_x^{Q_0}$, where $x = R_1, R_2, C_1, C_2$.

2.

Convert all your Butterworth filters from previous problems into Chebyshev filters. Compare the results using Altium Designer.

3.

Equalise the delay in the Butterworth lowpass filter you designed for Exercise 3 of the Topic 2 notes. Confirm your design by examining the phase response using Altium Designer.

6 Electromagnetic Compatibility

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6.1 Principles of EMC

Electromagnetic compatibility refers to the capability of two or more electrical devices to operate simultaneously without interference. A system that is electromagnetically compatible therefore satisfies the following criteria:

1. It does not cause interference with other systems.
2. It is not susceptible to emissions from other systems.
3. It does not cause interference with itself.

Additionally the system must meet regulatory requirements.

The interference that EMC refers to is known as electromagnetic interference (EMI). A common name for EMI is “noise”. Noise can be natural (e.g. lightning, solar disturbances) or human (e.g. relays, radio, radar) in origin.

One class of EMI is called radio frequency interference (RFI). RFI is propagated as an electromagnetic wave at radio frequencies (e.g. ignition coils, switch arcs).

There are always three elements involved in a noise problem: a *noise source* (line transients, relays, magnetic field etc.) a *coupling medium* (capacitance, inductance, wire, air) and a *receiver* (a circuit that is susceptible to the noise).

The EMC problem

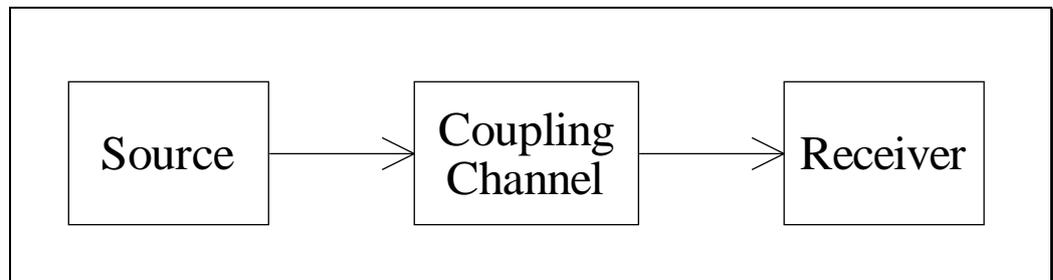


Figure 6.1 – Basic Composition of the EMC Problem

To solve a noise problem (make something electromagnetically compatible), one or more of these elements must be removed, reduced or diverted. Their role in the problem must be understood before the problem can be solved.

6.2 Types of Sources

Noise in any electronic system can originate at a large number of sources, including digital circuits, power supplies, adjacent equipment; noise sources can even include improperly connected shields and ground wires that were intended to combat noise.

6.2.1 Supply Line Transients

Anything that is switched will cause a transient on the supply lines. This can range from a digital circuit switching between a high and low state, or an appliance connected to a GPO.

The basic mechanism behind supply line transients is shown below:

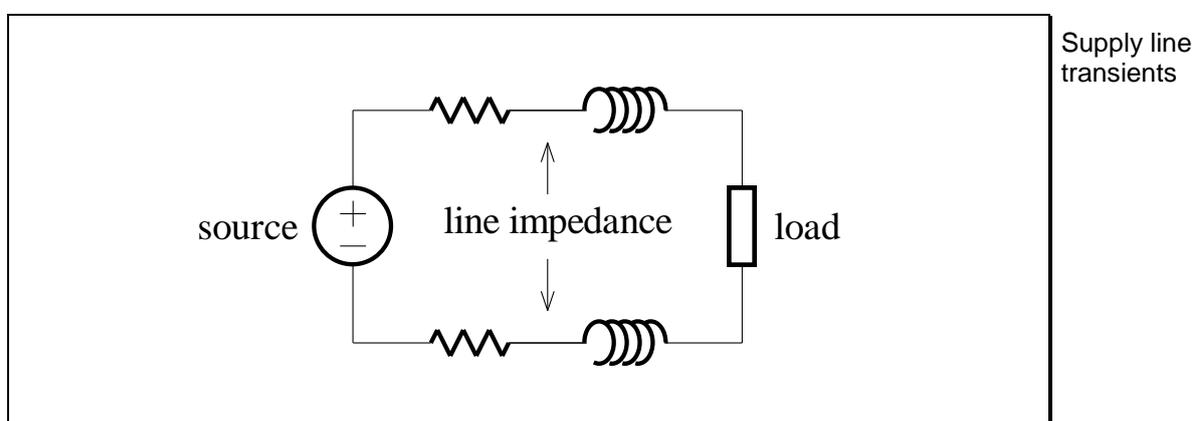


Figure 6.2 – Supply Line Transients

Any change in the load causes a transient due to the line inductance. Supply line voltage dips can cause reset conditions in microprocessors, cause distortion in analog outputs, and at the very worst, even destroy components.

6.2.2 EMP and RFI

Anything that produces arcs or sparks will radiate electromagnetic pulses (EMP) or radio frequency interference (RFI). Arcs and sparks occur in automotive ignition systems, electric motors, switches, static discharges, etc. In switches, the transients on the supply line will cause an opening switch to throw a spark.

6.2.3 ESD

Electrostatic discharge (ESD) is the spark that occurs when a person picks up a static charge (e.g. from walking on carpet) and then discharges it onto a metallic (door handle) or electronic device (CMOS chip). ESD can be very damaging to an electronic system - it can blow craters in silicon.

6.2.4 Intentional Sources

The power supply is an intentional signal transmitted to most electronic devices. Although it is intended to supply power, it can also couple into other signal paths.

Digital circuits are by their very nature switching circuits. They also operate at high frequencies. They therefore have supply line transient problems and emit RFI.

6.3 Coupling

There are four main ways that noise is coupled into a system. Different noise problems require different solutions. Adding a capacitor or shield will not solve every problem.

6.3.1 Common Impedance (“Ground”) Coupling

One of the most common methods of coupling noise is through poor design. Consider the following digital and analog system:

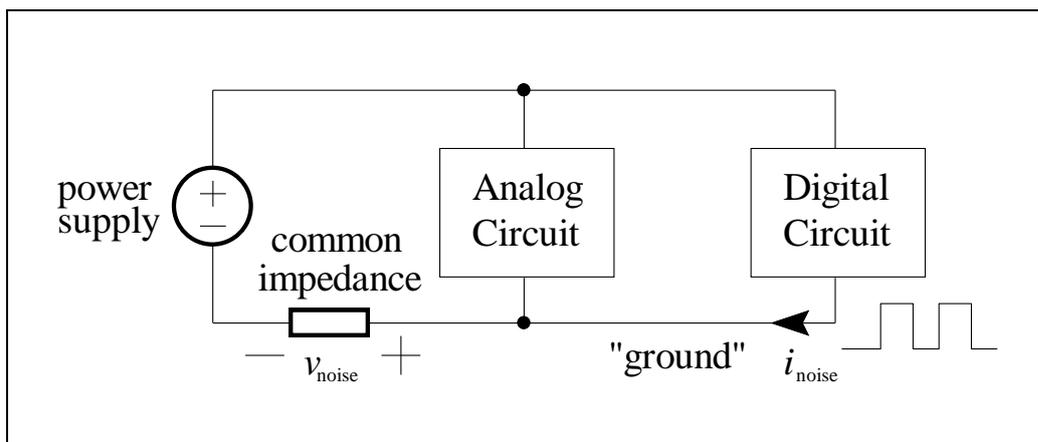


Figure 6.3 – How noise is developed by a common impedance

The “ground” point in this arrangement could be connected to op-amp non-inverting terminals (for example) and output transistors of digital logic (when they are driving low). The “ground” point has a tangible impedance to the power supply common. The noise current causes a noise voltage across the common impedance which will present itself as a noise voltage to the analog circuit.

6.3.2 Capacitive Coupling

Capacitive coupling involves the passage of interfering signals through mutual (or “stray”) capacitances that are never shown on the circuit diagram, but we know are there. This type of noise is often associated with fast rise and fall times or high frequencies (a capacitor is like a short circuit to high frequencies).

Stray capacitance couples “noise”

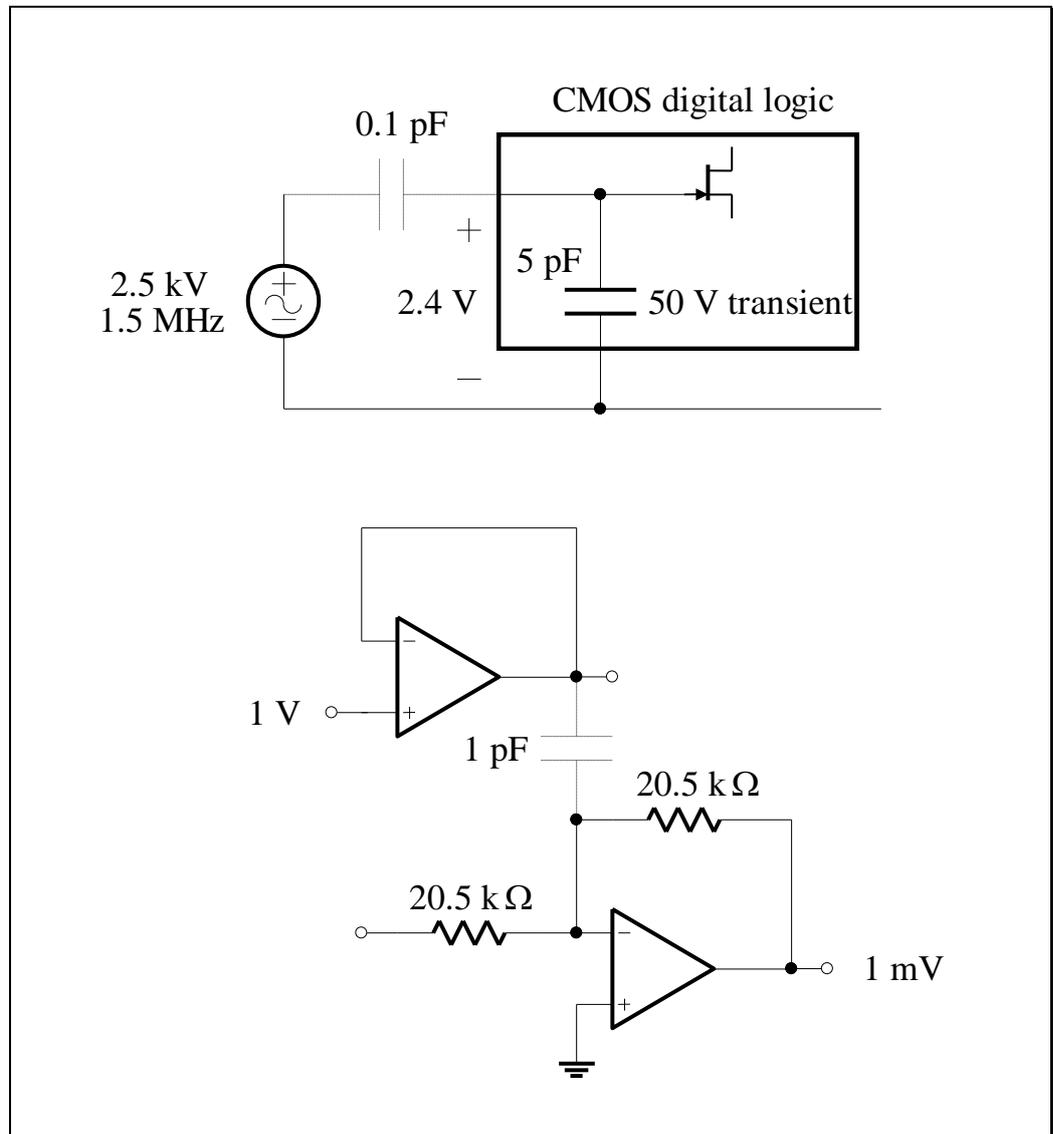


Figure 6.4 – Stray capacitance couples “noise” into other circuits

It is amazing how small mutual capacitance can cause serious problems.

6.3.3 Inductive Coupling

Inductive coupling is where a magnetic field from some external source links with a current loop in the victim circuit.

A current exists only in a loop. Sometimes it is difficult to determine where the current is (e.g. a ground plane), and we often ignore its path on a schematic due to the “common” and op-amp symbols. The physical geometry of the loop formed by the current is the key to understanding inductive coupling.

Any current creates a magnetic field. We know from Ampère’s Law that the field strength is dependent on the current enclosed by our path of integration in circling the current. A current loop therefore creates a magnetic field.

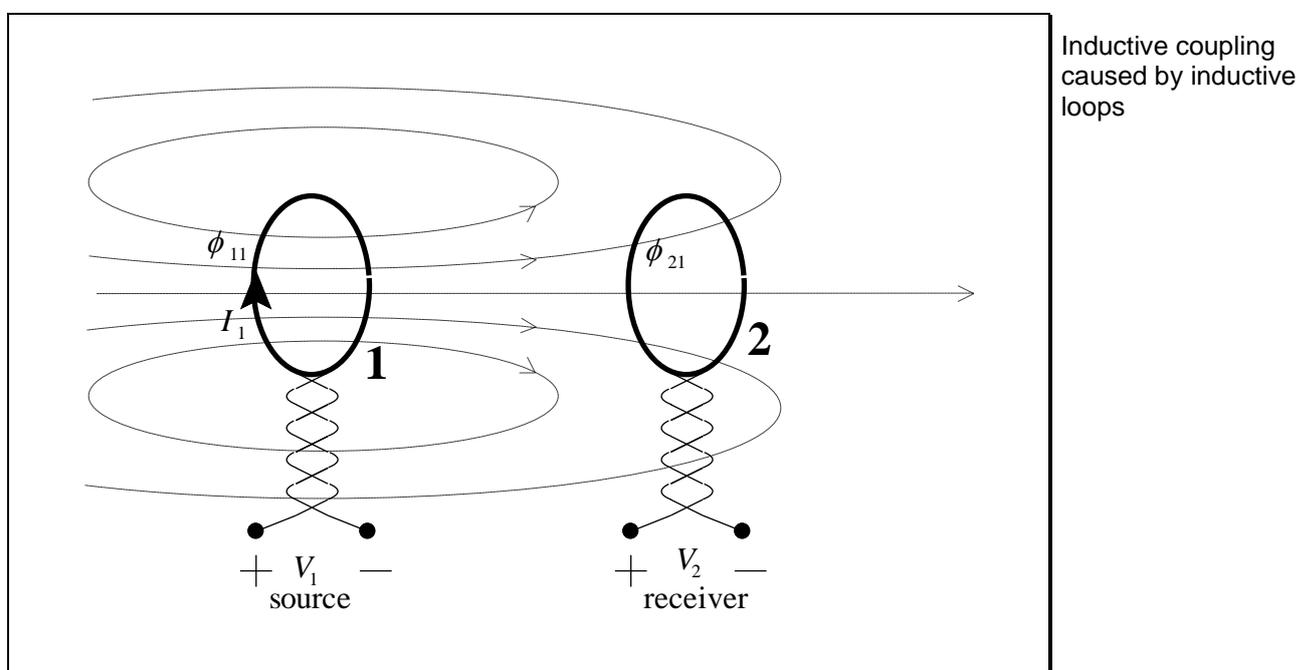


Figure 6.5 – Inductive coupling caused by current loops

If a time varying magnetic field links with a conductive loop, then Faraday’s Law applies and a voltage will be induced in the loop.

Minimising inductive coupling does not necessarily involve magnetic materials. If two conductors, each carrying a current I in opposite directions, are in close proximity then the external magnetic field tends to cancel.

6.3.4 Radiated Coupling

A time-varying electric field generates a time-varying magnetic field and vice versa. Far from the source of a time-varying EM field, the ratio of the amplitudes of the electric and magnetic fields is always 377Ω . Close to the source of the fields, however, this ratio can be quite different, and dependent on the nature of the source. The region in space where the ratio is near 377Ω is called the far field, and the region where the ratio is significantly different from 377Ω is called the near field.

The near field goes out about $1/6$ of a wavelength from the source. At 1 MHz this is about 50 m and at 10 MHz it's about 5 m. This means that if an EMI source is in the same room with the victim circuit, then it's most likely to be a near field problem. The reason this matters is that in the near field an RF interference problem could be almost entirely due to E -field coupling or H -field coupling, and this influences the way in which we combat this type of noise.

In the near field of a whip antenna, the E/H ratio is higher than 377Ω , which means it's mainly an E -field generator. A wire-wrap post or a test point terminal can be a whip antenna. Interference from a whip antenna would be by electric field coupling, which is basically capacitive coupling.

In the near field of a loop antenna, the E/H ratio is lower than 377Ω , which means it's mainly an H -field generator. Any current loop is a loop antenna. Interference from a loop antenna would be by magnetic field coupling, which is basically the same as inductive coupling.

Even so, in the near field and far field, the fact that the EMI is being radiated means we have to treat this type of coupling differently.

6.4 Combating EMI

There is a whole range of techniques that help combat EMI. They range from minimising the generation of noise voltages, reducing the coupling, and making a graceful recovery after being subjected to an electromagnetic disturbance.

Some techniques, like grounding and shielding, attack the EMI problem from many sides. *Consideration of EMC at the design stage, such as PCB layout, can prevent many noise problems from ever occurring.*

6.4.1 Combating Capacitive Coupling

Reducing Mutual (Stray) Capacitance on a PCB

- Lay tracks as far apart as possible on a PCB (separate high- and low-level signals).

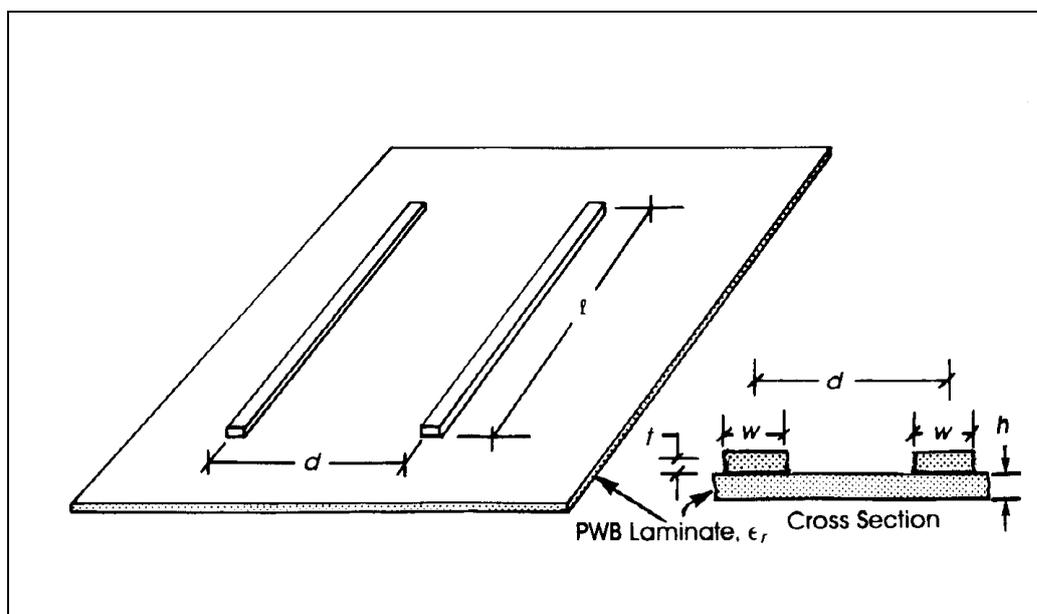


Figure 6.6 – Separate tracks to reduce capacitance

6.10

- Use a ground plane.

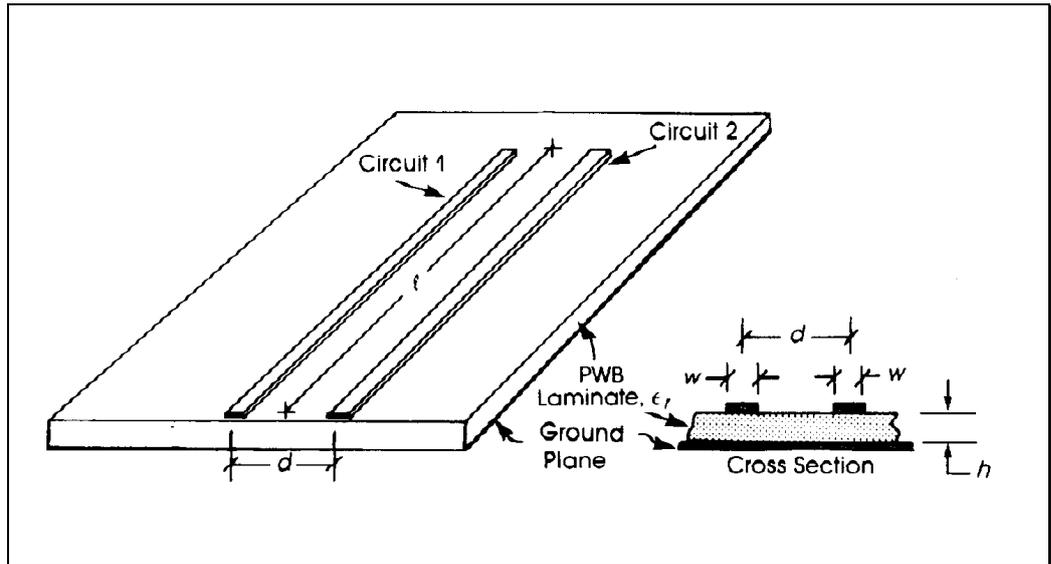


Figure 6.7 – Use a ground plane to minimise mutual capacitance

Properly Implemented Shields

- Connect shields to the common at the source.

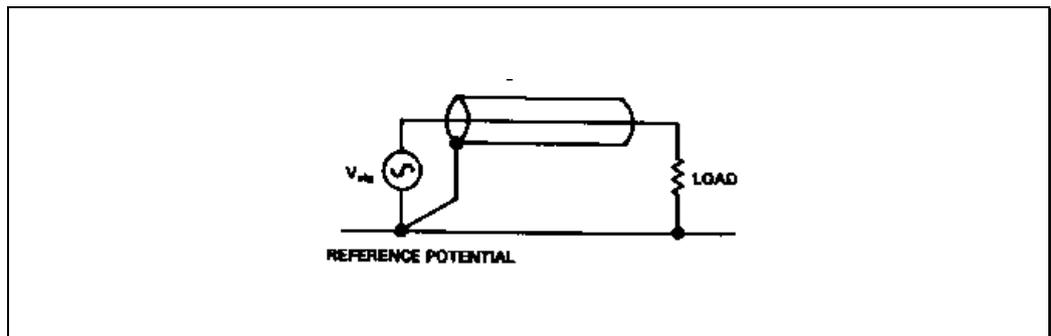


Figure 6.8 – Grounding a cable shield

- Don't connect both ends of the shield to "ground".

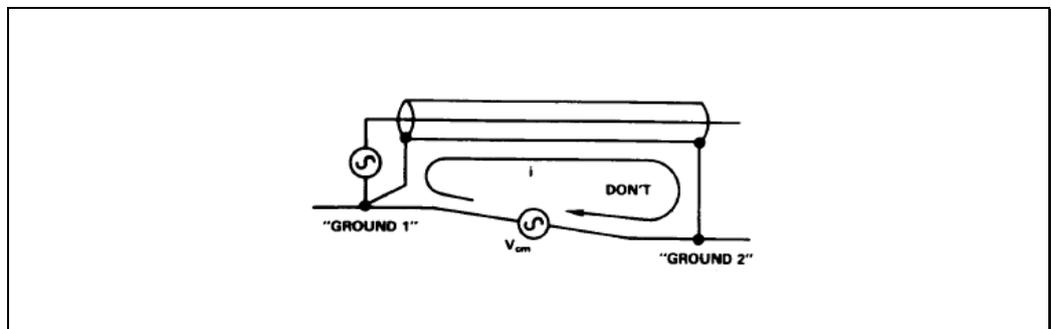


Figure 6.9 – Don't connect the shield to ground at more than one point

- Don't allow shield current to exist (conflicts with combating inductive coupling).

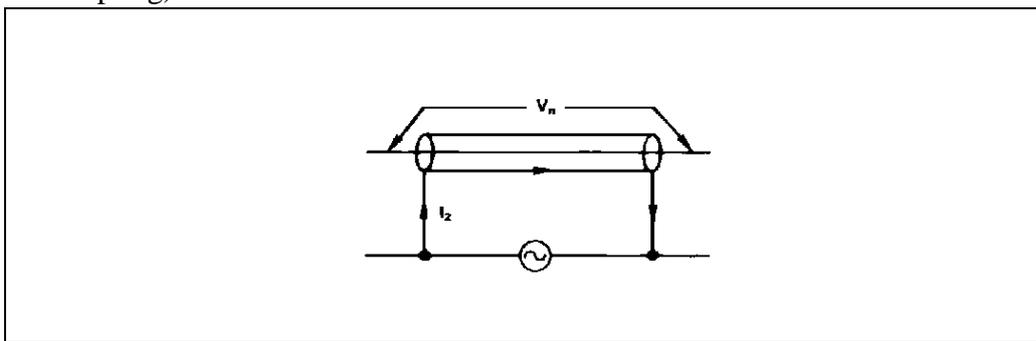


Figure 6.10 – Magnetic pickup from current through a cable shield

- Don't allow the shield to be at a voltage with respect to the reference potential.

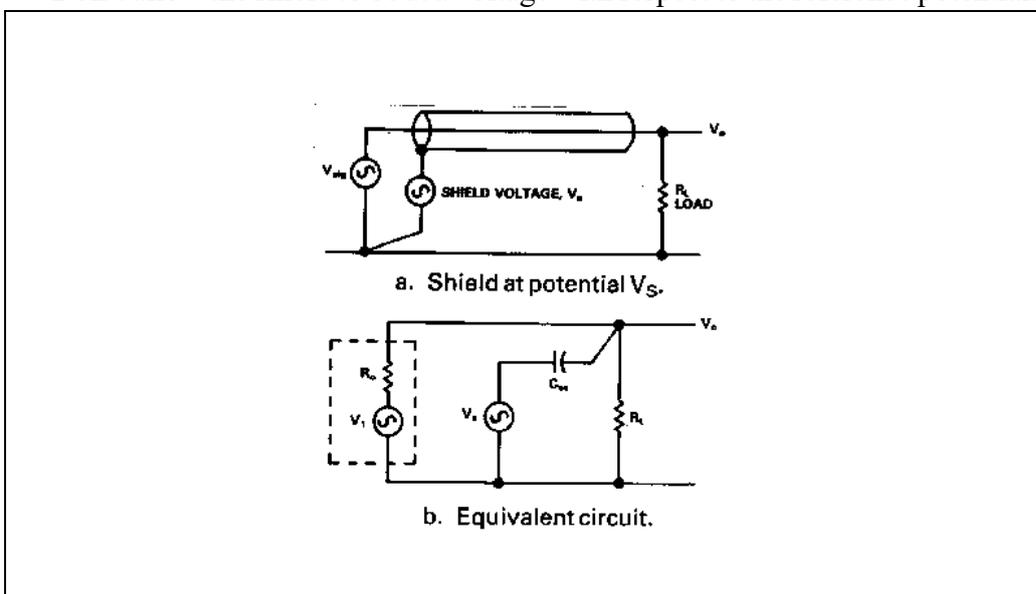


Figure 6.11 – Don't allow the shield to be at a voltage with respect to the reference potential

6.12

- Know by careful study how the noise current that has been captured by the shield returns to “ground”.

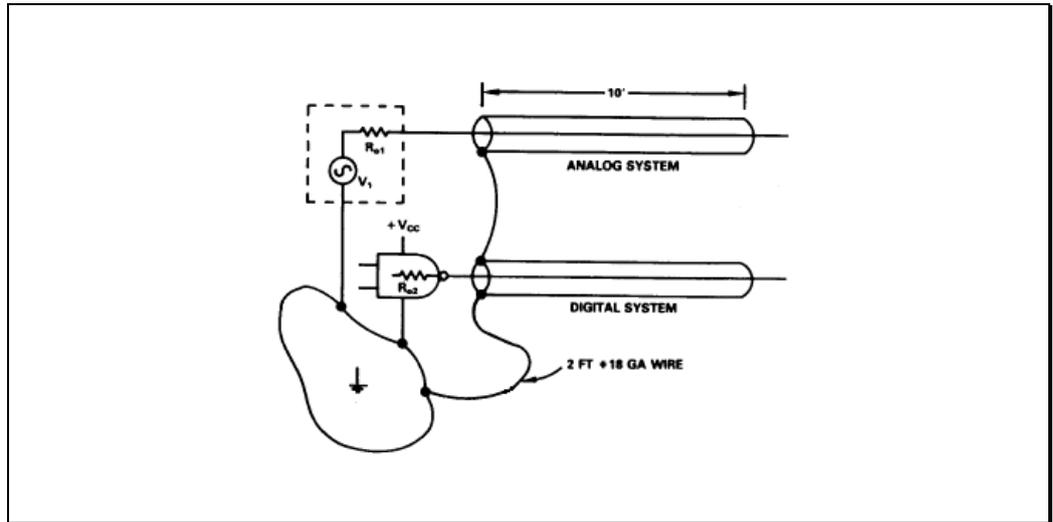


Figure 6.12 – A situation that generates transient shield voltages

- Use a Faraday shield to isolate whole circuits.

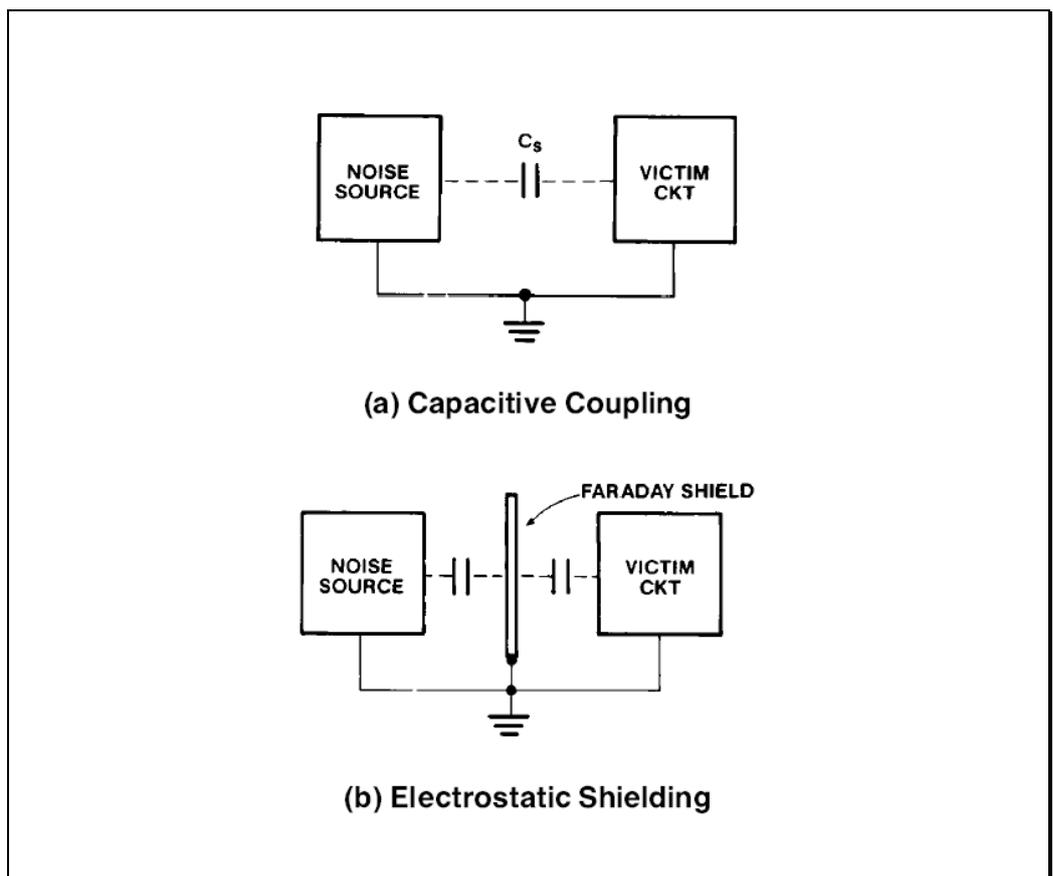


Figure 6.13 – Use of Faraday shield

6.4.2 Combating Inductive Coupling

Reducing Mutual Inductance on a PCB

- Minimise current loop areas.
- Use a ground plane (or gridded-ground).

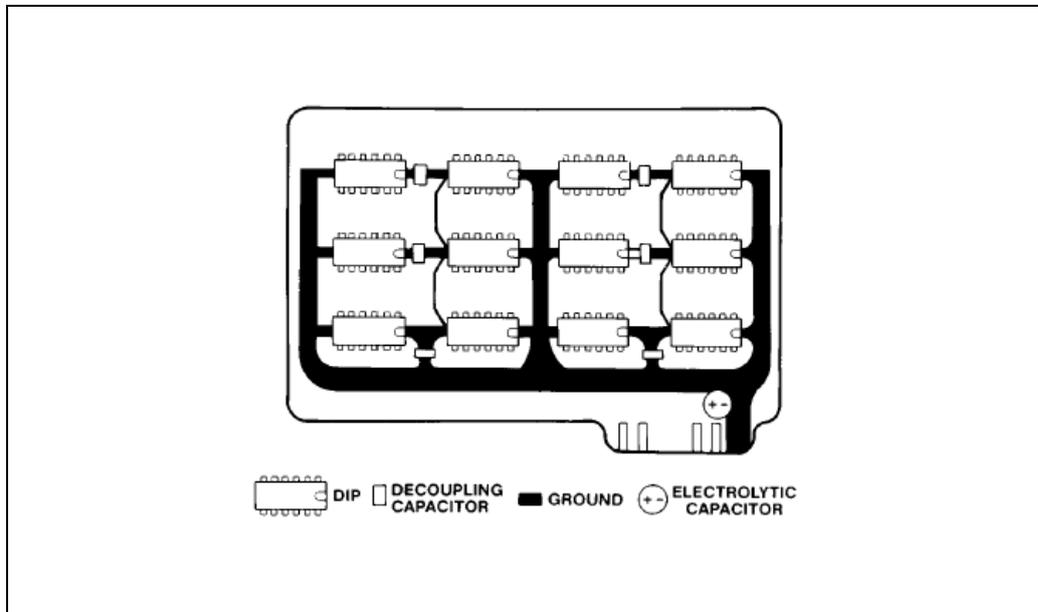


Figure 6.14 – PCB with gridded ground

- Orient susceptible loops at right angles to the magnetic field, if possible.

Shields

- Use coaxial cable.

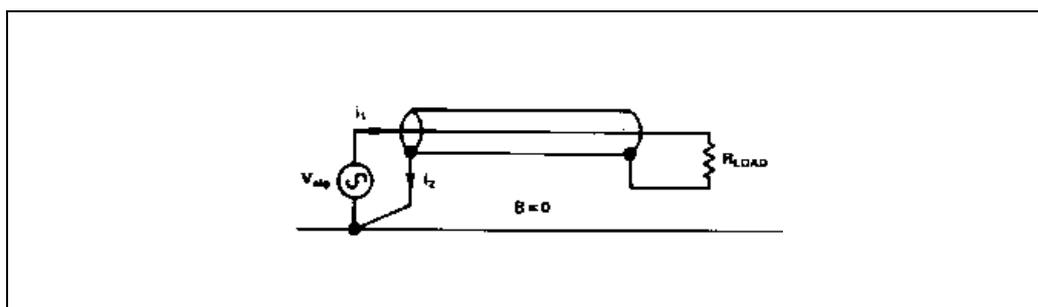


Figure 6.15 – Use a shield for return current to a noisy source

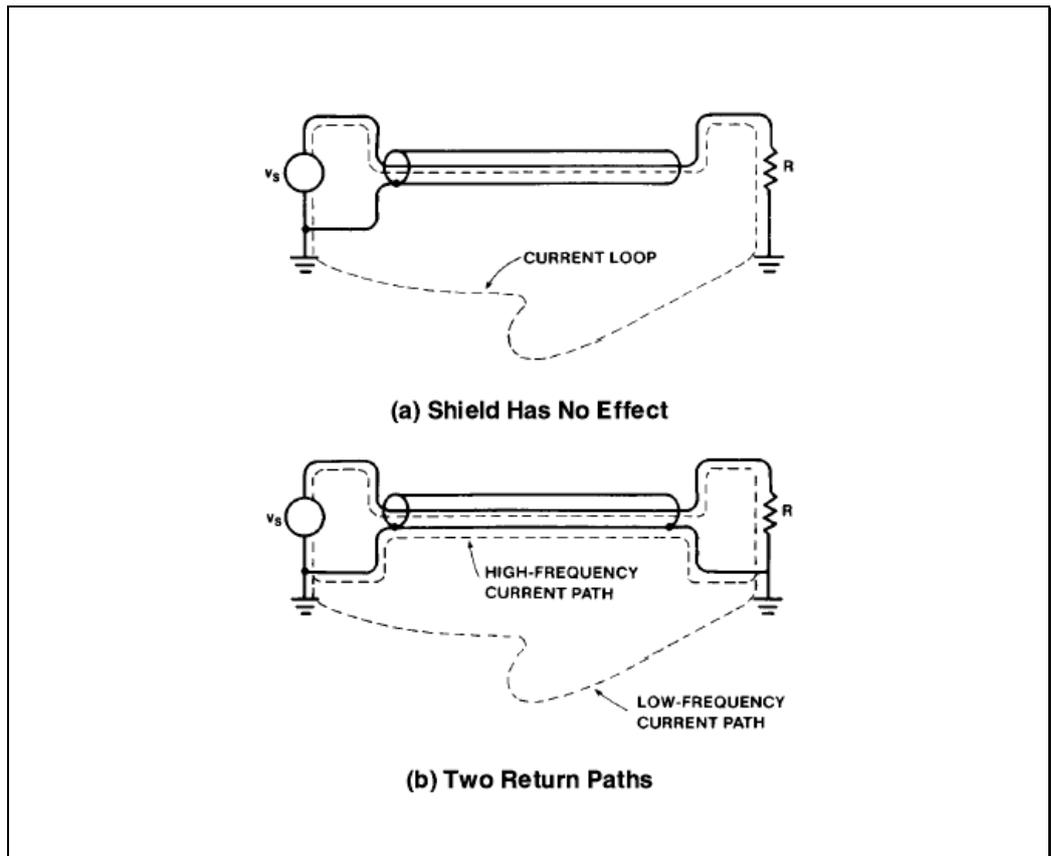


Figure 6.16 – Use of coaxial cable

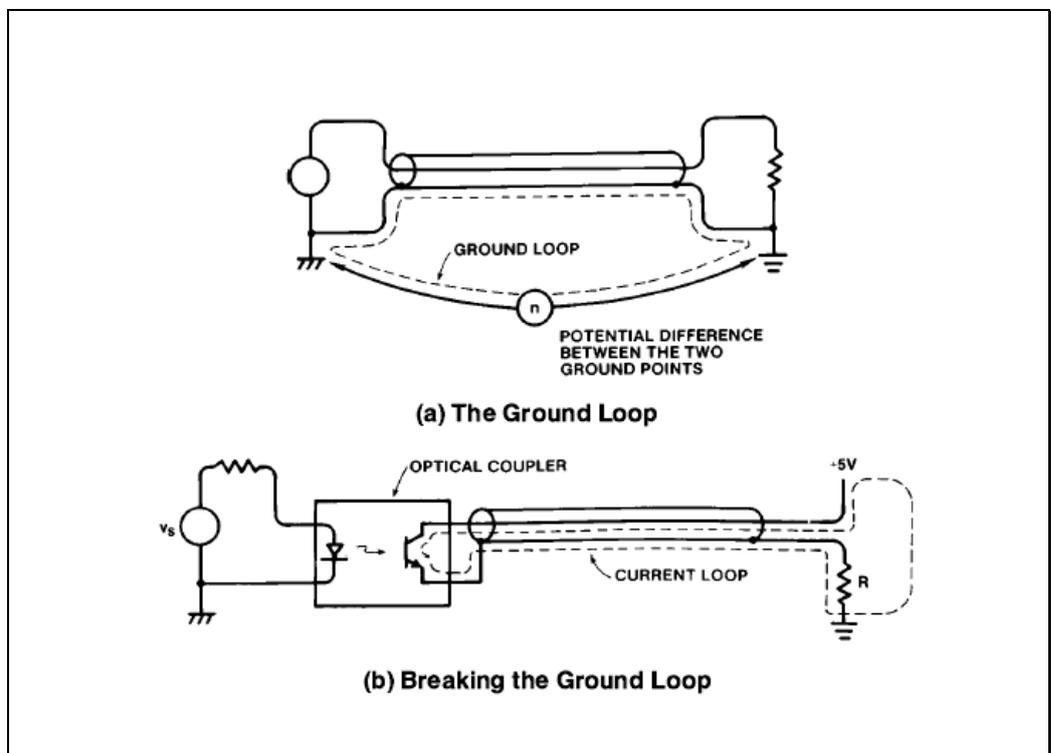


Figure 6.17 – Use of optical coupler

- Use a twisted pair of wires.

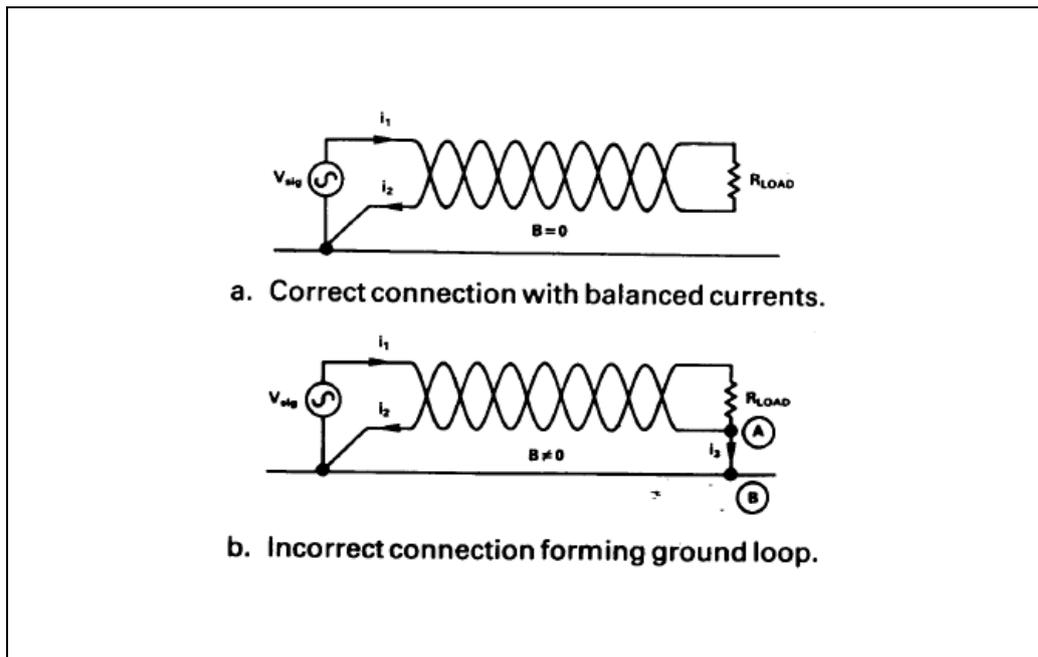


Figure 6.18 – Connections of a twisted pair

- Use an appropriate shielding material for the frequency and field strength.

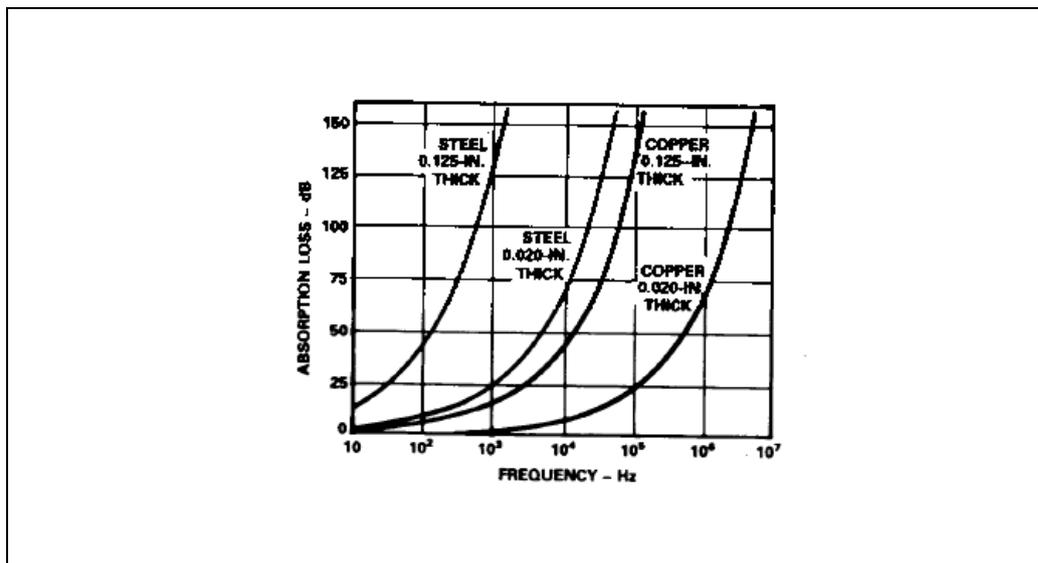


Figure 6.19 – Absorption loss vs. frequency for two thicknesses of copper and steel

6.16

- Use steel or mumetal at power frequencies.

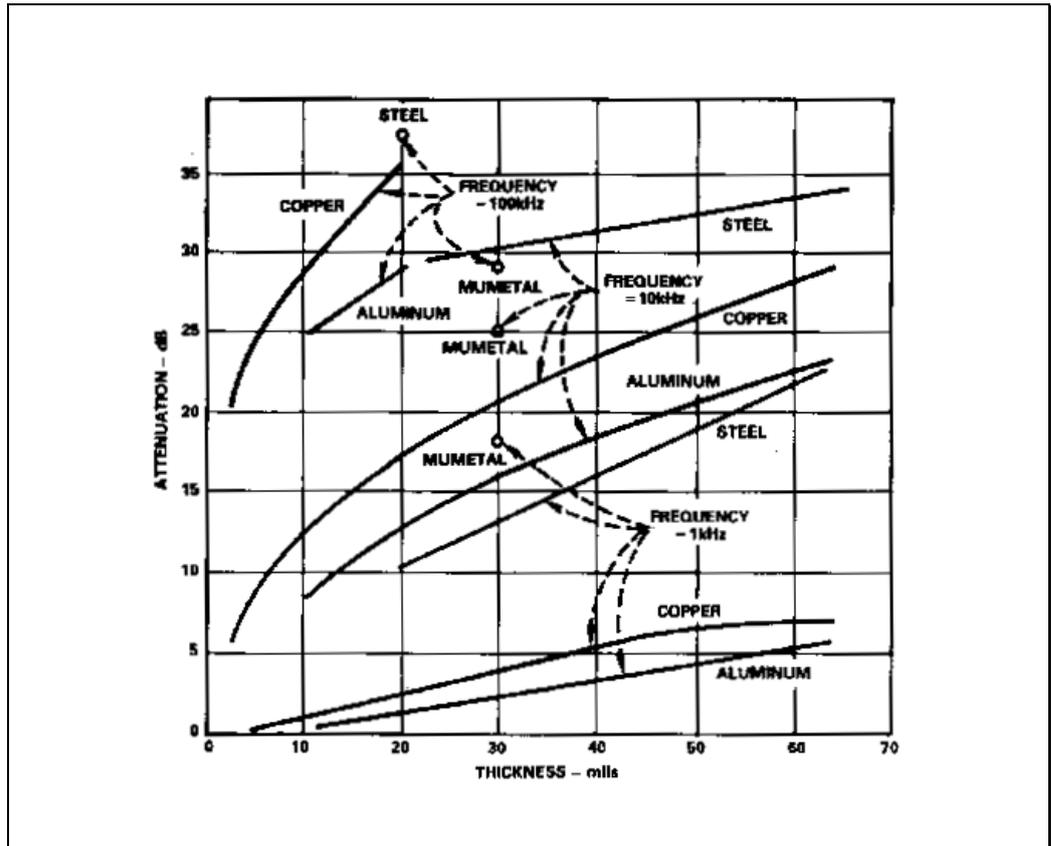


Figure 6.20 – Absorption loss vs. frequency for two thicknesses of copper

6.4.3 RF Shielding

- Use copper or aluminium for *E*-field shielding (rare).

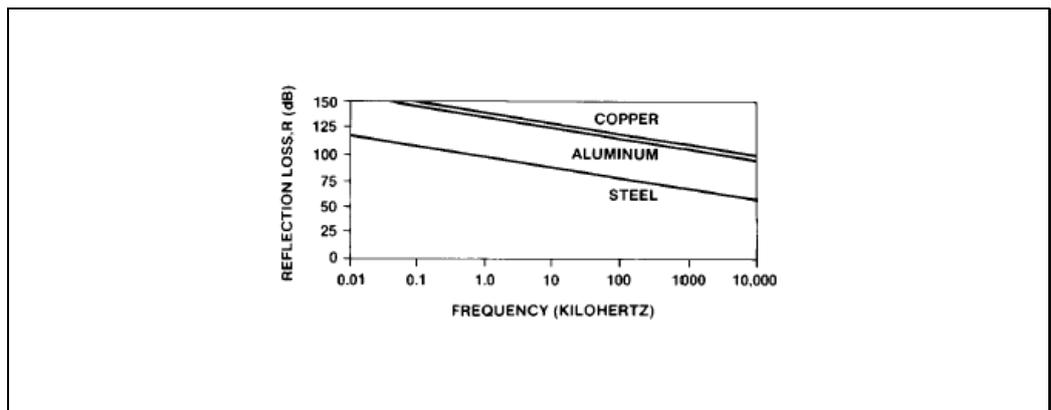


Figure 6.21 – E-field shielding

- Use steel for *H*-field and far field shielding (common).

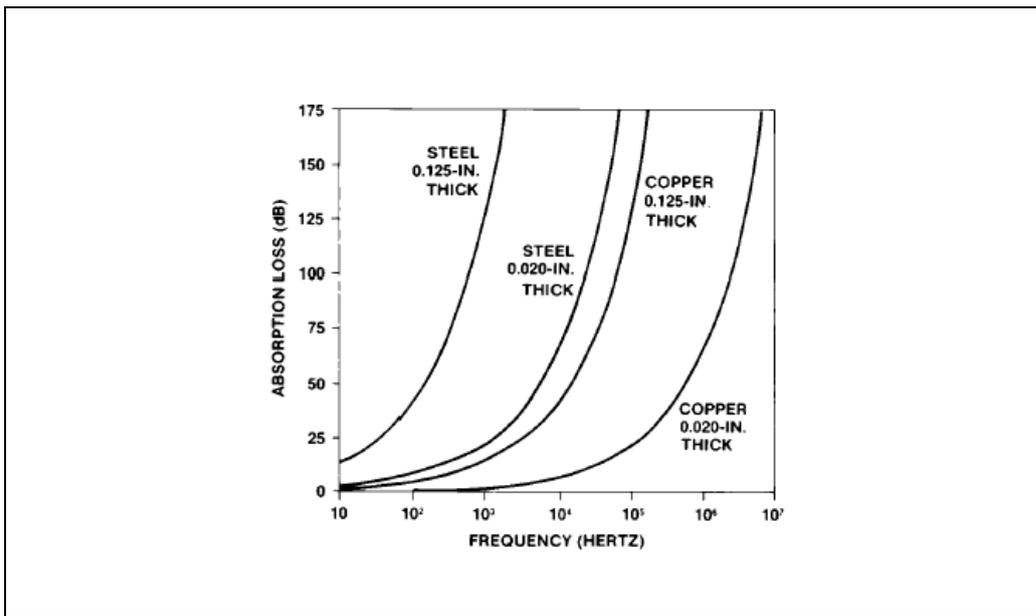


Figure 6.22 – H-field shielding

- Use steel if in doubt.

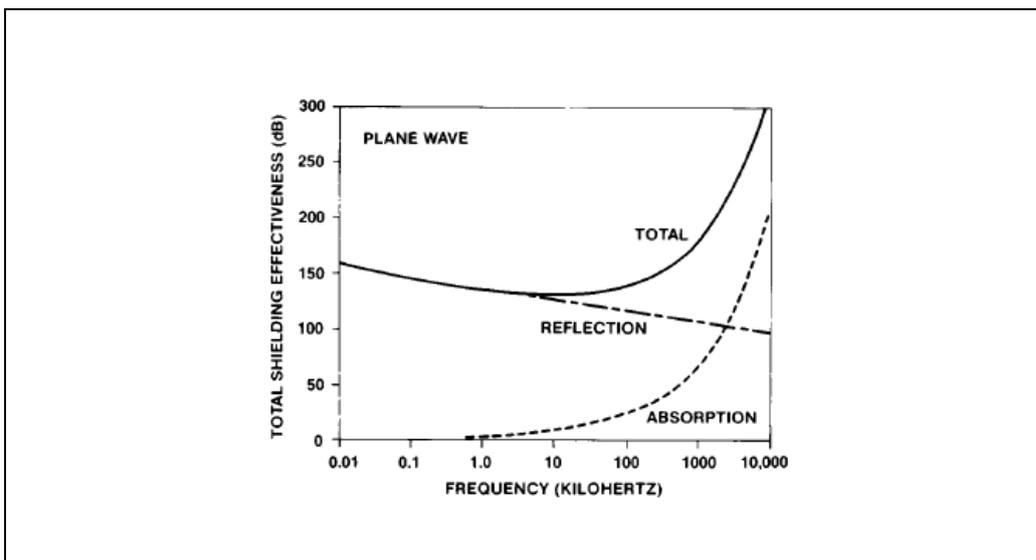


Figure 6.23 – E- and H-field shielding

6.18

- Avoid seams, joints and large holes in the shield.

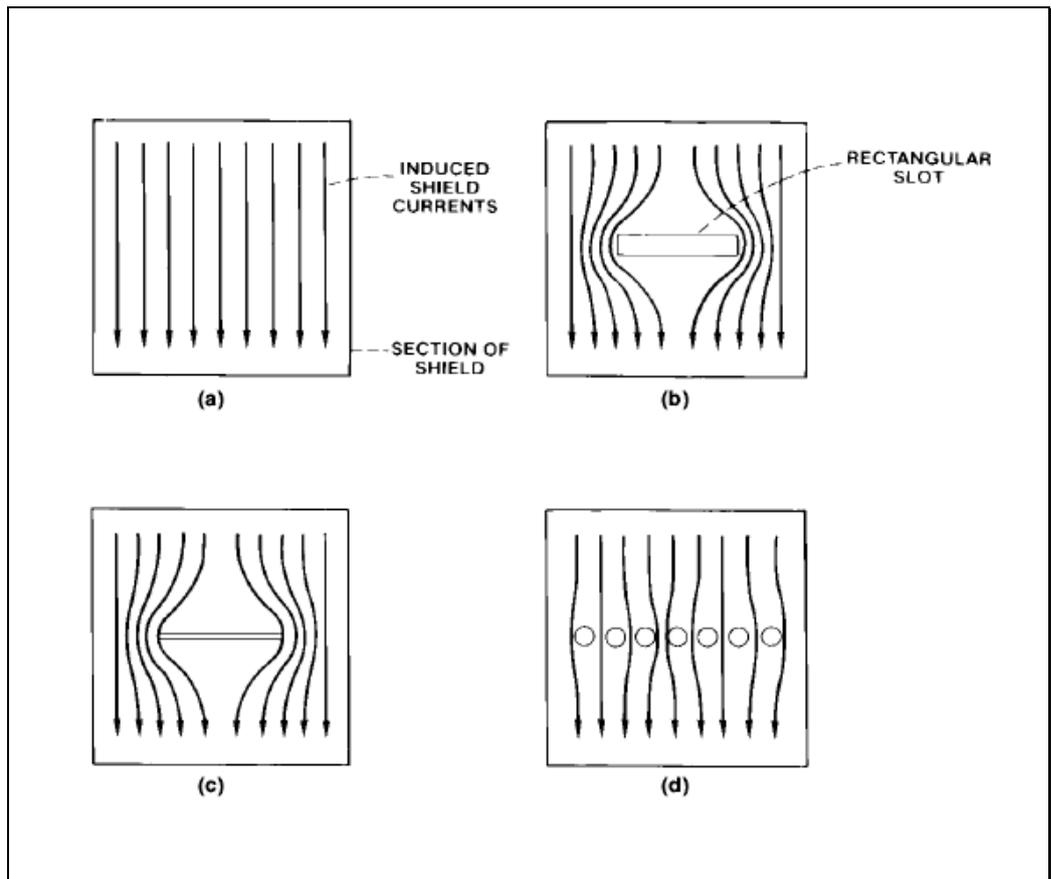


Figure 6.24 – Effect of shield discontinuity on magnetically induced shield current

6.4.4 Grounds

- Use a combination of serial and parallel ground-wiring.

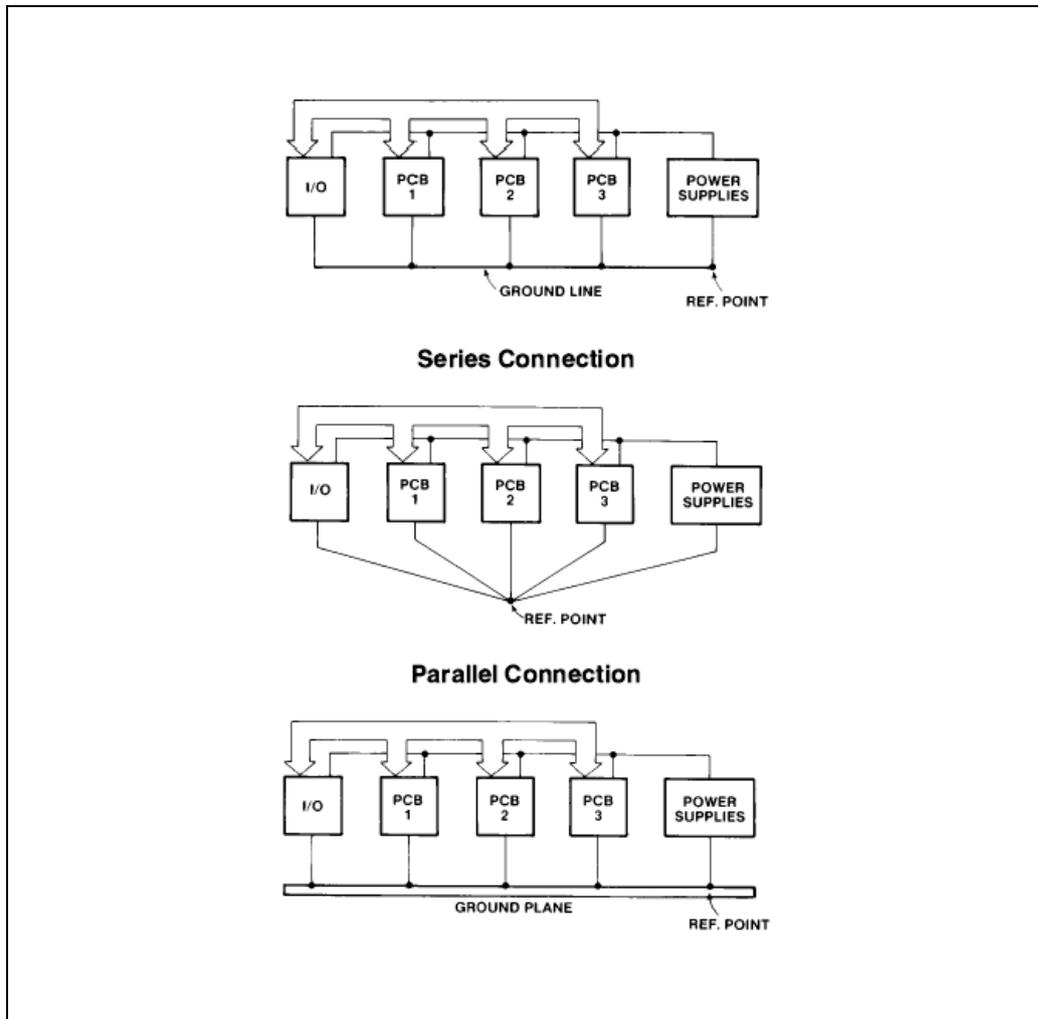


Figure 6.25 – Three ways to wire the grounds

- Separate digital grounds and analog grounds.

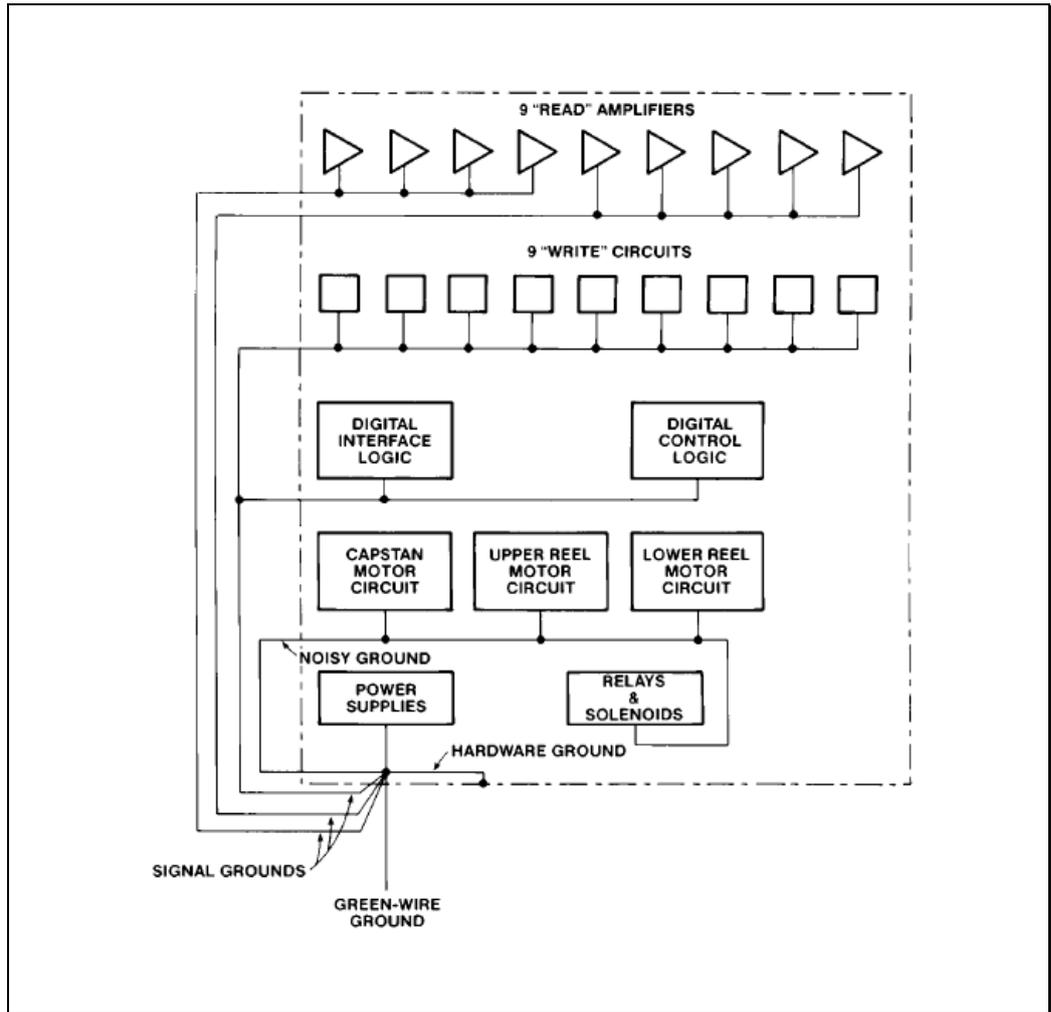


Figure 6.26 – Ground systems in a 9-track digital recorder

- Separate power grounds and signal grounds.

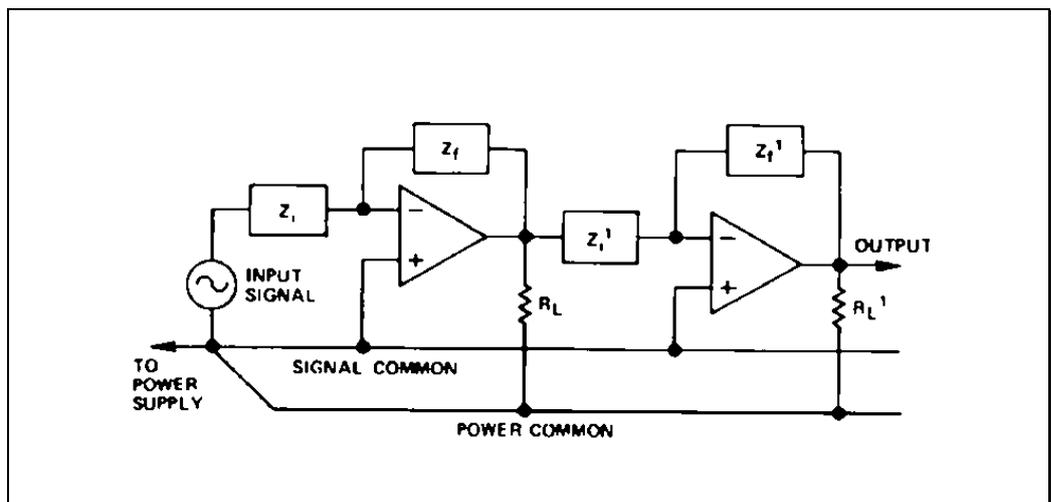


Figure 6.27 – Minimizing common impedance coupling

- Use a ground plane.

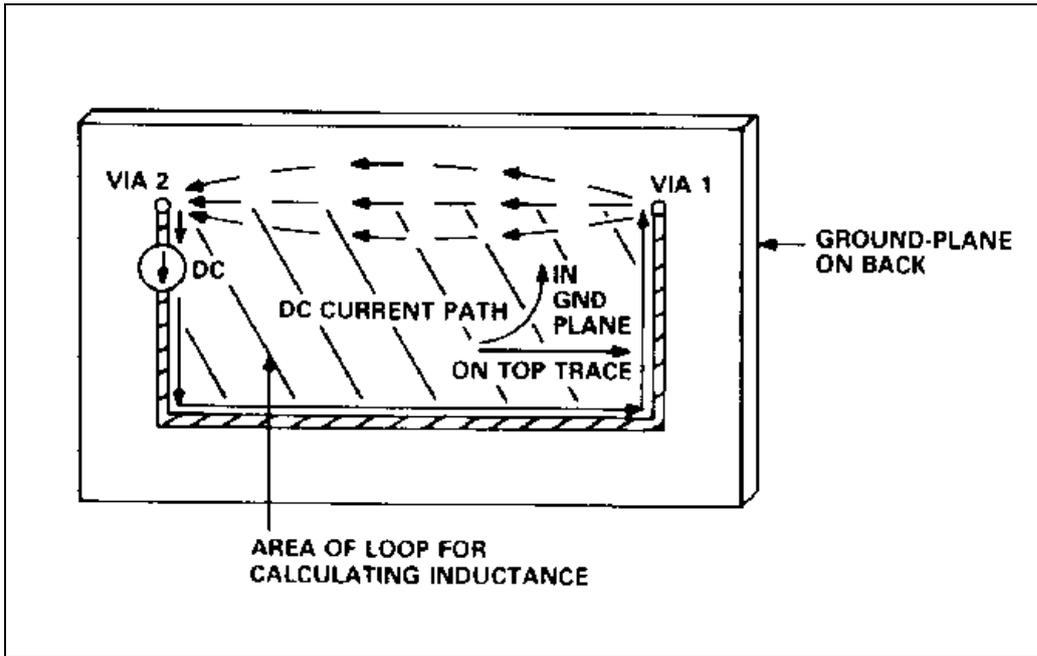


Figure 6.28 – DC current path

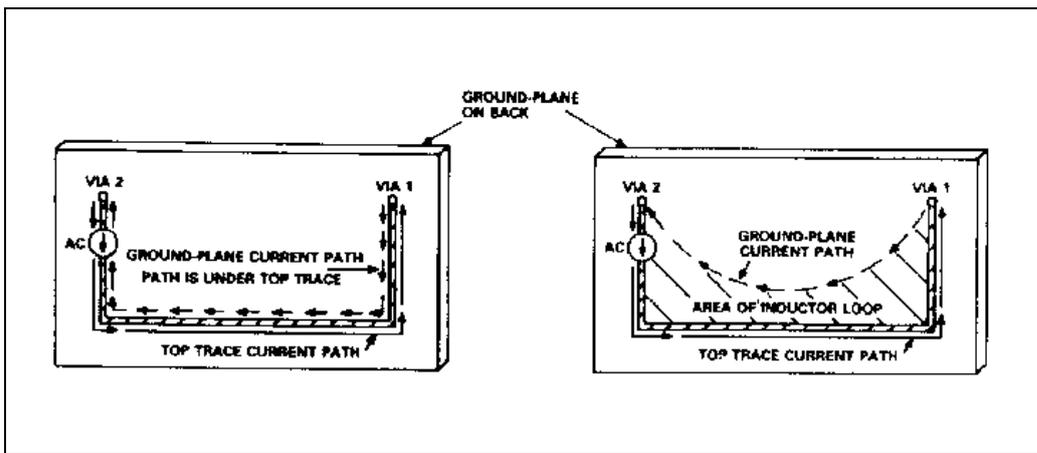


Figure 6.29 – AC current path without (left) and with (right) resistance in ground plane

- Connect signal grounds to AC power ground for safety.

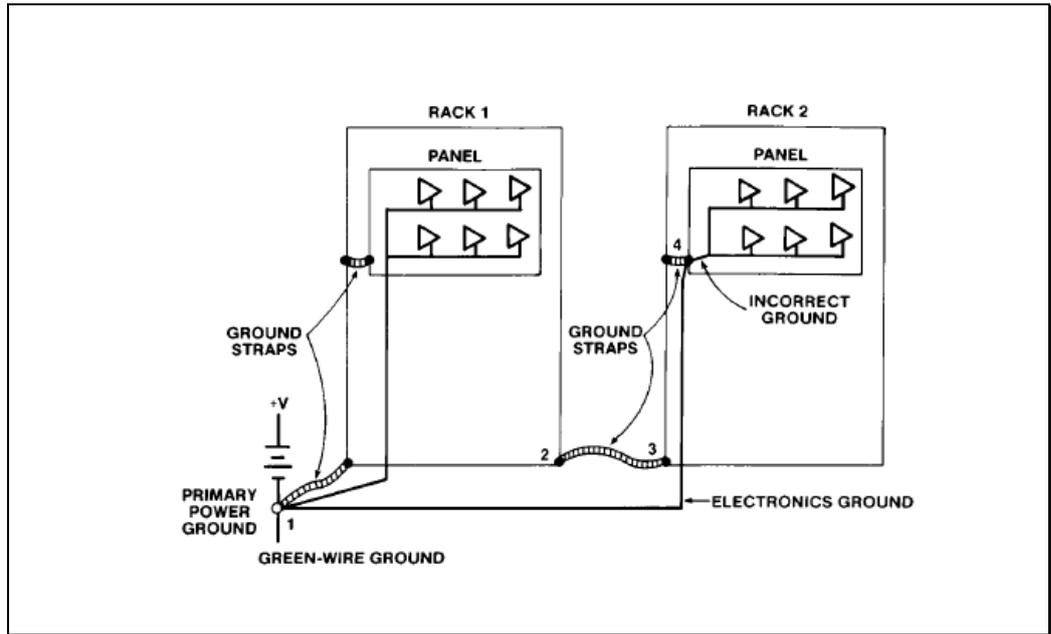


Figure 6.30 – Electronic circuits mounted in equipment racks should have separate ground connections. Rack 1 shows correct grounding, Rack 2 shows incorrect grounding

6.4.5 Power Supply Distribution and Decoupling

- Use decoupling capacitors on all I.C.s.

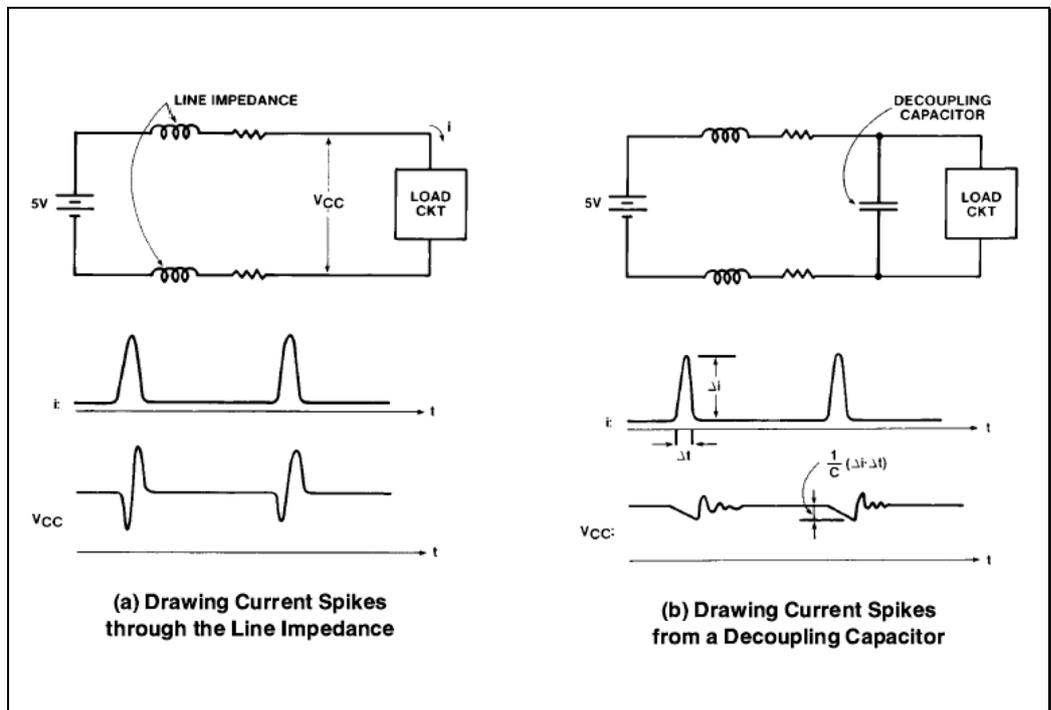


Figure 6.31 – What a decoupling capacitor does

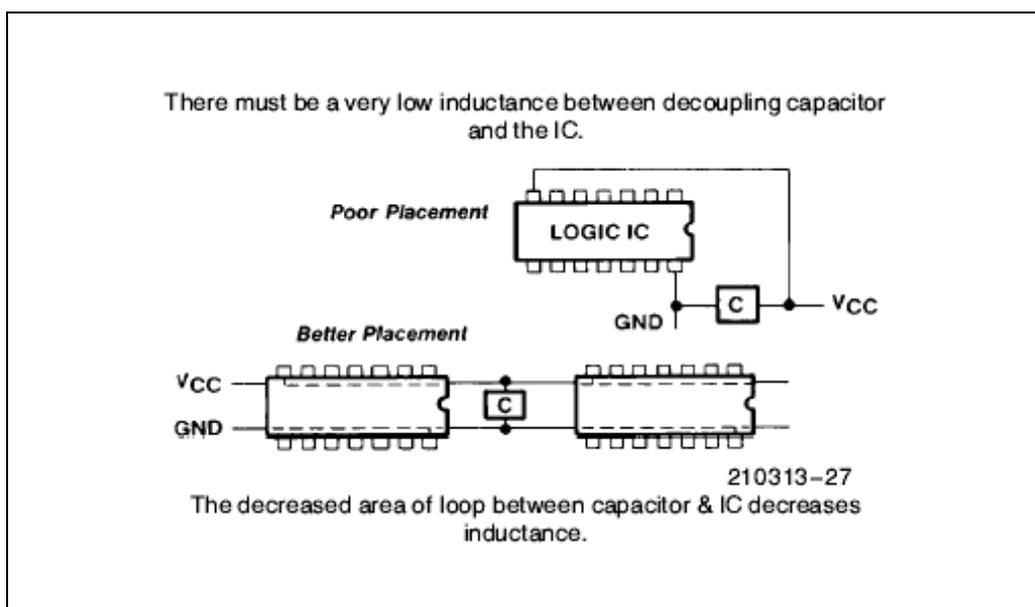


Figure 6.32 – Placement of decoupling capacitors

- Use a large decoupling capacitor (usually electrolytic) for the whole PCB.
- Use regulators.
- Use transient suppressors.

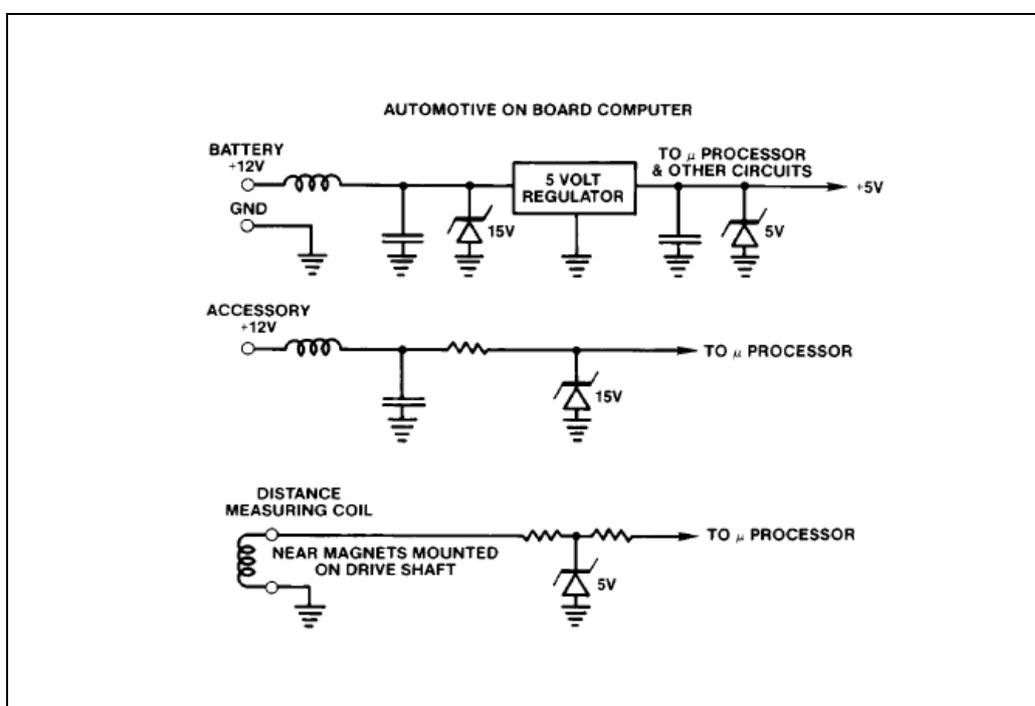


Figure 6.33 – Use of transient suppressors in automotive applications

6.24

- Minimise loop areas to decoupling capacitors.

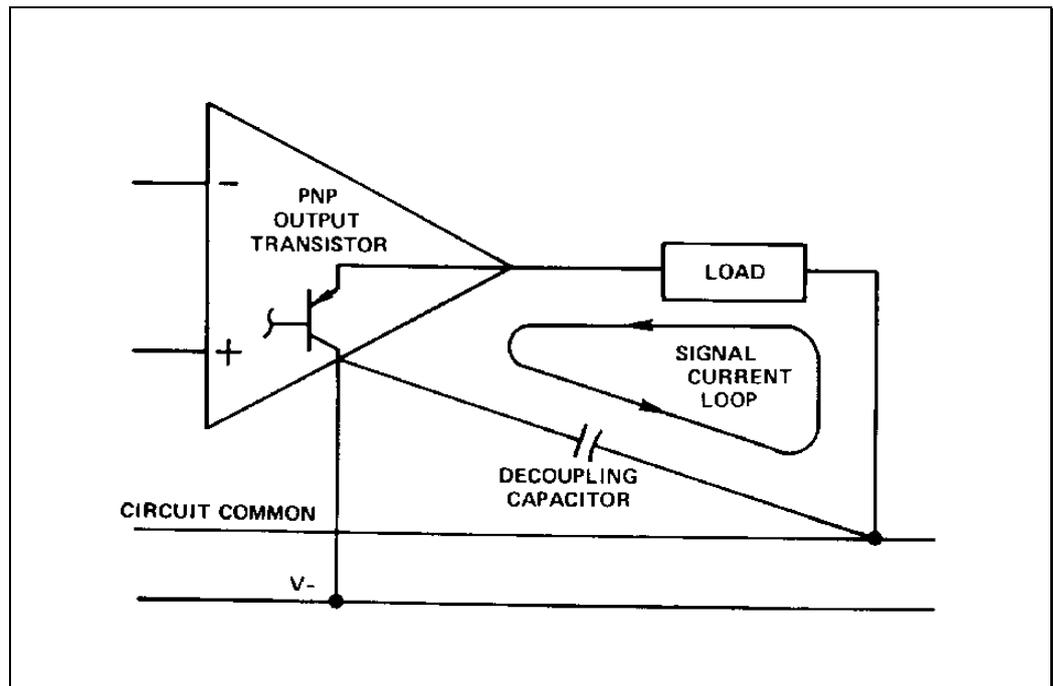


Figure 6.34 – Decoupling of negative supply for a grounded load

- Use large tracks for power distribution on PCBs (minimise impedance).

6.5 Regulatory Standards

In the early 1990s, countries in Europe started taking the issue of electromagnetic compatibility seriously as the number of interfering and susceptible devices increased, especially in the automotive and medical environments. The European Union (EU) established numerous standards in the late 1990s which were later adopted by the International Electrotechnical Commission (IEC). These IEC standards form the basis of standards in countries all around the world. The EU enforced EMC compatibility in 1997 by passing laws barring products from the marketplace that were not compliant with the standards. Products that are compliant are labelled with a “CE marking”.

The management of electromagnetic emissions is important in ensuring the fair and equitable use of the electromagnetic spectrum as a public resource. There are a number of regulations and standards which apply to the management of spectrum and electromagnetic compatibility (EMC) for industrial and consumer electronic devices.

The Australian Communications and Media Authority (ACMA) is the government agency tasked with enforcing these EMC regulations under the Radiocommunications Act 1992. Under the Act, suppliers of industrial and consumer electronic devices which are manufactured, imported or sold in the Australian market must register with the National Equipment Registration System national database operated by the Electrical Regulatory Authorities Council (ERAS) in accordance with AS/NZS 4417.1, and meet the following two regulatory arrangements:

1. Meet the EMC standards specified in the Radiocommunications (Electromagnetic Compatibility) Standard 2008, and
2. Carry appropriate labelling specified in the Radiocommunications Labelling (Electromagnetic Compatibility) Notice 2008.

6.5.1 The Regulatory Compliance Mark

The Regulatory Compliance Mark (RCM) is labelling that is required to be shown on devices which are compliant with the Regulations.

The RCM system requires Australian based manufacturers, importers or suppliers to register prescribed products on the Electrical Equipment Safety System (EESS) national database. The EESS incorporates a National Database that is the gateway for in-scope electrical equipment registration in Australia and New Zealand.

The RCM is a single compliance mark and can only be used by Australian/New Zealand suppliers after establishing compliance with all applicable regulations including EMC, Telecoms, Radiocomms, Electromagnetic Radiation (EMR) and electrical safety.

Those using the RCM for ACMA compliance must first register as the ACMA Responsible Supplier on the EESS website. Next, they must identify the ACMA applicable standards and obtain the appropriate test reports. Products that do not have wireless or telecoms function are relatively straightforward and only require an EMC Test report to an EMC standard listed on the ACMA list of applicable standards.

Products that include a wireless transmitter such as Bluetooth, Wi-Fi, portable radios etc. must also comply with the relevant ACMA Radiocommunications (Radcom) standard and the ACMA Electromagnetic Radiation (EMR) standard. If a product also includes connection to the mobile phone network, satellite telecom network or the Public Switched Telephone Network (PSTN), it must comply with the relevant standards prescribed in the Telecommunications Labelling Notice (TLN) 2014. If it includes a Mobile Phone Network interface, determining the applicable ACMA requirements is often fraught with risk. It may be prudent to consult with an ACMA accredited Certification Body.

There are some exemptions from the ACMA RCM labelling requirements and these are listed in the ACMA EMC Labelling Notice. RCM Labelling of some battery-powered equipment is voluntary; however, care should be taken when

interpreting the 'Battery Powered' definition. The battery must be internal to the product and it must not be possible to operate when it is being charged by an external supply. Devices that use an automobile 12 V supply are not exempt. Nor are devices that have a wireless transmitter such as Bluetooth or Wi-Fi.

A documentary proof of compliance (compliance folder) must be kept at the disposal of the ACMA for audit purposes. It must include a signed Declaration of Conformity (DoC), the relevant test reports and a description of the product. The labelled product can then be sold in Australia and New Zealand.

Most mains-powered products other than Prescribed Articles are Compliance Level 1 under ERAC regulations. Registration is not mandatory; however, compliance with an appropriate safety standard is required under various state and federal laws. A valid safety test report should be kept on file in case of regulator request.

Domestic appliances and AC adapters/chargers are considered high-risk devices and are classified as Compliance Level 3 devices. A complete list of Compliance Level 3 (in-scope) equipment can be found in AS/NZS 4417.2:2012. Under the EESS RCM rules, Compliance Level 3 products must be registered on the ERAC National Data Base by the Responsible Supplier (also the importer). A Certificate of Conformity must first be obtained from an accredited Certification Body. Imported Level 3 products must be registered by each importer (must be in Australia/NZ), even if another importer has registered the same product. Overseas entities cannot register as a Responsible Supplier.

The importer based in Australia/NZ will be solely responsible for safety compliance of the product. This responsibility cannot be delegated by a Local Representative Agreement with an overseas manufacturer. However, the importer can delegate the registration process to their Authorised Representative (consultant), who must be located in Australia/NZ. Registration and certification fees can be avoided by using a charger/adaptor that is sourced from an Australian- or New Zealand based supplier located in one of the two countries.

6.28

Under section 2.2 of the Radiocommunications (Electromagnetic Compatibility) Standard 2008, prototype devices are exempt from the regulatory requirements stipulated under the above Act. In addition to the regulatory compliance requirements under Law, the EMC technical standards which apply to designs, including prototypes, need to be satisfied. Products that are EMC compliant have a RCM label placed on them which allow them to be sold in the Australian and New Zealand market.

In the United States of America, EMC compliance is handled by the Federal Communications Commission's "Part 15 – Radio Frequency Devices". Products that conform to the guidelines receive an FCC marking.

EMC compliance markings for Western markets are shown below:

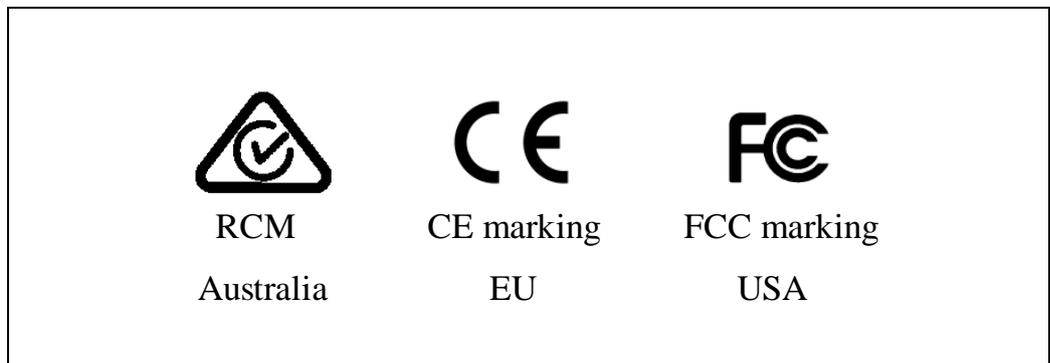


Figure 6.35 – EMC compliance markings

The most common standards which are used for compliance testing are:

- EN 55024:1998 *Information technology equipment – Immunity characteristics* – Limits and methods of measurement
- EN 61000-3-2:2000 *Electromagnetic compatibility (EMC) – Part 3-2: Limits* – Limits for harmonic current emissions (equipment input current up to and including 16 A per phase)
- EN 61000-3-3:1995 *Electromagnetic compatibility (EMC) – Part 3-3: Limits* – Limitation of voltage changes, voltage fluctuations and flicker in public low-voltage supply systems, for equipment with rated current ≤ 16 A per phase and not subject to conditional connection
- FCC Part 15 – *Radio Frequency Devices*
- AS/NZS CISPR 22:2006 : *Information technology equipment - Radio disturbance characteristics* – Limits and methods of measurement
- AS/NZS CISPR 14.1:2010 : *Electromagnetic compatibility (EMC) - Requirements for household appliances, electrical tools and similar apparatus* - Emission
- AS/NZS 61000.6.1:2006 : *Electromagnetic compatibility (EMC) – Generic standards* – Immunity for residential, commercial and light-industrial environments
- AS/NZS 4417.1:2012 : *Regulatory compliance mark for electrical and electronic equipment*

Products that do not comply with local regulations are *illegal* and cannot be placed in that market.

In Australia, if a product's compliance is called into question, the ACMA will carry out investigations. For wilful violations, penalties may apply to both individuals and companies. A primary offence for the supply of a non-standard device may result in the seizure and forfeiture of stock and up to \$160,000 criminal penalties. Imprisonment is also possible under the Crimes Act and offences are also committed for breaches of the RCM labelling provisions under Trade Mark and Copyright regulations. Similarly harsh penalties are applicable in most other countries that have EMC regulations.

6.6 References

Australian Communications and Media Authority,

<http://www.acma.gov.au/Industry/Suppliers/Equipment-regulation>

Brokaw, Paul & Barrow, Jeff: “Grounding for Low- and High-Frequency Circuits - Know Your Ground and Signal Paths for Effective Designs, Current Flow Seeks Path of Least Impedance - Not Just Resistance”, *The Best of Analog Dialogue*, Analog Devices, Inc., 1991, pp 193-195.

Rich, Alan: “Understanding Interference-Type Noise - How to Deal with Noise without Black Magic - There Are Rational Explanations for - and Solutions to - Noise Problems”, *The Best of Analog Dialogue*, Analog Devices, Inc., 1991, pp 120-123.

Rich, Alan: “Shielding and Guarding - How to Exclude Interference-Type Noise, What to Do and Why to Do It - A Rational Approach”, *The Best of Analog Dialogue*, Analog Devices, Inc., 1991, pp 124-129.

Walker, Charles S.: *Capacitance, Inductance and Crosstalk Analysis*, Artech House, Boston, 1990.

Williams, T: *EMC for Product Designers 2nd Ed.*, Reed Educational and Professional Publishing Ltd., 1996.

Williamson, Tom: “Designing Microcontroller Systems for Electrically Noisy Environments”, *Embedded Controller Applications Handbook*, Intel Corporation, 1988.

<http://developer.intel.com/design/mcs96/applnotes/210313.htm>

7 Printed Circuit Boards

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7.1 Overview

For electronic assemblies, printed circuit boards (PCBs) are substrates providing mechanical support as well as electrical interconnections. A PCB can be rigid or flexible, with single or multiple layers of conductors separated by insulating layers.

A sinusoid is the sum of a forward rotating phasor and a backward rotating phasor

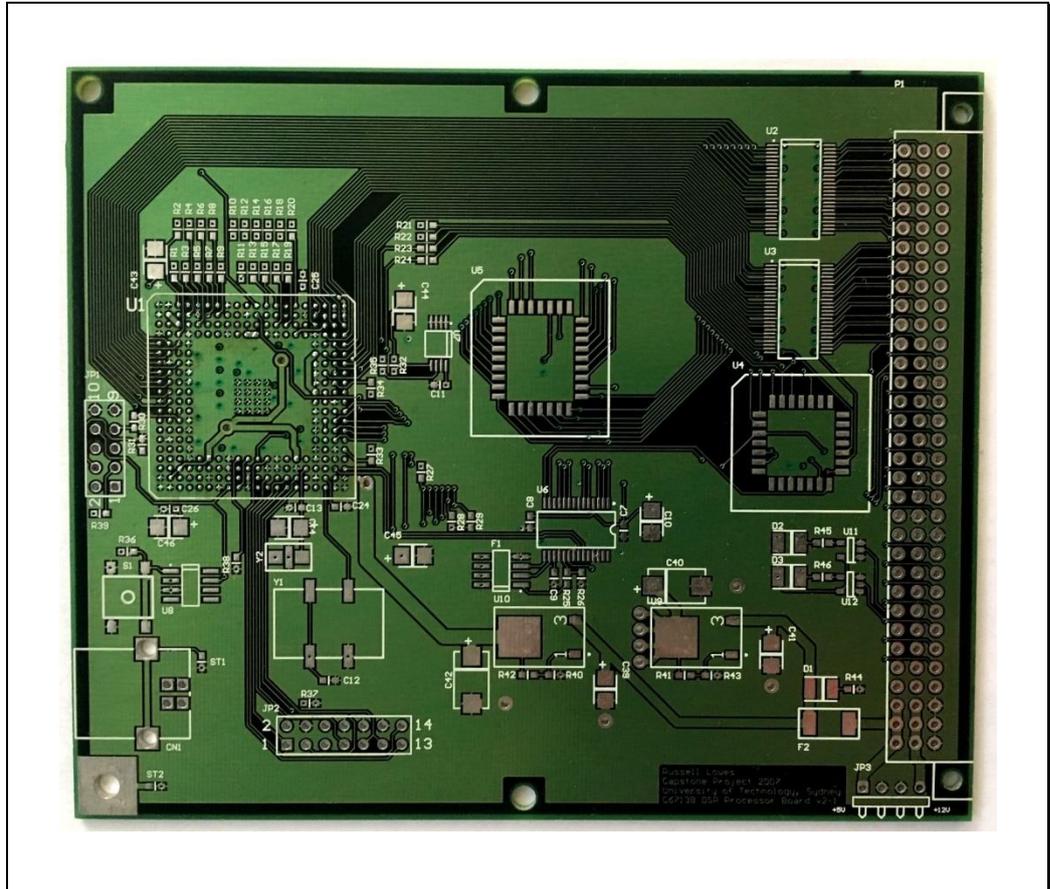


Figure 7.1 – A Printed Circuit Board (PCB)

PCB manufacturing originated in the USA. Therefore, Imperial (non-SI) units (oz, mil, in) were universally adopted for specifying PCBs. It has only been relatively recently, with the rise in Chinese manufacturing, that the units used in the industry are becoming increasingly metric.

Manufacturer Web sites may still use Imperial units – in this case refer to the following table:

Table 7.1 – Imperial Units and Their Conversions to SI Units

Imperial Unit	SI Unit	Usage
1 inch	25.4 mm	Board dimensions
1 mil = 0.001 inch	25.4 μm	Dielectric thicknesses and conductor width / spacing
1 oz	35 μm	Conductor (commonly copper) thickness (the weight of the conductor in 1 square foot of material)

7.4

7.1.1 General Dimensional Specifications

The *finished* thickness of a PCB comes in several standards: 0.8 mm, 1.0 mm and 1.6 mm. Non-standard thicknesses are readily available for high-volume orders. Actual board thicknesses range from 0.25 mm to 3 mm.

The maximum dimensions of a PCB are typically 500 mm x 400 mm, but this depends on the manufacturer.

Irregular shapes and slots are readily available, since they are routed:

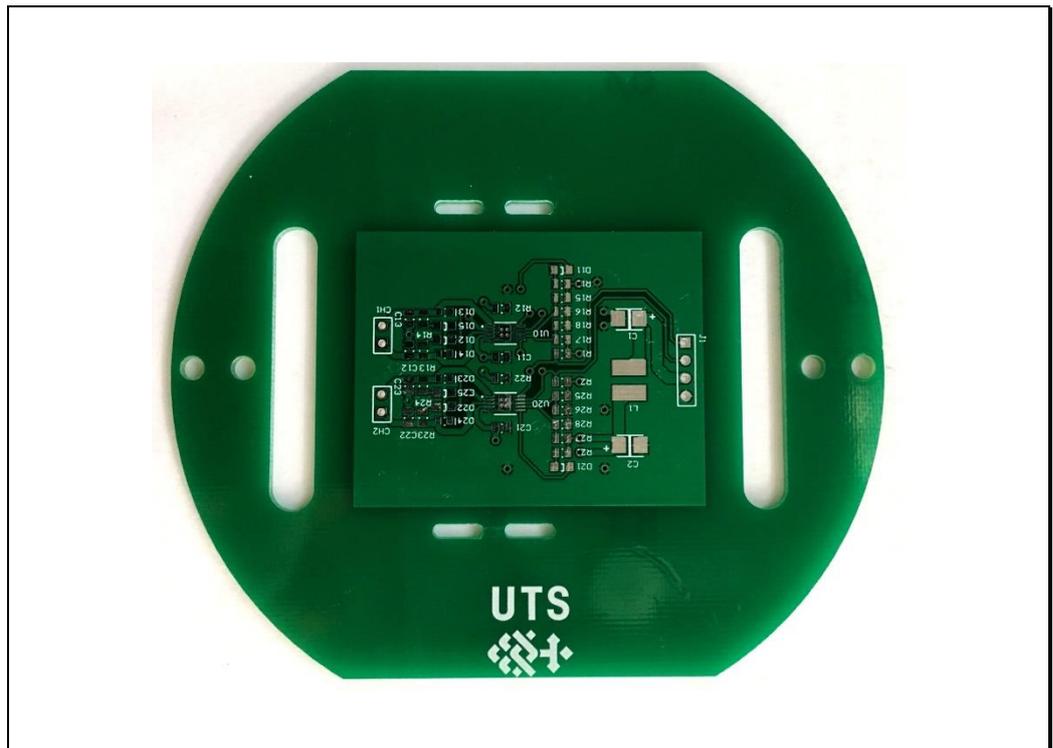


Figure 7.2 – A Rounded PCB with Slots

7.1.2 Stackup

The typical PCB *stackup* consist of alternating layers of core and prepreg:

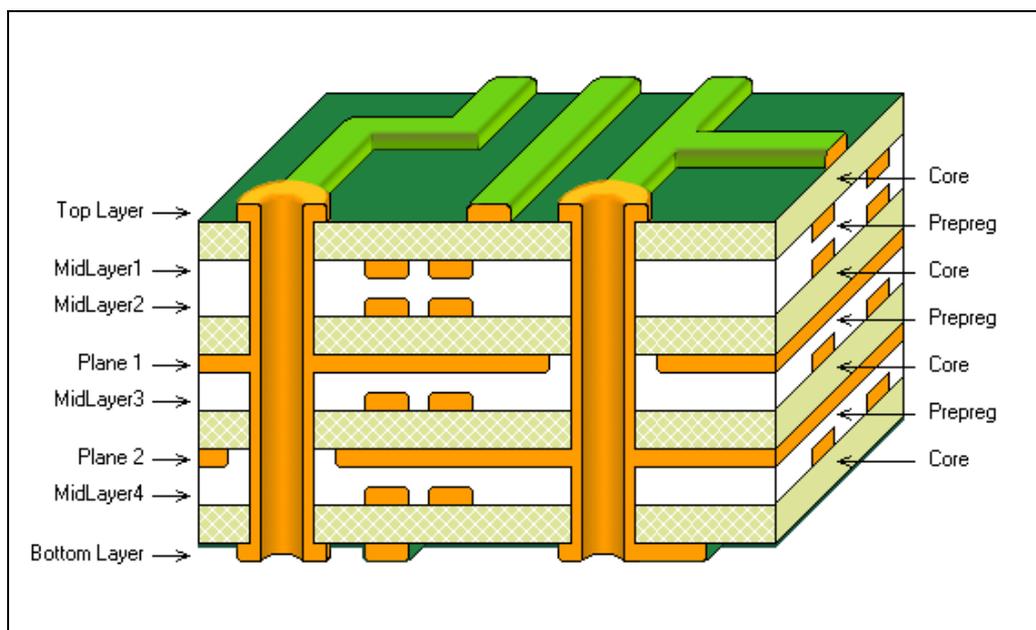


Figure 7.3 – Cross-section of a Typical 8-Layer PCB Stackup

The *core* is a thin piece of dielectric with copper foil bonded to both sides. Core dielectric is cured fiberglass-epoxy resin.

The *prepreg* is uncured fiberglass-epoxy resin. Prepreg will cure (i.e. harden) when heated and pressed.

The outermost layers are prepreg with copper foil bonded to the outside (surface foils).

To avoid crosstalk: Wires on adjacent signal layers are routed mostly orthogonally.

The stackup is symmetric about the centre of the board in the vertical axis to avoid mechanical stress in the board under thermal cycling.

7.2 Conductors

The conductor material is typically copper (*Cu*).

The number of layers can vary from single to 20-layers (and more).

Dedicated supply layers are also called “ground layers” / “ground planes” and “power planes”. Power planes are typically built on the thinnest core available from a fabrication vendor to maximize the capacitance between the planes. They often use thicker copper layers than signal layers to reduce resistance. Power plane provide stable reference voltages for signals, distribute power to all devices and control cross-talk between signals.

The most popular multilayer boards us: 4-8 signal layers plus 4-8 ground layers. Prototypes may only use 2 or 4 layers.

Conductor thicknesses typically vary between 17.5 to 105 μm . 17.5 (0.5 oz) and 35 μm (1 oz) are standard for inner layers.

There is a trend towards 8.75 μm (0.25 oz) (particularly for laminated IC packages).

The width and spacing of the conductors is usually $\geq 0.13\text{ mm}$ (5mil), which is limited by the manufacturer’s capabilities.

The *sheet resistance* of a conductor is used to estimate track resistances. Consider a track on a layer of a PCB:

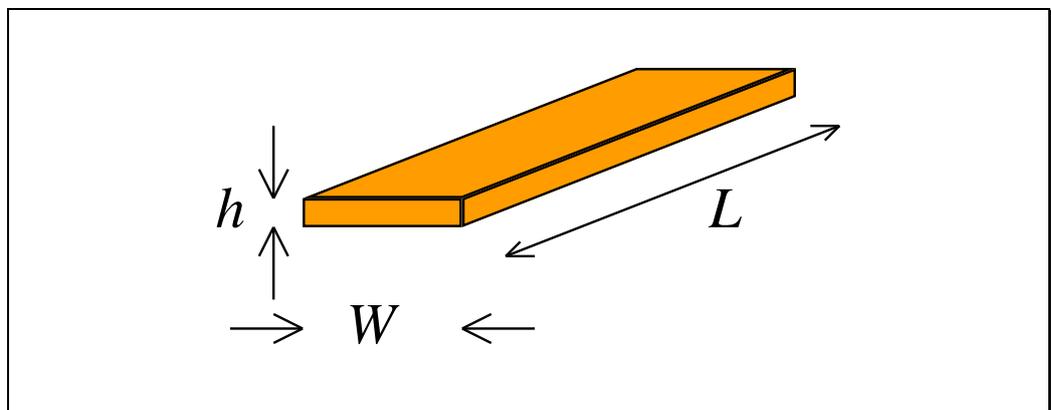


Figure 7.4 – PCB track showing dimensions

The resistance is:

$$R = \frac{\rho L}{A} = \frac{\rho}{h} \frac{L}{W} = R_s \frac{L}{W} \quad (7.1)$$

where the sheet resistance is defined as:

$$R_s = \frac{\rho}{h} \quad (7.2) \quad \text{Sheet resistance defined}$$

The concept of sheet resistance can be used to size tracks to achieve a certain resistance, and to quickly calculate the resistance of a track. The sheet resistance for copper at $T = 300$ K for various thicknesses (h) is given in the table below:

Table 7.2 – Sheet Resistance for Copper at Different Thicknesses

Thickness	R_s ($\mu\Omega$)
17.5 μm (0.5 oz)	971
35 μm (1 oz)	486
70 μm (2 oz)	243
105 μm (3 oz)	162

To determine the resistance of a copper track, you just need to multiply the sheet resistance by the L/W ratio of the track.

7.3 Insulators

There are several dielectric materials used in PCB manufacturing. Most of them support a relatively controlled permittivity which makes them suitable for high-frequencies (such as digital logic and RF) where *distributed parameter* circuit theory must be used and conductors act like *transmission lines*.

7.3.1 FR-4

The dielectric material used in PCBs is predominantly fiberglass epoxy-resin, with the most common, widely available and relatively low cost material usually going by the acronym “FR-4” which is a NEMA grade designation for glass-reinforced epoxy laminate material. FR-4 is a composite material composed of woven fiberglass cloth with an epoxy resin binder that is flame resistant (self-extinguishing). "FR" stands for flame retardant. FR-4 is a rigid structure, and can handle temperatures up to around 130°C (before it starts becoming flexible and ultimately melting).

7.3.2 CEM

Composite Epoxy Material (CEM) are a group of composite materials typically made from woven glass fabric surfaces and non-woven glass core combined with epoxy synthetic resin. They are extremely low-cost, have copper on one side only, and are generally found in mass-produced consumer items. The holes can be punched, rather than drilled. An example of a printed circuit assembly using CEM is shown below:

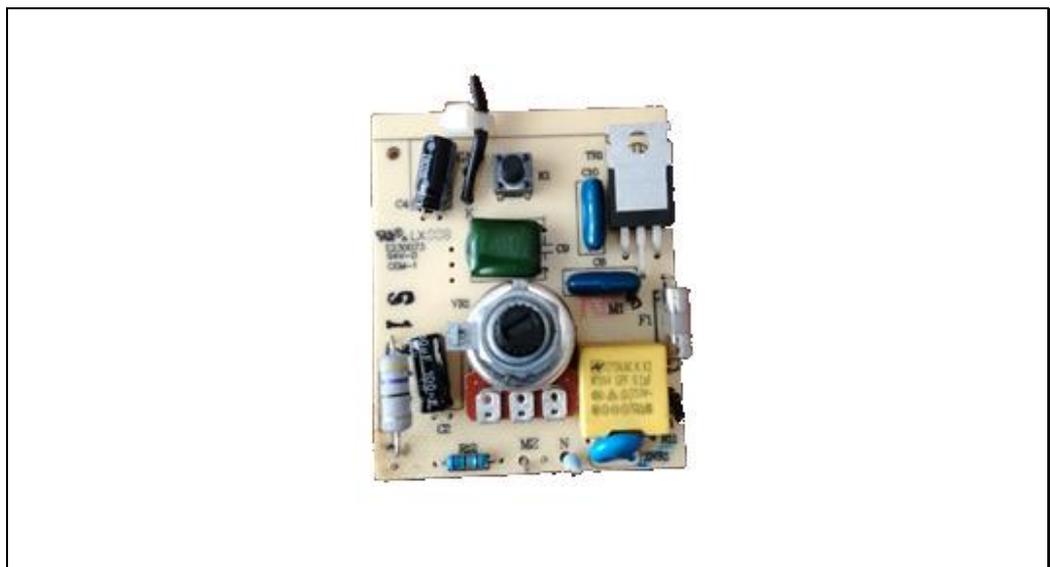


Figure 7.5 – A CEM Printed Circuit Assembly

7.3.3 Polyamide

Polyamide PCBs can be made flexible or rigid and can tolerate a high temperature. They are generally used to wire between PCBs, such as connecting a main PCB to a display panel:

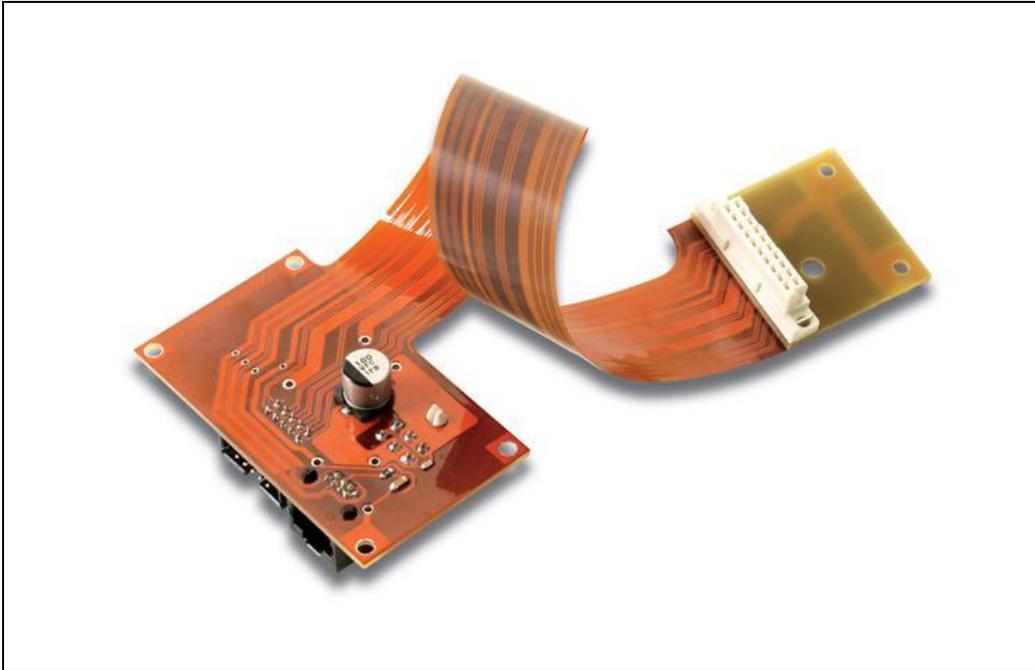


Figure 7.6 – A Polyamide Flexible PCB

7.3.4 PTFE

Polytetrafluoroethylene (PTFE) is the scientific name for the very well-known material trademarked by DuPont called Teflon. PTFE materials are used in high-frequency applications (above 5 GHz) such as wireless communications and satellites. At RF and microwave frequencies, the dielectric constant of standard FR-4 material (approx. 4.5) is often too high, resulting in significant signal loss during transmission across the PCB. PTFE materials boast dielectric constant values as low as 3.5 or below, making them ideal for overcoming the high-speed limitations of FR-4. In addition to a reduction in dielectric constant, these materials also offer very impressive thermal characteristics for high temperature PCB applications, where they can be used as high as 250°C.

7.4 Vias

You interconnect layers in a PCB with the use of *vias*. There are a variety of via types, shown in the figure below:

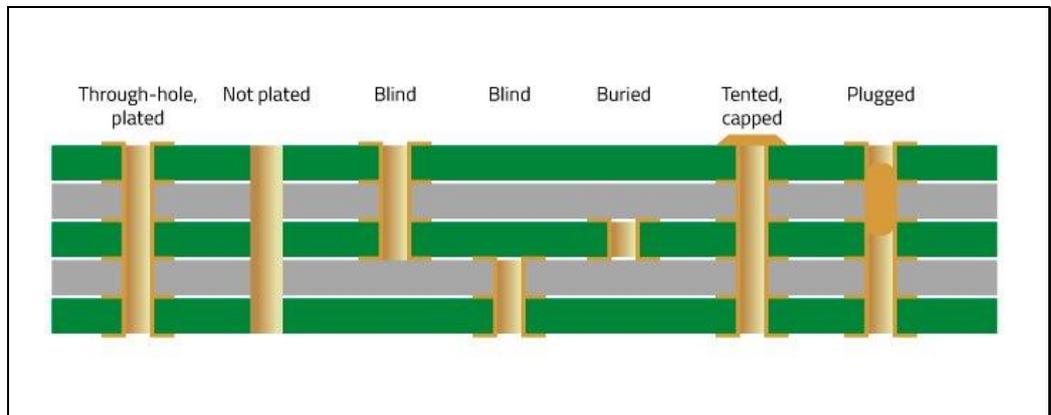


Figure 7.7 – Styles of Via

The via type and dimensions impact on manufacturability.

Vias require pads on each layer to which they connect. Because the holes are not guaranteed to be perfectly aligned with the copper traces there needs to be an annulus of copper around the plated hole. This is to ensure that the copper won't be broken by the drilling operation. Pads on inner vias are larger than outer pads to allow for greater dimensional tolerances.

Vias reduce wiring density and are therefore expensive.

Where a via passes through a plane (i.e. not connected to the plane) a clearance hole is required. Where a via is supposed to connect to a plane, sometimes a thermal relief structure is required (usually four small metal bridges between via and plane) to reduce heat flowing from the plane to the track.

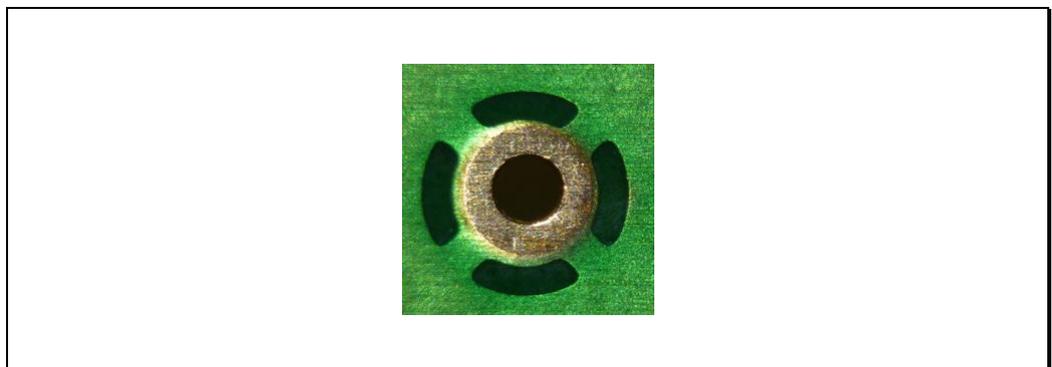


Figure 7.8 – A Via with Thermal Relief

There are two special vias available for high-volume PCBs:

- Blind vias (connection of outer layer to inner layer)
- Buried vias (connection of inner layers - preferably on same core!)

The advantages are:

- Increased wiring density (vias don't occupy all layers)
- Product safety (creepage and clearance distances for electrical insulation)

The disadvantages are:

- Restricted choice of suppliers
- More complex process leading to increased cost and reduced reliability

7.5 Manufacturing Process

The manufacturing process steps for a typical rigid multilayer PCB (representing about 70% of all PCBs manufactured) is:

1. PCB data acquisition
2. Preparation of PCB laminate (core)
3. Inner layer image transfer
4. Laminate layers
5. Drilling and cleaning holes
6. Make holes conductive
7. Outer layer image transfer
8. Surface finish
9. Final fabrication

7.12

7.5.1 PCB data acquisition

Gerber files, drill files and fabrication drawings are transferred to the PCB manufacturer.

The file are reviewed, and if all is well, they will proceed with the order.

The manufacturer will then embark on PCB tooling:

- Photo-tool for image transfer (image created by PCB software is reproduced on film using laser photoplotters)
- Drill files
- Profile routing files
- CNC route file
- All tooling is stepped and repeated for optimum utilisation of standard panels (24in x 18in)

7.5.2 Preparation of PCB laminate (core)

- Dielectric material: Woven glass fiber or paper

The material depends on the function of the PCB. Some materials perform better in some environments than others (heat, humidity). Some materials are more suitable for particular manufacturing processes (e.g. hole punching). Others again are chosen for electric properties (permittivity). Most widely used: FR-4 / CEM.

- Coat/impregnate dielectric material with resin & harden
- Copper foil is rolled or electrolytically deposited on the base laminate
- Core material is sheared to panel size
- Core material is cleaned mechanically and/or chemically

Removal of surface contamination required to promote subsequent adhesion of photoresist (PR)

At the completion of this step, the PCB looks like:



Figure 7.9 – Preparation of PCB Laminate

7.5.3 Inner Layer Image Transfer (photo-lithography)

The purpose of this step is to transfer the circuit image to the core through a “print-and-etch” process:

- Coat copper foils with photoresist (PR)

Negative PR: Light-sensitive organic PR polymerises (“hardens”) when exposed to light. Polymerised PR will resist etching.

- Place phototool and expose to light.

After exposure, PR layer is developed. Polymerised areas remain, unexposed areas are washed away.

- Etching

Selectively remove exposed copper areas. Etching is performed with conveyorised equipment (etchant flood rinse, several water rinses). Common etchants: Acidic cupric chloride and ammoniacal alkaline solution.

At the completion of this step, the PCB looks like:

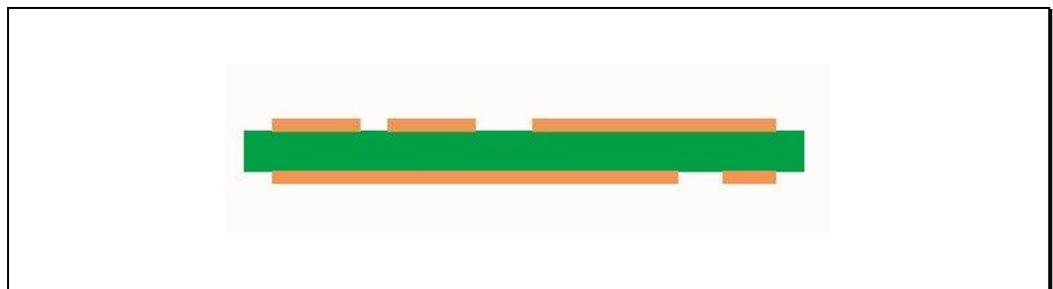


Figure 7.10 – PCB after Inner Layer Image Transfer

7.5.4 Laminate Layers

Cores are pinned in a stack with sheets of prepreg ("pre-impregnated" composite fibers where an epoxy is already present) that are in the "B-stage" (partially cured) separating the copper layers. Outer layers are made with a foil of copper.

Prepreg is available in different styles with varying amounts of resin and glass fibers. This allows the manufacturer to control the thickness between layers and the thickness of the overall PCB.

The horizontal alignment is critical! The stack is pinned between two heavy metal plates creating a "book". The book is put in a heated hydraulic press for about 2h.

The lamination step looks like:

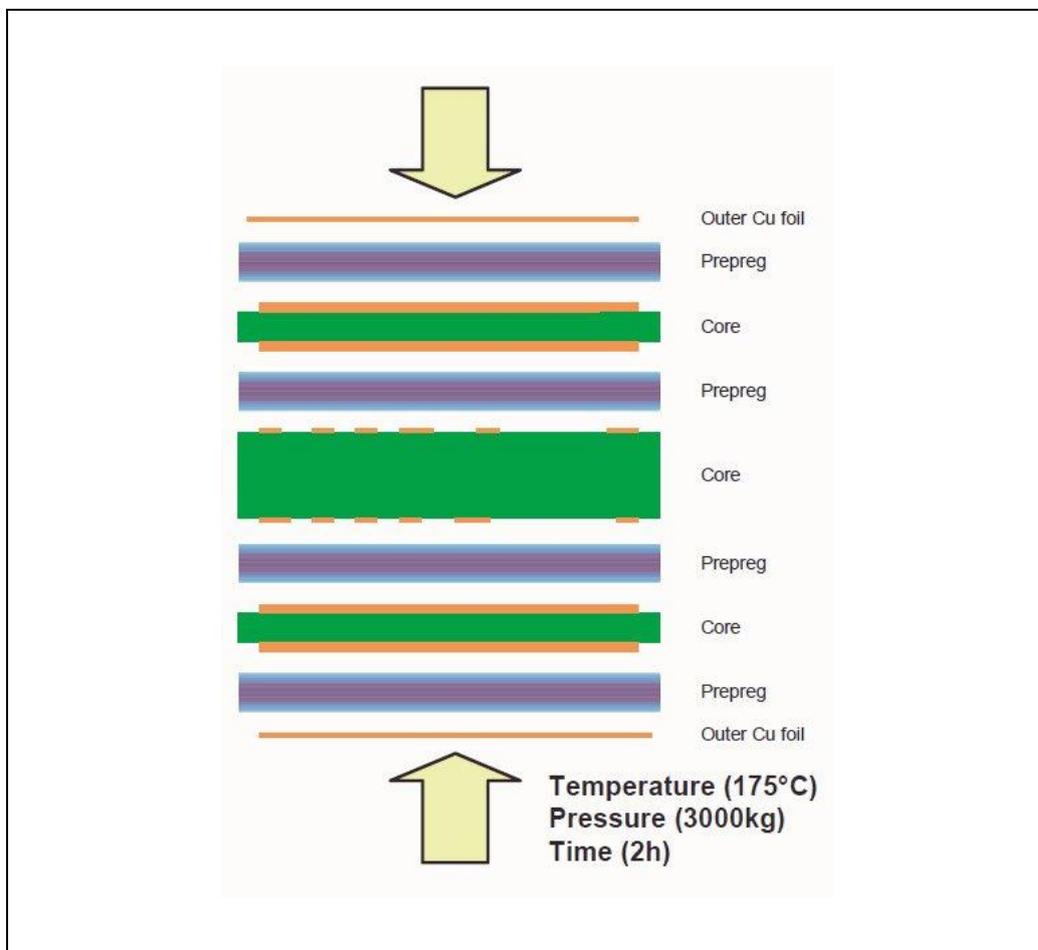


Figure 7.11 – Laminating PCB Layers

7.5.5 Drilling and Cleaning

Holes are drilled through the PCB to interconnect layers (with vias), and to allow the insertion of plated-through-hole (PTH) components.

- Drilling performed with CNC equipment using drill files.

Alternative methods to drilling exist (punching, laser).

- Multiple panels can be drilled together.

Drilling of complex boards can take several hours per load.

- Desmear

Desmear removes the melted resin smear

- Etchback

Etch glassfibers. Goal: Copper on the inner layers protrude out into the barrel of the hole.

- Panels are deburred/scrubbed after drilling

At the completion of this step, the PCB looks like:

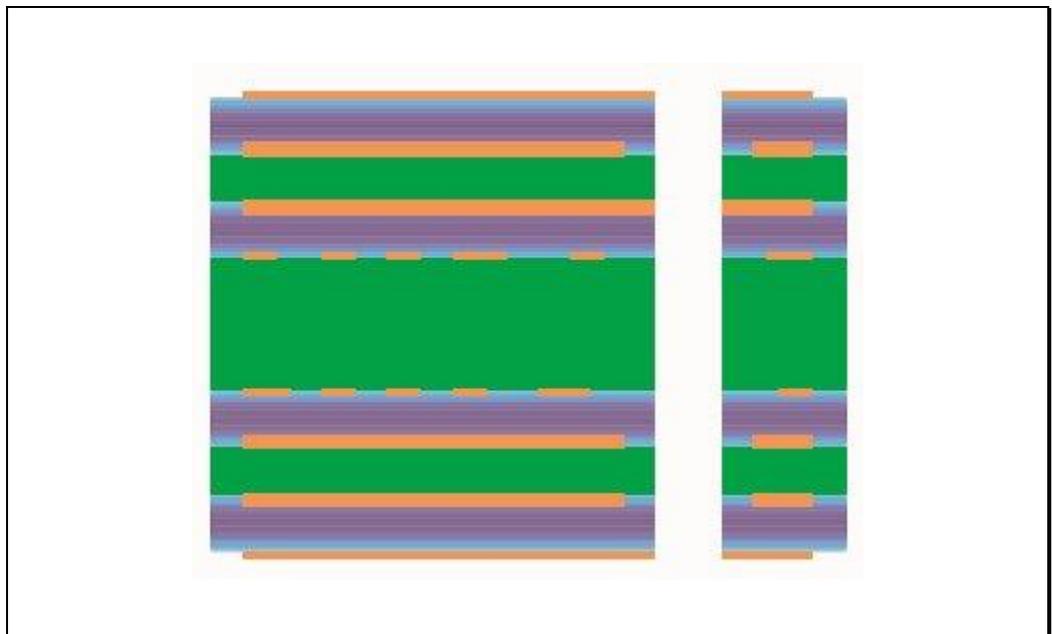


Figure 7.12 – PCB after Drilling and Cleaning

7.5.6 Make Holes Conductive

The PCB substrate is not conductive. Therefore a non-electrolytic deposition method is required. In a later process step, electroplating to the required thickness can be performed.

- Process: Electroless copper

Electroless copper is reliable but alternative methods exist. Electroless copper has some significant disadvantages (like exposure to formaldehyde, which is a carcinogen).

- Electroless copper bath deposits up to 3 μm of copper
- Constituents of electroless copper:

Sodium hydroxide, formaldehyde, EDTA and copper salt. Complex reaction catalysed by palladium, formaldehyde reduces the copper ion to metallic copper.

At the completion of this step, the PCB looks like:

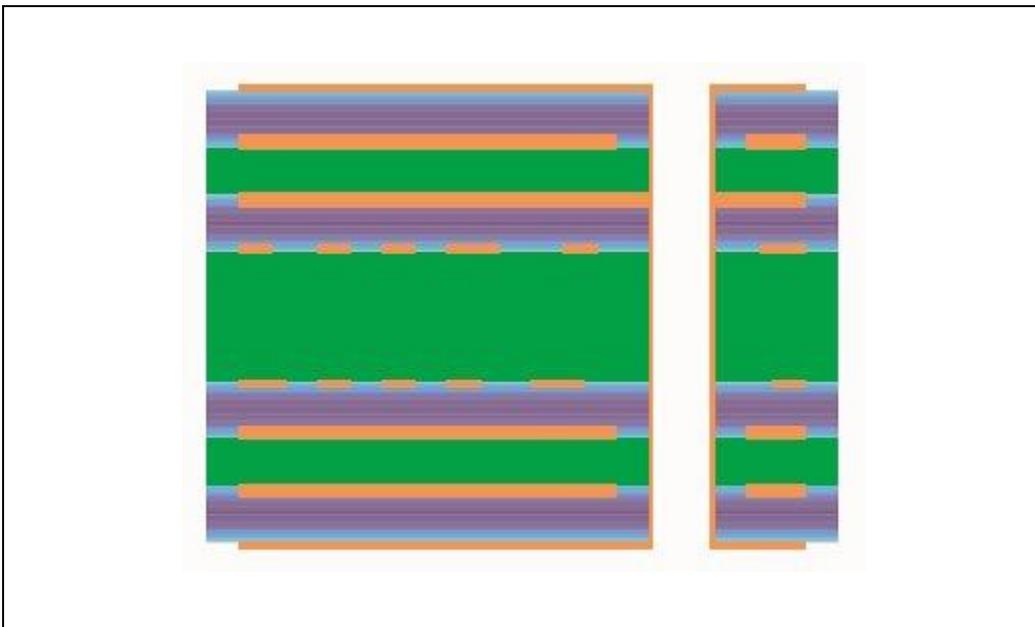


Figure 7.13 – PCB after making Holes Conductive

7.5.7 Outer Layer Image Transfer

The most common process is: Print, pattern plate, and etch:

- Coat copper foils with photoresist (PR)
- Place phototool and expose to light

Outer layer phototools are positive images of the circuit. Circuit image is developed away exposing the copper. PR remaining on the panel will act as plating resist.

- Pattern plating (copper electroplating)

Outer layers will be plated to a thickness of 1.5 mil (to ensure a minimum thickness of 1 mil in the holes). Copper electroplating takes place in a copper sulfate bath. Plating is performed at roughly 30 A/ft². Plating duration is roughly 1h.

- Plate metallic etch resist
- Etching

At the completion of this step, the PCB looks like:

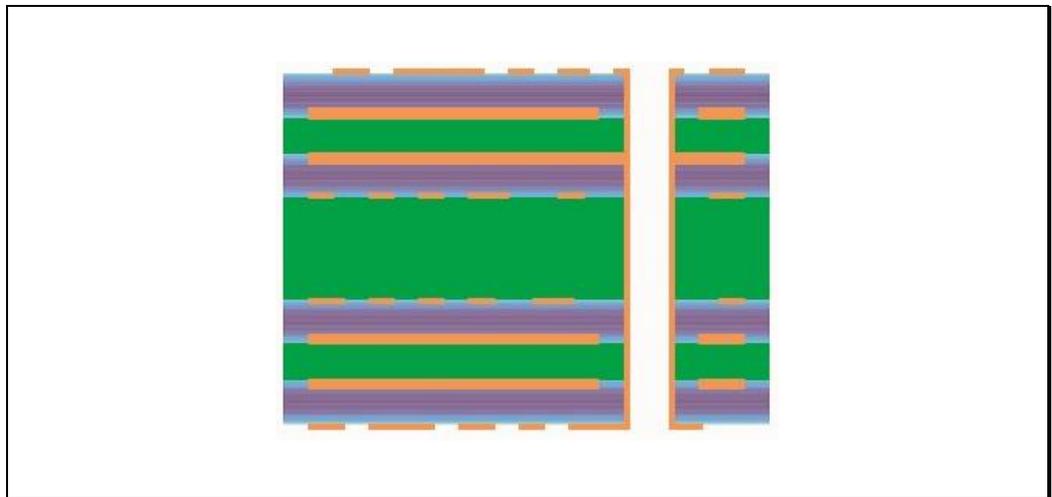


Figure 7.14 – PCB after Outer Image Layer Transfer

7.5.8 Surface Finish

The purpose of this stage is to prevent copper oxidation and to facilitate solderability.

- Most popular surface finish process: SMOBC/HASL:SMOBC:

Solder-mask-over-bare-copper. HASL: hot-air-solder-levelling

- Solder mask pre-clean (mechanical scrub)

- Application of solder mask

Purpose of solder mask: Insulate those portions where no solder is required. Most popular mask type: LPI (liquid photoimageable).

- Application of flux

Provides oxidation protection. Affects heat transfer during solder immersion.

- HASL

Panels are dipped into molten solder (237°C). Panels are then rapidly carried past jets of hot air. Exposed copper is coated with solder. Masked areas remain solder free. Panels are then cleaned in hot water.

7.5.9 Final fabrication

In this step mechanical features are added to the board (mounting holes, cut-outs, etc.)

- Routing done through CNC machines
- De-panelisation
 - Partial de-panelisation. Most of the circuit is routed out of the panel, but tabs remain to hold the circuit in place. This allows the assembly machine to populate multiple boards at once. Afterwards, the circuit can be snapped or broken out of the panel. Such panels are called “breakaways”, “snaps”, or “arrays”.
 - The alternative to partial de-panelisation is to have the panel V-scored. V-scoring is done through a thin rotating scoring blade that will route across the top and the bottom of the panel with about 30% of the thickness of the panel. V-scoring allows more circuits per panel (no spacing is required for routing bits).

7.6 Panels

PCBs are fabricated in panels to minimise the cost to the PCB manufacturer and assembly manufacturer.

Typical panel dimensions are 610 x 460 mm.

Overall board dimensions are kept much smaller than a full panel.

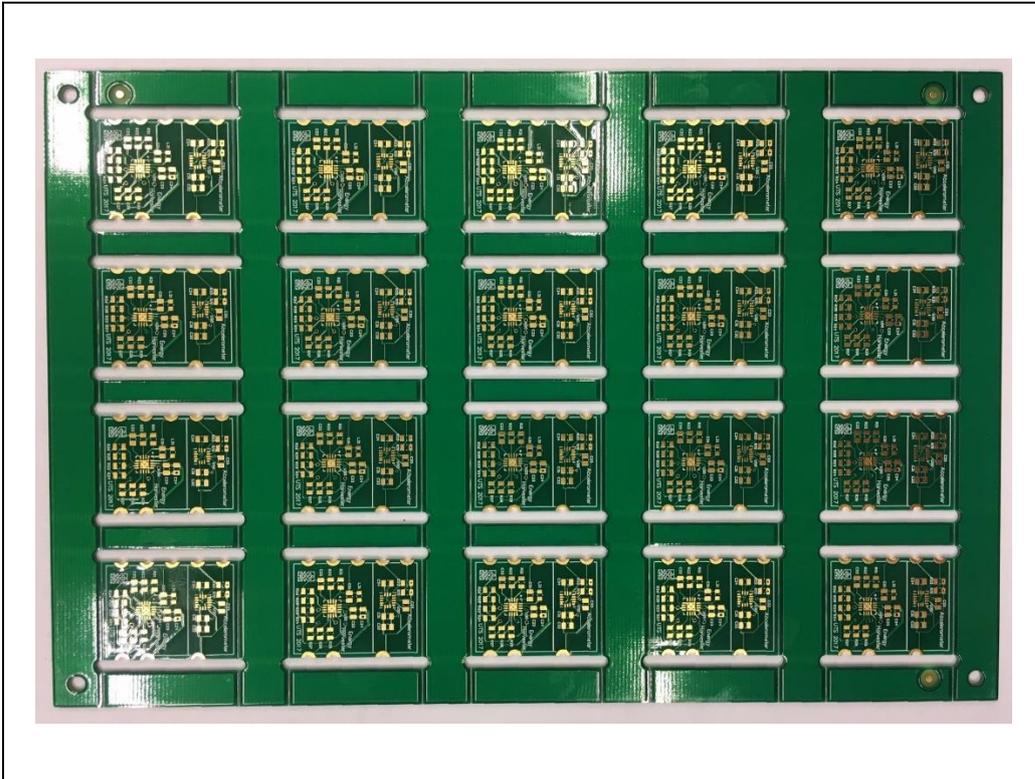


Figure 7.15 – A PCB panel

7.7 Printed Circuit Assembly

A printed circuit assembly (PCA) is a PCB that has been populated with components. The typical manufacturing process is:

- Silk-screen solder paste onto the panel
- Pick-and-place (P&P) components (machine or manually)
- Heavier components and components on the bottom side of the board need to be glued down (epoxy-based glue)
- Soldering in IR reflow oven (providing the required reflow temperature profile of gradually increasing, sustaining, and removing heat)
- Visual inspection (manual or vision systems)
- Electrical testing using automated test equipment (ATE)

For normal manufacturing, whole panels are done at once. For engineering prototyping work, only 1 or 2 boards are hand-soldered.

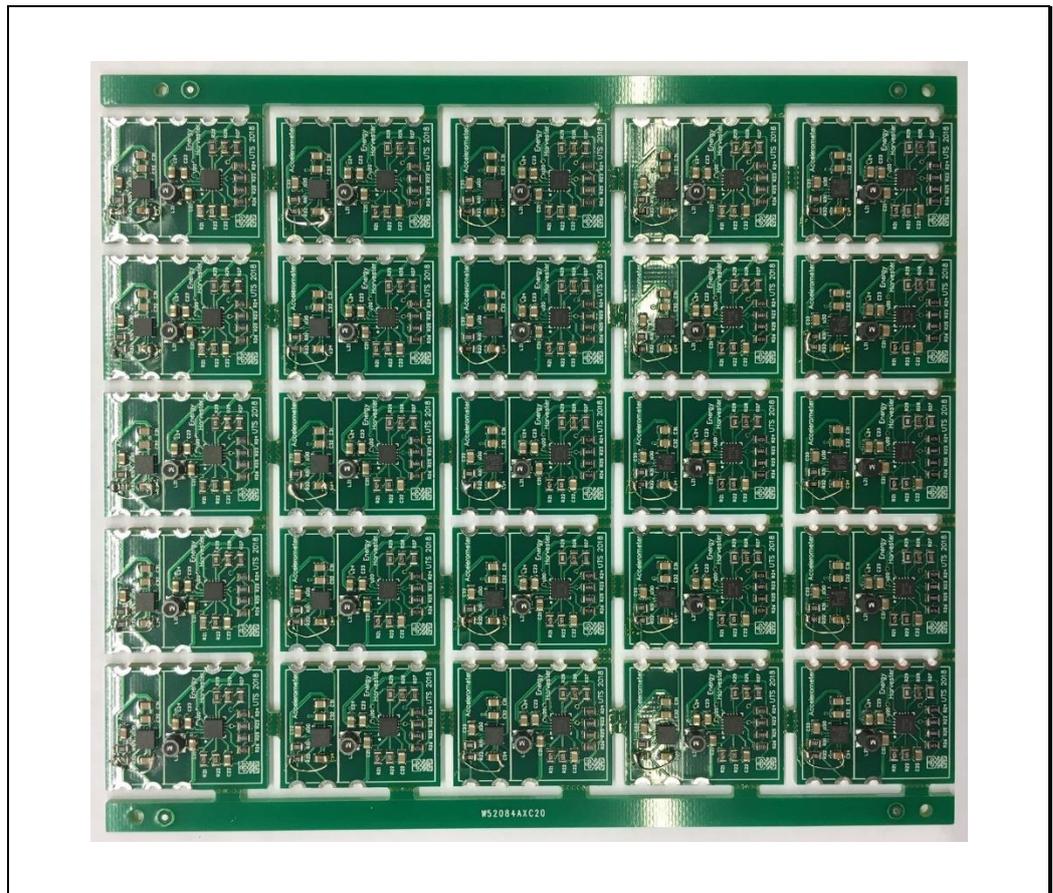


Figure 7.16 – A panel of completed PCAs

8 Direct Filter Realisations

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8.2 Frequency Transformations	8.11
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Introduction

All our design strategies so far have hinged around the fact that we will be implementing the filter with a *cascade* of first-order and second-order circuits. We can do this with op-amp circuits because they provide buffering.

Active filters cannot be used all the time

Sometimes we cannot use op-amps in a filter. For example, the gain of op-amps decreases with frequency, so they cannot be used for high frequency (>1MHz) applications. Sometimes we require the filter to handle large voltages, or power, and so again the op-amp cannot be used. Sometimes we cannot even justify the use of a power supply for the op-amps. In these cases we have to resort to filters made up of passive elements only.

We will now examine various methods of implementing a transfer function *directly* - that is, without splitting it up into a cascade of smaller transfer functions.

8.1 Doubly Terminated Lossless Ladders

The very first filters were made up of inductors and capacitors arranged in ladder form:

Lossless ladder defined

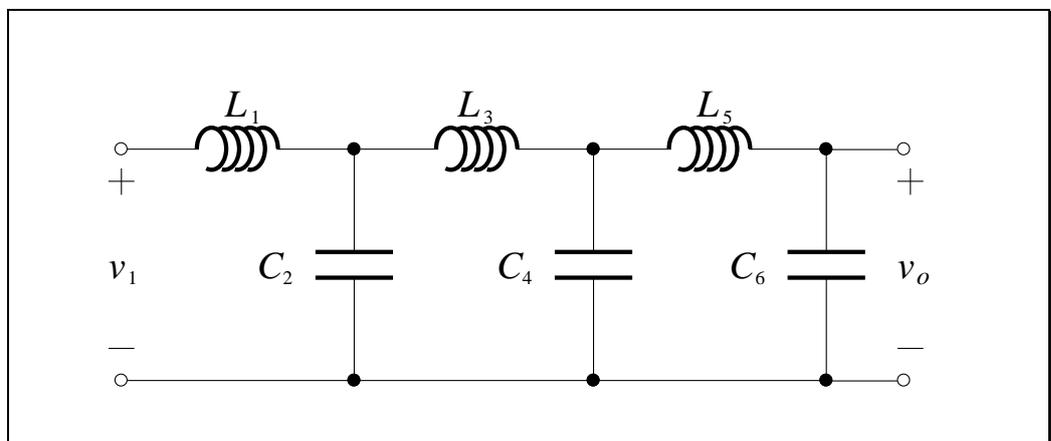


Figure 8.1

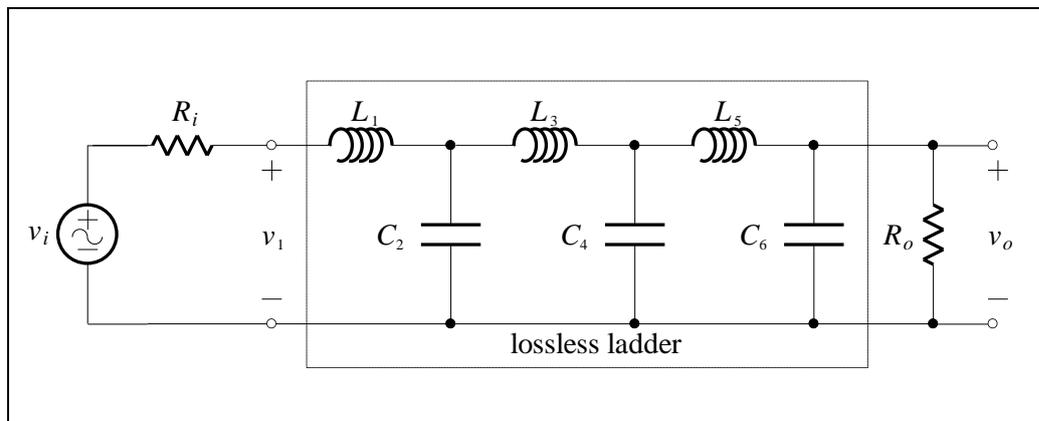
Since it is made up of inductors and capacitors only, it is also known as a *lossless ladder* circuit. At DC, the gain of the ladder is obviously 1. The gain drops off at high frequency, so it is actually a lowpass filter. It can be shown that the transfer function of such a ladder is:

$$\mathbf{T}(s) = \frac{1}{\mathbf{Q}(s)} \quad (8.1)$$

where $\mathbf{Q}(s)$ is a polynomial of degree n , n being the total number of inductors plus capacitors. Such a transfer function is referred to as an *all-pole* function. For the lossless ladder by itself, the polynomial $\mathbf{Q}(s)$ has only even terms, such as s^2 , s^4 etc.

A lossless ladder is an all-pole lowpass filter, with even s terms only

If there is a load at the output side, and also a resistor at the input side to represent the resistance of the source, then we have a *doubly terminated lossless ladder* circuit:



Doubly terminated lossless ladder defined

Figure 8.2

The transfer function for a general doubly terminated lossless ladder has the form:

$$\mathbf{T}(s) = \frac{K}{\mathbf{Q}(s)} \quad (8.2)$$

8.4

A doubly terminated lossless ladder has even and odd s terms

The addition of the two resistors does not increase the order of the denominator polynomial. However, the polynomial now contains both even and odd terms. The K in the numerator takes account of the fact that at DC the circuit forms a potential divider:

The gain of a doubly terminated lossless ladder at DC

$$\mathbf{T}(j0) = \frac{K}{\mathbf{Q}(j0)} = K = \frac{R_o}{R_i + R_o} \tag{8.3}$$

The lossless ladder is a simple structure since it consists of a series inductor, then a parallel capacitor, then a series inductor etc. We will derive a synthesis strategy where the values of the inductors and capacitors are readily obtained. The circuit we will use for synthesis is shown below:

Lossless ladder design

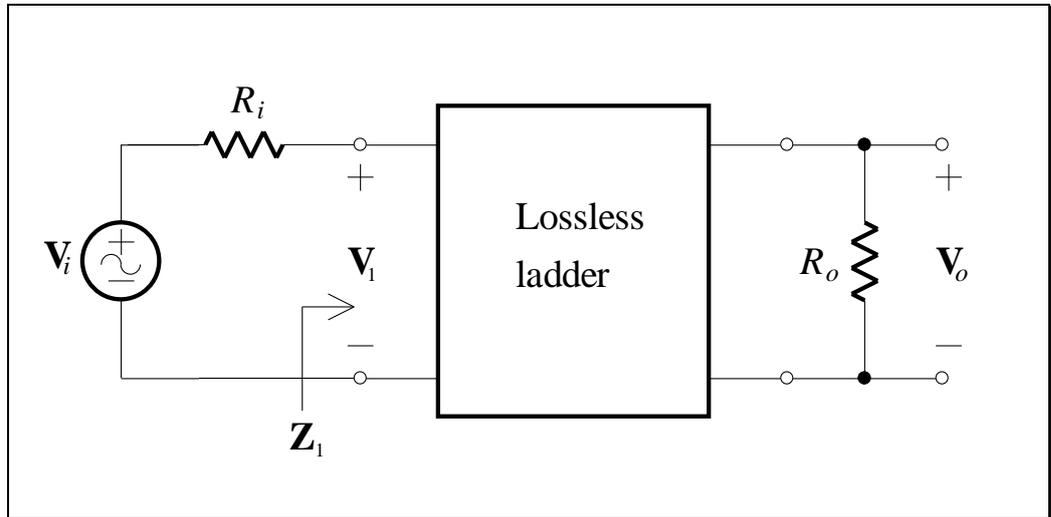


Figure 8.3

The input impedance of the lossless ladder plus terminating resistor R_o is identified as Z_1 :

$$\mathbf{Z}_1 = R_1 + jX_1 \tag{8.4}$$

Since the ladder is lossless, we can equate the input average power to the output average power:

$$\frac{R_1 |\mathbf{V}_i(j\omega)|^2}{|R_i + \mathbf{Z}_1|^2} = \frac{|\mathbf{V}_o(j\omega)|^2}{R_o} \quad (8.5) \quad \text{begins with equating input and output power,}$$

From this we get the magnitude squared of the desired transfer function:

$$\left| \frac{\mathbf{V}_o(j\omega)}{\mathbf{V}_i(j\omega)} \right|^2 = |\mathbf{T}(j\omega)|^2 = \frac{R_o R_1}{|R_i + \mathbf{Z}_1|^2} \quad (8.6)$$

Now we introduce an auxiliary function:

$$|\mathbf{A}(j\omega)|^2 = 1 - 4 \frac{R_i}{R_o} |\mathbf{T}(j\omega)|^2 \quad (8.7) \quad \text{then we define an auxiliary function in terms of the transfer function}$$

and noting that:

$$|R_i + \mathbf{Z}_1|^2 - |R_i - \mathbf{Z}_1|^2 = 4R_i R_1 \quad (8.8)$$

we get:

$$|\mathbf{A}(j\omega)|^2 = \mathbf{A}(\mathbf{s})\mathbf{A}(-\mathbf{s}) \Big|_{\mathbf{s}=j\omega} = \frac{|R_i - \mathbf{Z}_1|^2}{|R_i + \mathbf{Z}_1|^2} \quad (8.9)$$

We as usual separate $\mathbf{A}(\mathbf{s})$ from $\mathbf{A}(-\mathbf{s})$ to give:

$$\mathbf{A}(\mathbf{s}) = \pm \frac{R_i - \mathbf{Z}_1}{R_i + \mathbf{Z}_1} \quad (8.10)$$

8.6

We can now specify Z_1 in terms of the auxiliary function:

The input impedance of the lossless ladder plus termination in terms of the auxiliary function

$$\mathbf{Z}_1 = R_i \frac{1 - \mathbf{A}(s)}{1 + \mathbf{A}(s)} \quad \text{or} \quad \mathbf{Z}_1 = R_i \frac{1 + \mathbf{A}(s)}{1 - \mathbf{A}(s)} \quad (8.11)$$

With this equation, we can determine the lossless ladder circuit for a given transfer function.

Determining the ladder to give a Butterworth response

To illustrate, consider the problem of determining a Butterworth response for which there are normalised terminations $R_i = R_o = 1$. The transfer function of the doubly terminated lossless ladder is given by Eq. (8.2) and for a Butterworth response with equal terminations we have:

$$|\mathbf{T}(j\omega)|^2 = \frac{K^2}{|\mathbf{Q}(j\omega)|^2} = \frac{1/4}{|\mathbf{B}(j\omega)|^2} \quad (8.12)$$

where \mathbf{B} is the Butterworth polynomial. The auxiliary function, by Eq. (8.7) is:

$$|\mathbf{A}(j\omega)|^2 = 1 - \frac{1}{1 + \omega^{2n}} = \frac{\omega^{2n}}{1 + \omega^{2n}} \quad (8.13)$$

Since:

$$\omega^{2n} = (j\omega)^n (-j\omega)^n = \mathbf{s}^n (-\mathbf{s})^n \quad (8.14)$$

then:

The auxiliary function for a Butterworth response

$$\mathbf{A}(s) = \frac{\mathbf{s}^n}{\mathbf{B}(s)} \quad (8.15)$$

From Eq. (8.11) we finally obtain:

$$\mathbf{Z}_1 = \left(\frac{1 - \mathbf{A}}{1 + \mathbf{A}} \right)^{\pm 1} = \left(\frac{\mathbf{B} - \mathbf{s}^n}{\mathbf{B} + \mathbf{s}^n} \right)^{\pm 1} \quad (8.16)$$

Input impedance for
a Butterworth ladder
filter

EXAMPLE 8.1 Doubly Terminated Lossless Ladder Design

We wish to design a doubly terminated lossless ladder for a Butterworth response with $n = 3$. We know that $\mathbf{B}(s) = s^3 + 2s^2 + 2s + 1$, so \mathbf{Z}_1 is found directly from Eq. (8.16) as:

$$\mathbf{Z}_1 = \left(\frac{2s^2 + 2s + 1}{2s^3 + 2s^2 + 2s + 1} \right)^{\pm 1}$$

Take the case where the power is -1. Then the numerator is of a higher order than the denominator, and we can express this as:

$$\mathbf{Z}_1 = s + \frac{s + 1}{2s^2 + 2s + 1} = s + \frac{1}{2s + \frac{1}{s + 1}}$$

which is of the form:

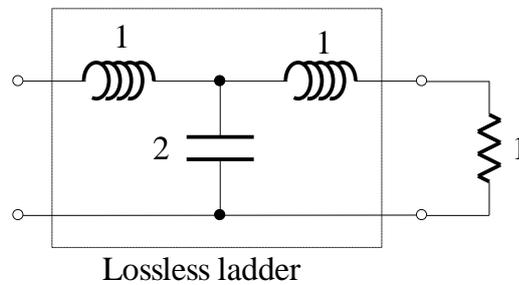
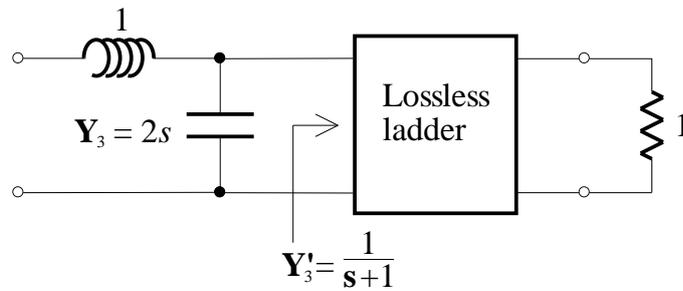
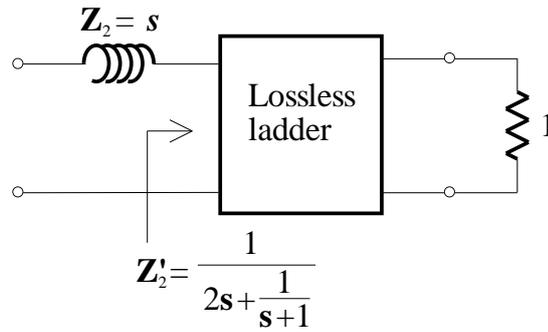
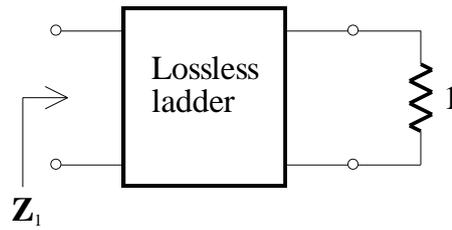
$$\mathbf{Z}_1 = \mathbf{Z}_2 + \frac{1}{\mathbf{Y}_3 + \frac{1}{\mathbf{Z}_4}}$$

The form of the
ladder impedance

8.8

There is a pattern we can follow in drawing the circuit with this impedance:

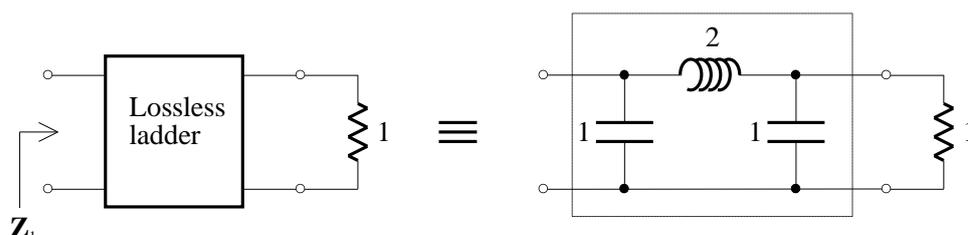
Determining the ladder directly from the impedance



If we take the power in the first equation as 1, we get:

$$Z_1 = \frac{1}{s + \frac{1}{2s + \frac{1}{s+1}}}$$

Show that the circuit in this case is:



There are two ways to build a ladder circuit - they are duals

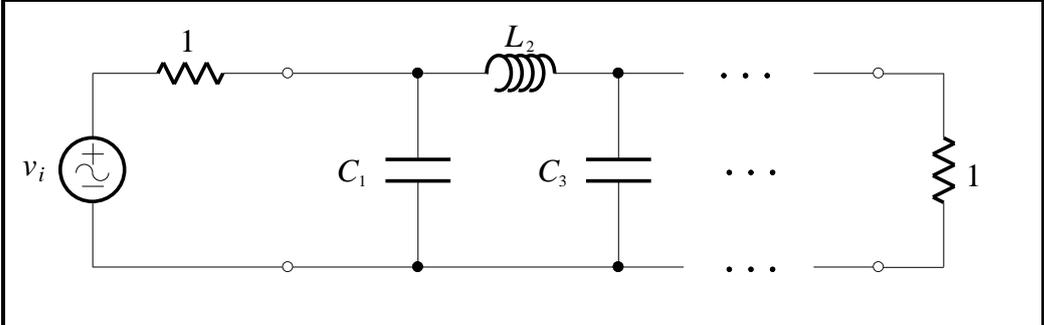
The “T” and “Π” (pi) circuits in the figures above are *duals* of each other. The two realisations are equivalent as far as the transfer function is concerned, so we would choose one over the other depending on whether inductors or capacitors are easily obtained. The “T” circuit is called a *minimum-capacitance* realisation, the “Π” circuit is called a *minimum-inductance* realisation.

8.10

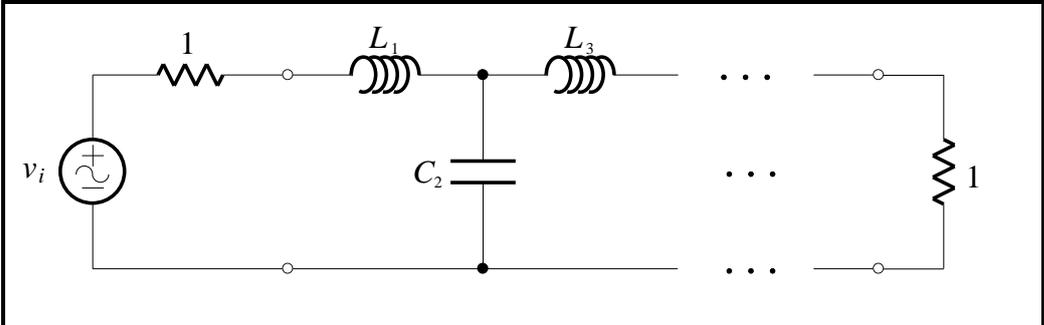
We ordinarily use tables to design ladder filters

Tables of element values exist for all the common filter responses – Butterworth, Chebyshev, Cauer etc. We would not normally derive the element values for a lossless ladder filter - we would use a table, such as the one below:

Table 8.1 – Element values for normalised lowpass Butterworth filters



n	C_1	L_2	C_3	L_4	C_5
2	1.414	1.414			
3	1.000	2.000	1.000		
4	0.7654	1.848	1.848	0.7654	
5	0.6180	1.618	2.000	1.618	0.6180
n	L_1	C_2	L_3	C_4	L_5



8.2 Frequency Transformations

We can use the frequency transformations based on Foster reactance functions to convert a lowpass prototype circuit into some other kind of circuit, such as a bandpass circuit. We begin with a doubly terminated lossless ladder lowpass prototype circuit:

We can apply frequency transformations to the lowpass ladder filter

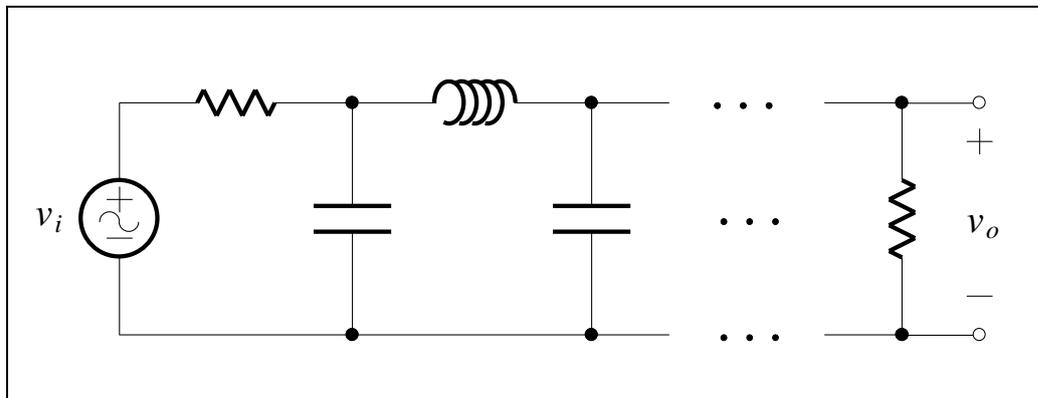


Figure 8.4

Let the L 's and C 's in the prototype circuit have the subscript p . For every inductor in the prototype, we have:

$$\mathbf{Z} = \mathbf{S}L_p$$

(8.17) The prototype impedance of an inductor

where \mathbf{S} is the lowpass prototype complex frequency. For every capacitor:

$$\mathbf{Y} = \mathbf{S}C_p$$

(8.18) and admittance of a capacitor

8.12

8.2.1 Lowpass to Highpass

In transforming from highpass to lowpass prototype, we use the frequency transformation:

$$\mathbf{S} = \frac{\omega_0}{\mathbf{s}} \quad (8.19)$$

Substituting this into Eqs. (8.21) and (8.22) gives:

$$\mathbf{Z} = \frac{\omega_0}{\mathbf{s}} L_p = \frac{1}{\mathbf{s}C} \quad (8.20)$$

swap roles in a highpass filter

$$\mathbf{Y} = \frac{\omega_0}{\mathbf{s}} C_p = \frac{1}{\mathbf{s}L} \quad (8.21)$$

The frequency transformation converts *elements* of the lowpass prototype. In this case, each inductor is turned into a capacitor, given by:

and are frequency scaled

$$C = \frac{1}{\omega_0 L_p} \quad (8.22)$$

Similarly, each capacitor is turned into an inductor, given by:

$$L = \frac{1}{\omega_0 C_p} \quad (8.23)$$

8.2.2 Lowpass to Bandpass

For the bandpass case, the frequency transformation is:

$$\mathbf{S} = \frac{1}{bw} \frac{s^2 + \omega_0^2}{s} \tag{8.24}$$

Again, substituting this into Eqs. (8.17) and (8.18) gives:

$$\mathbf{Z} = s \frac{L_p}{bw} + \frac{\omega_0^2 L_p}{sbw} = sL_1 + \frac{1}{sC_1} \tag{8.25}$$

$$\mathbf{Y} = s \frac{C_p}{bw} + \frac{\omega_0^2 C_p}{sbw} = sC_2 + \frac{1}{sL_2} \tag{8.26}$$

For bandpass transformations, elements are converted into series and parallel LC circuits

We see that each inductor is transformed into a series LC circuit, and each capacitor transforms into a parallel LC circuit.

The frequency transformation of elements is summarised in the table below:

Table 8.2 – L and C Transformations

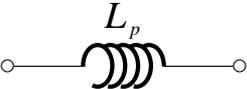
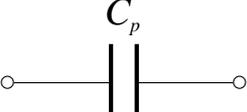
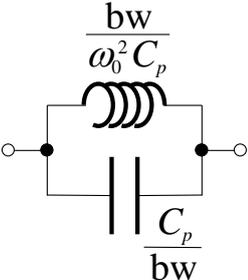
Prototype (lowpass) elements	Highpass elements	Bandpass elements
	$\frac{1}{\omega_0 L_p}$ 	$\frac{L_p}{bw}$ and $\frac{bw}{\omega_0^2 L_p}$ 
	$\frac{1}{\omega_0 C_p}$ 	$\frac{bw}{\omega_0^2 C_p}$ and $\frac{C_p}{bw}$ 

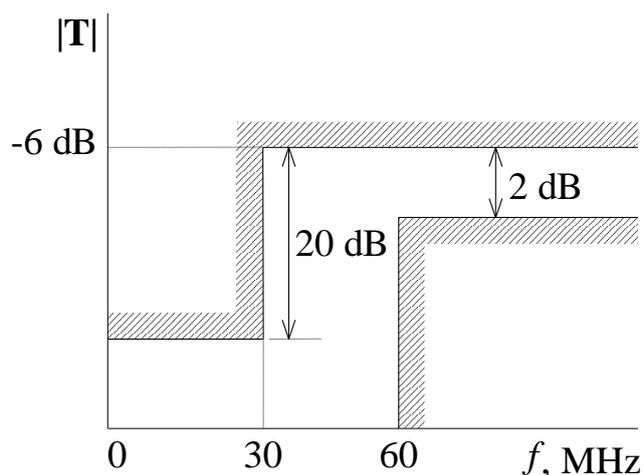
Table of lowpass prototype element transformations

8.14

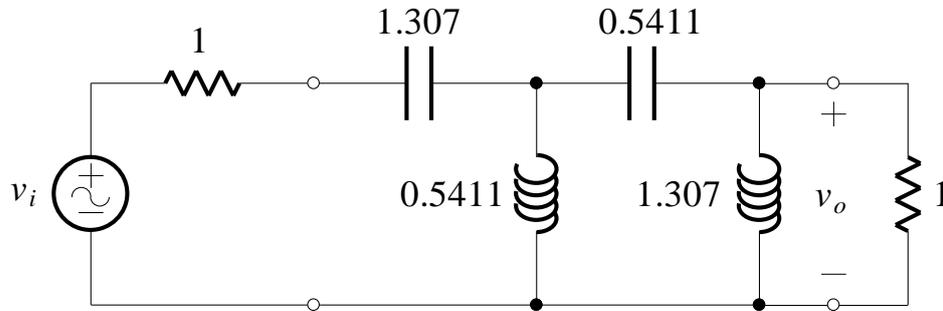
EXAMPLE 8.2 Passive Highpass Filter Design

A local amateur radio station is broadcasting on 30 MHz near a television receiver which, due to saturation effects within its RF amplifier stage is experiencing interference at the second harmonic (channel 1 at 60 MHz). We decide to introduce an “antenna trap” into the 50Ω antenna feed transmission line to the TV receiver. We also decide that the TV can cope with an 8 dB attenuation in channel 1, and that the radio transmission should be further attenuated by 18 dB.

At these high frequencies, we have to use passive elements, so we decide upon a doubly terminated lossless ladder highpass filter with a Butterworth response, which we know has an attenuation of 6 dB in the passband due to the terminating resistors. The specification is therefore:

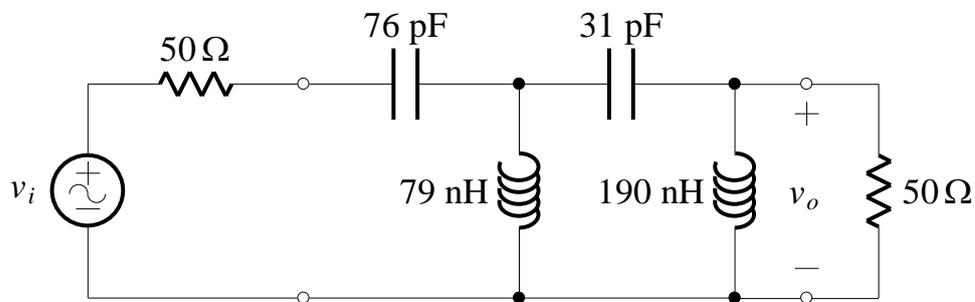


Using standard design procedure, we find that $n = 4$ satisfies these requirements. Now we use Table 8.2 to find the circuit that will implement a fourth-order Butterworth filter, and we use Table 8.2 to perform a lowpass to highpass transformation on the lossless ladder elements. The result of this is the normalised circuit:



Again, following normal design procedure, we calculate $\Omega_0 = 1.097$ and so $k_f = 343.7 \times 10^6$. The filter is terminated in the 50Ω characteristic resistance of the antenna feed, so we must have $k_m = 50$.

With these scaling factors the circuit is:



8.3 Ladder Design with Simulated Elements

We would like to remove the inductors from the ladder filter

An inherent advantage of doubly terminated lossless ladder circuits is that the magnitude of the transfer function has an extremely low sensitivity to element values. The disadvantage is that it requires inductors, which are hard to build, are heavy, and are difficult to adapt to integrated circuits.

Instead of using inductors, we can use active elements to simulate them (and other elements). Consider the following circuit:

GIC defined

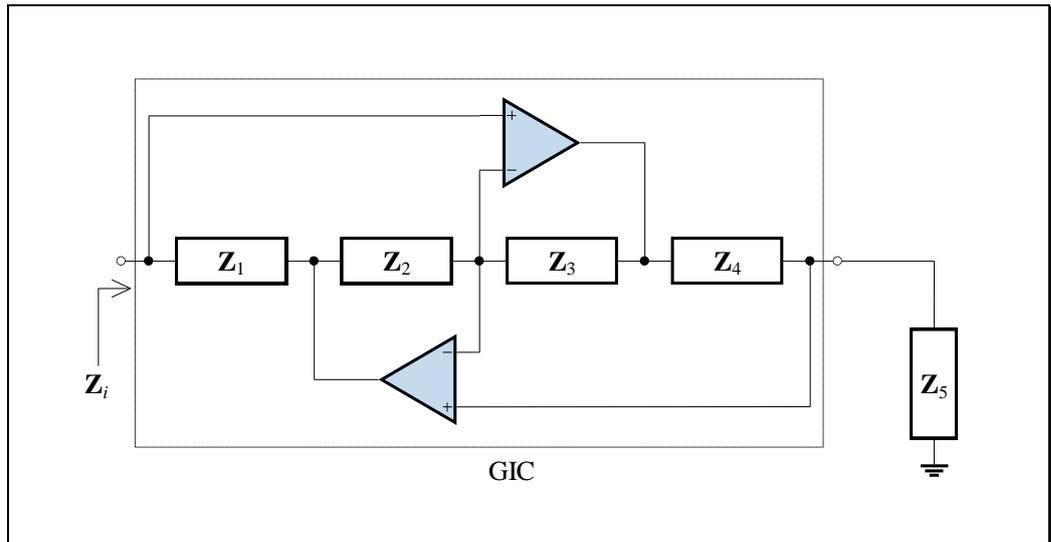


Figure 8.5

The circuit enclosed in dashed lines is known as a *generalised impedance converter* (GIC).

Show that the input impedance of this circuit is given by:

$$Z_i = \frac{Z_1 Z_3}{Z_2 Z_4} Z_5 \quad (8.27)$$

If we make the choice that $Z_4 = 1/sC_4$ and that all other elements are resistors, then:

$$Z_i = s \frac{R_1 R_3 R_5}{R_2} C_4 = sL$$

(8.28) The GIC is used to simulate an inductor

We may regard the GIC as a circuit that converts a resistor into an inductor.

Now we terminate the other end of the GIC:

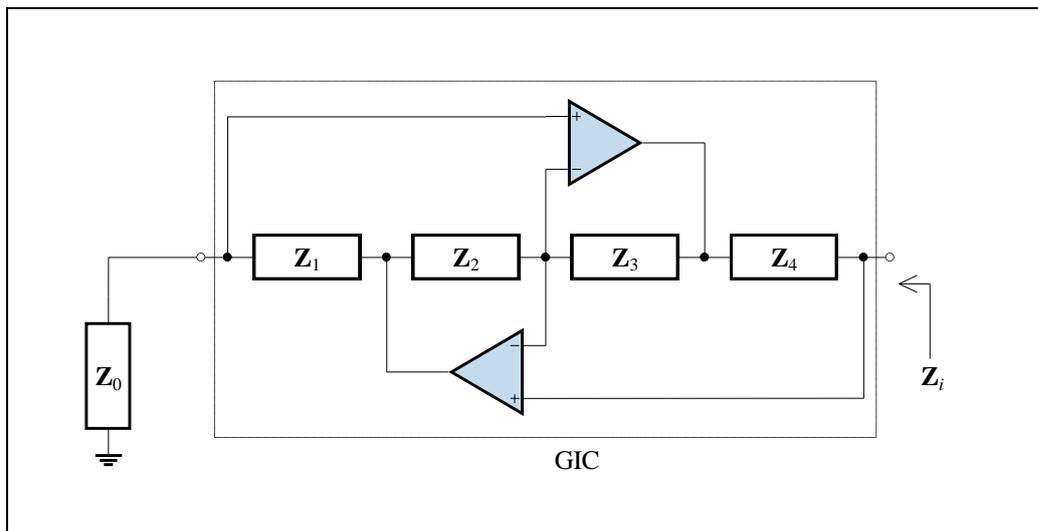


Figure 8.6

and evaluate the input impedance:

$$Z_i = \frac{Z_2 Z_4}{Z_1 Z_3} Z_0$$

(8.29)

8.18

With the same choice of elements in the GIC as before, and with $Z_0 = 1/sC_0$, this becomes:

$$Z_i = \frac{1}{s^2} \frac{R_2}{R_1 R_3 C_4 C_0} = \frac{1}{s^2 D} \tag{8.30}$$

This is a new element. If we let $s = j\omega$, then:

$$Z_i = \frac{-1}{\omega^2 D} \tag{8.31}$$

The GIC is used to create a new element - the FDNR

This impedance is negative and varies inversely with ω^2 . The circuit in Figure 8.6 is known as a *frequency-dependent negative resistor* (FDNR). We need a new symbol for this new element. It is shown below, together with the active circuit that simulates an inductor:

Active inductor circuit and FDNR circuit

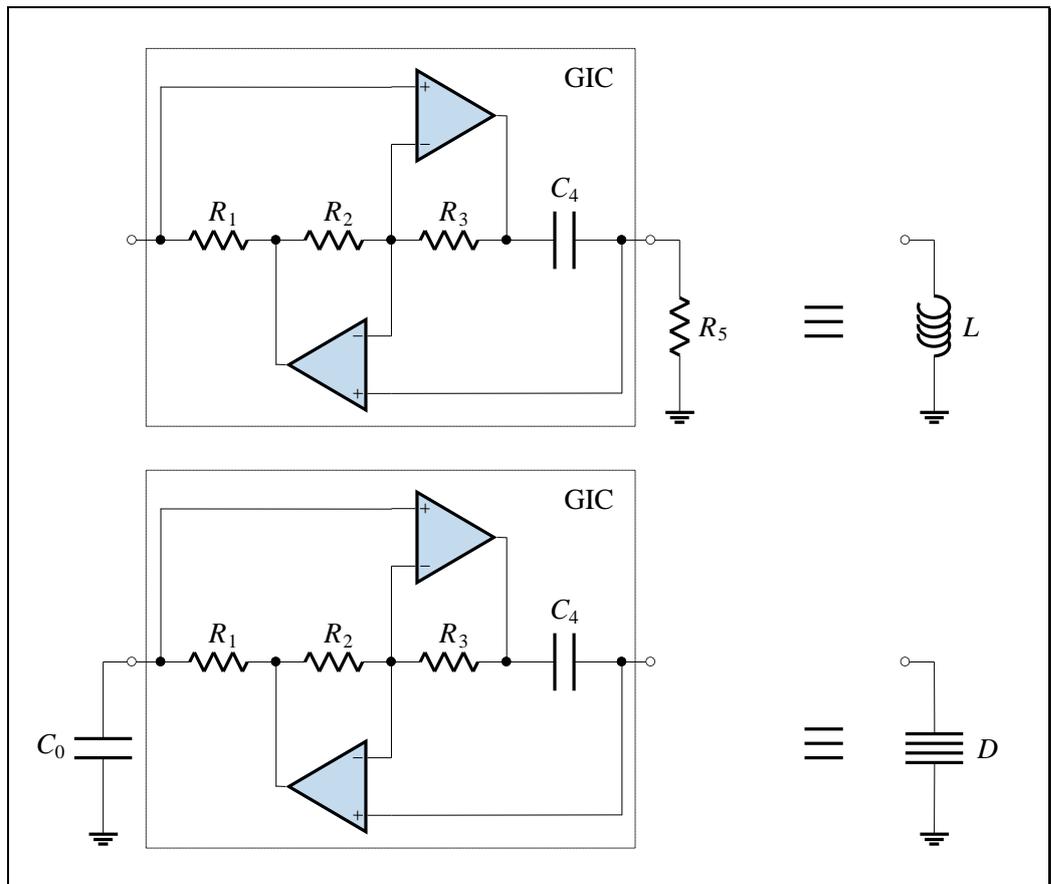


Figure 8.7

We know that magnitude scaling does not change a transfer function. Suppose we magnitude scale a circuit by the factor $1/s$. Then the normal impedances:

Magnitude scaling by $1/s$

$$\mathbf{Z}_R = R, \quad \mathbf{Z}_L = sL, \quad \mathbf{Z}_C = \frac{1}{sC} \quad (8.32)$$

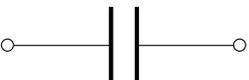
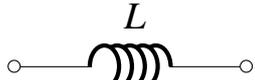
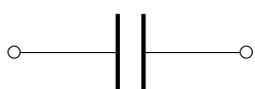
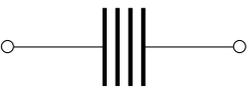
are scaled to:

$$\mathbf{Z}'_R = \frac{R}{s}, \quad \mathbf{Z}'_L = L, \quad \mathbf{Z}'_C = \frac{1}{s^2C} \quad (8.33)$$

We see that such scaling actually results in a transformation of elements. It is called a Bruton¹ transformation and is shown in the table below:

performs a Bruton transform on circuit elements

Table 8.3 – Bruton Transformations

Element	Bruton transformed element
	$C = 1/R$ 
	$R = L$ 
	$D = C$ 

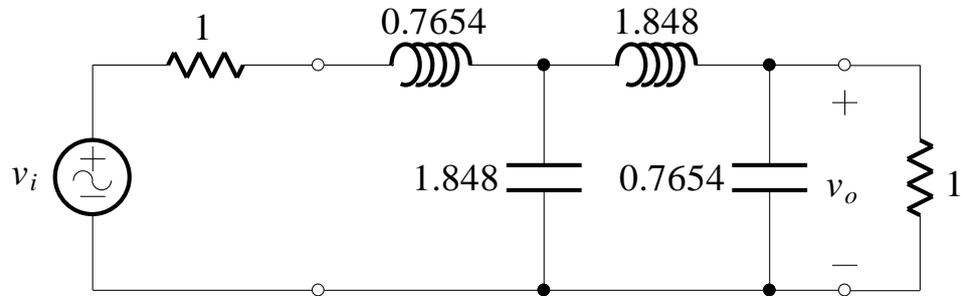
We now have a transformation that takes us away from inductors. The price paid for this is that one of the elements is an active element – the FDNR. It should be noted that the circuits in Figure 8.7 can only simulate an element that has one side connected to the common. This will determine when we use simulated inductors and when we use FDNRs.

We only simulate elements with one end connected to the common

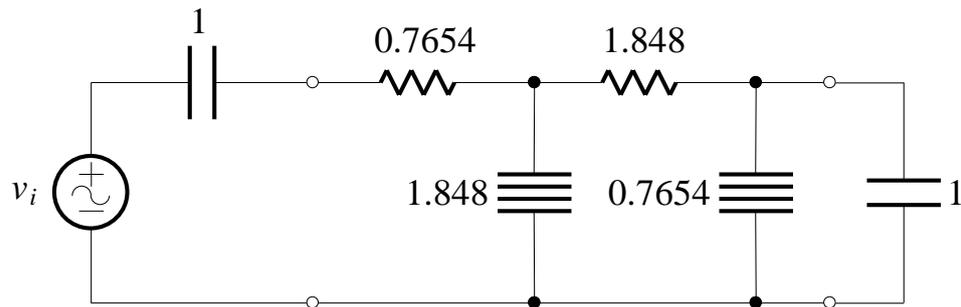
¹ L. T. Bruton, “Network Transfer Functions Using the Concept of Frequency-Dependent Negative Resistance,” *IEEE Trans. Circuit Theory*, vol. CT-16, pp. 406-408, 1968.

EXAMPLE 8.3 Simulated Element Lowpass Filter

Suppose we would like to implement a fourth-order Butterworth lowpass filter with a half-power frequency of 1 kHz and that only 10 nF capacitors are available. The prototype for such a filter is found from Table 8.1:



Since the inductors are not connected to the common, we will use the Bruton transformation of elements as in Table 8.3 to convert the capacitors to FDNRs. We obtain the following equivalent circuit:



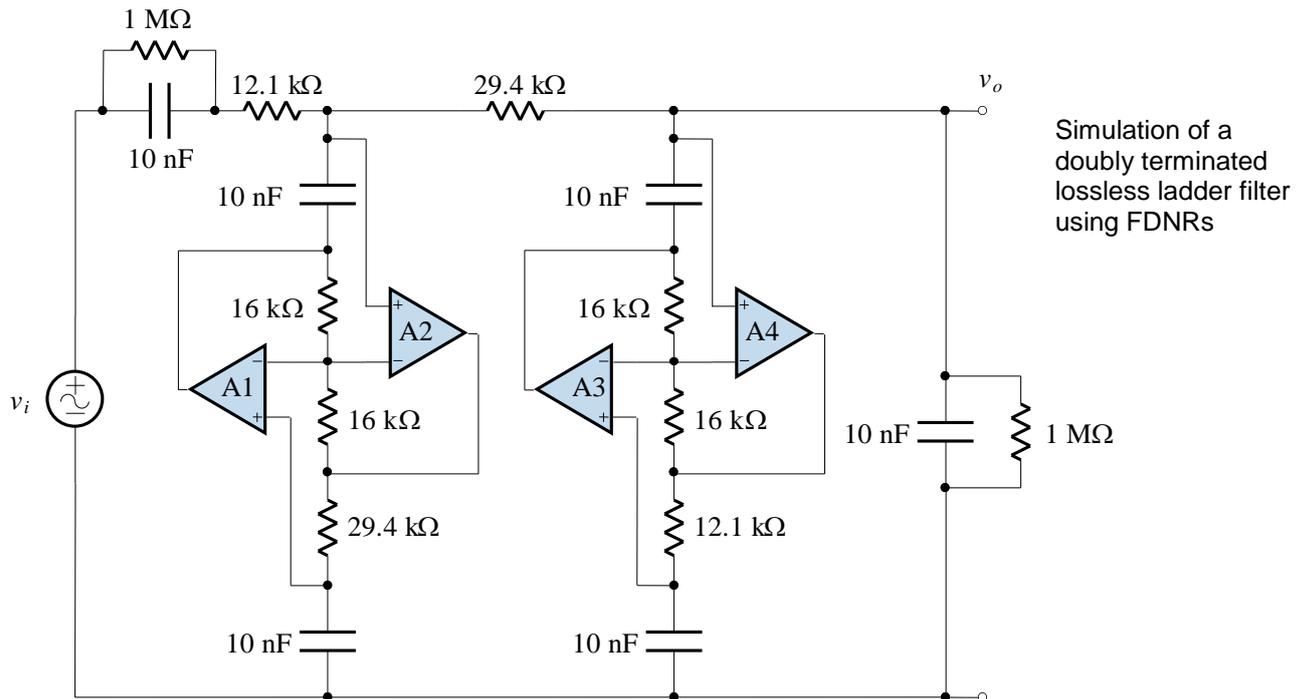
In realising the circuit, we will choose the GIC's elements to be:

$$C_0 = C_4 = 1, \quad R_2 = R_3 = 1, \quad R_1 = D$$

These are normalised values of course. With the specified half-power frequency and capacitor values we must have:

$$k_f = 2\pi \times 1000 \quad \text{and} \quad k_m = \frac{1}{k_f \times 10 \times 10^{-9}}$$

The circuit realisation is:



Since the circuit is based on the passive doubly terminated lossless ladder circuit, it will have the same sensitivity values as the passive circuit.

Note the $1\text{ M}\Omega$ resistors placed in parallel with the input and output capacitors. This is necessary for two reasons:

1. To allow for the correct DC response – the transfer function at DC would otherwise be $0/0$ (undefined). You can see intuitively that a DC path is required from input to output for a lowpass filter, and the input capacitor would otherwise act to block this DC path.
2. To provide a path for DC bias currents that exist in the non-inverting input terminals of op-amps A2 and A4. If there is no path for the DC bias currents to the circuit's common (or to either power supply) then the op-amps will saturate as they are not able to establish proper internal DC operating conditions. Typically, both the input and the output stage of the op-amps will be out of their linear region and the amplifiers won't work properly.

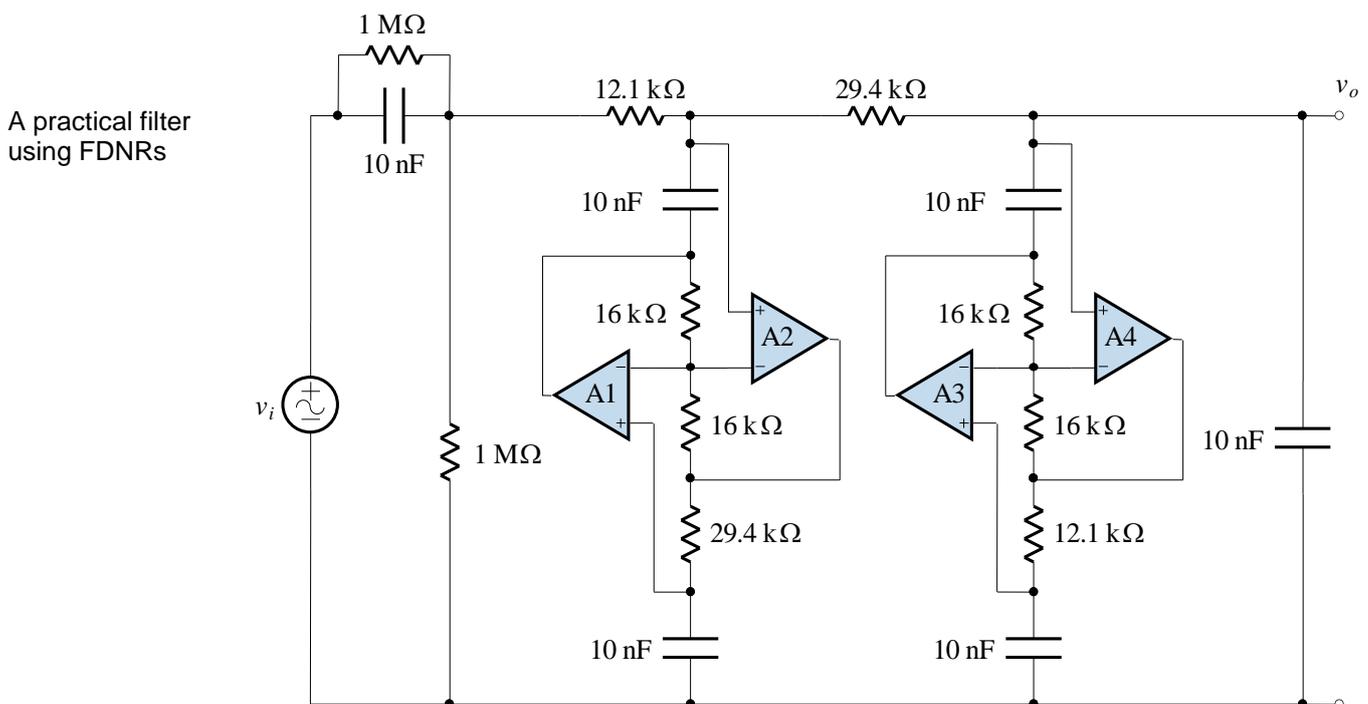
8.22

However, placing the two resistors in parallel with the input and output capacitors is problematic, because they create a voltage divider for DC and the DC gain ends up being:

$$\left. \frac{v_o}{v_i} \right|_{DC} = \frac{1\text{ M}}{1\text{ M} + 12.1\text{ k} + 29.4\text{ k} + 1\text{ M}} = 0.4898$$

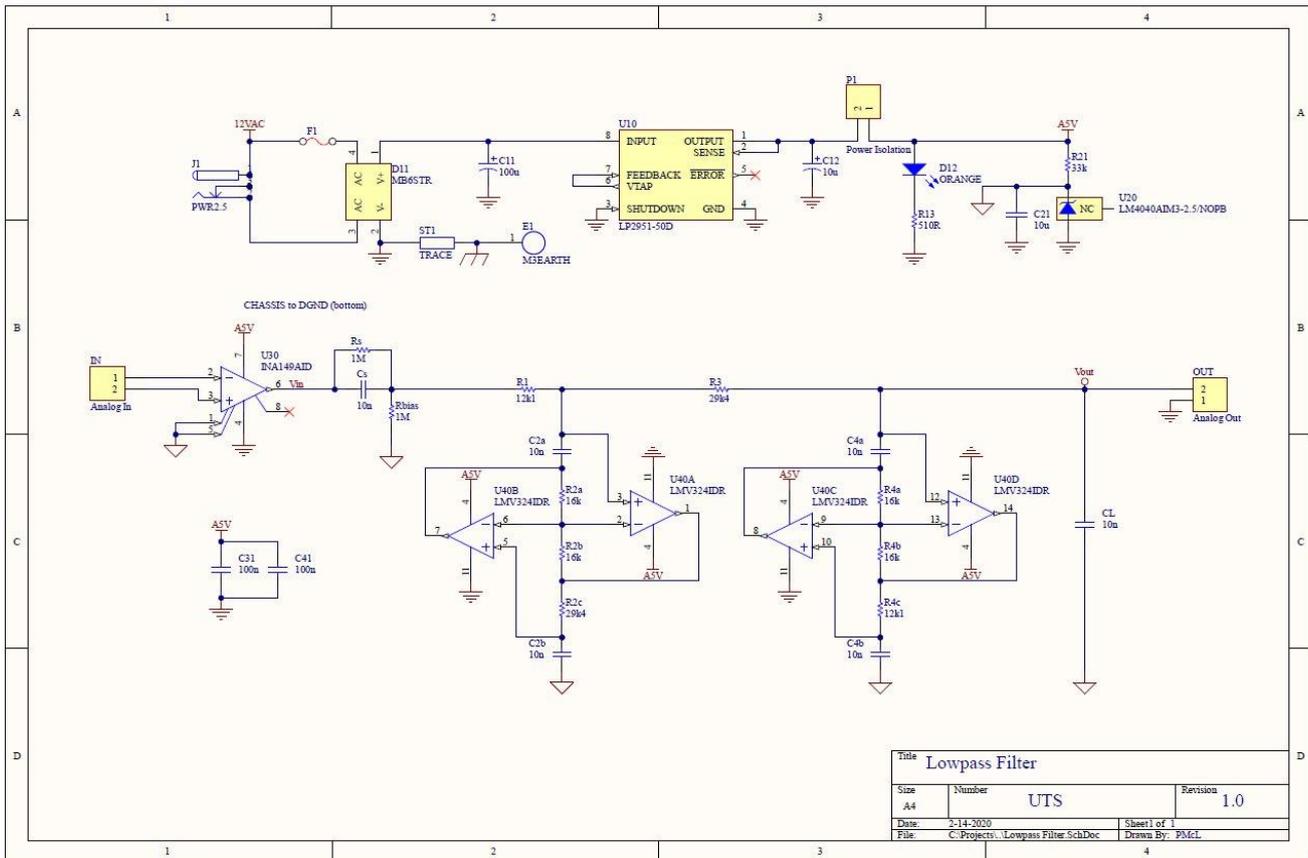
which is a 2% error compared to the theoretical AC gain of 0.5. Therefore, the two resistors must differ in value if the DC gain is to end up the same as the AC gain. This is inconvenient since these added resistors are high in value (so as not to disturb the original circuit) and precision high value resistors are only available in the relatively limited E24 series.

In practice, the configuration below works well:



The two equal-value resistors are now placed in the form of a DC attenuator at the input of the circuit. To minimise the impact on the overall frequency response, the value of these resistors needs to be around 25 times the “sum of values in the top resistor chain”. In this case, the resistors need to be around $25 \times (12.1\text{ k} + 29.4\text{ k}) = 1.038\text{ M} \approx 1\text{ M}\Omega$.

A schematic of a design that utilises the filter is shown below:

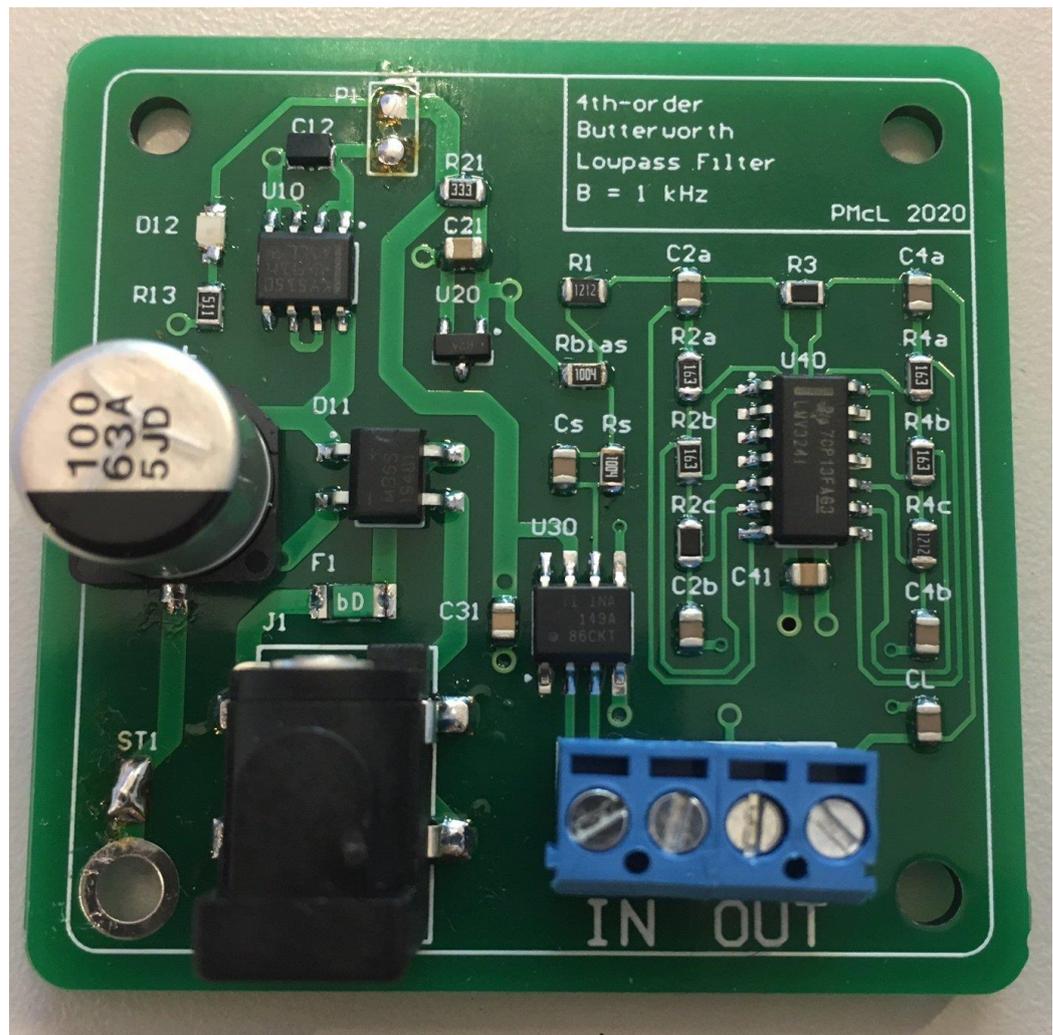


In this case there is INA149 difference amplifier at the front end to remove common-mode noise. The input power supply is 12 V AC, and the DC supply generated from it is +5V. The design utilises rail-to-rail single supply op-amps. Note the use of a precision voltage reference to generate +2.5 V which is attached to the REF input of the INA149 (pins 1 and 5). Also, the “common” for the filter is tied to +2.5 V (\downarrow) instead of to 0 V (\perp). The net effect is a level-shift so that the output, when taken with respect to 0V, exhibits a DC offset of +2.5 V.

Since the INA149 utilises a +5 V supply and a +2.5 V reference, the output is restricted to 1.5 V to 3.5 V, i.e. the maximum input signal this filter can handle is 2 V peak-to-peak.

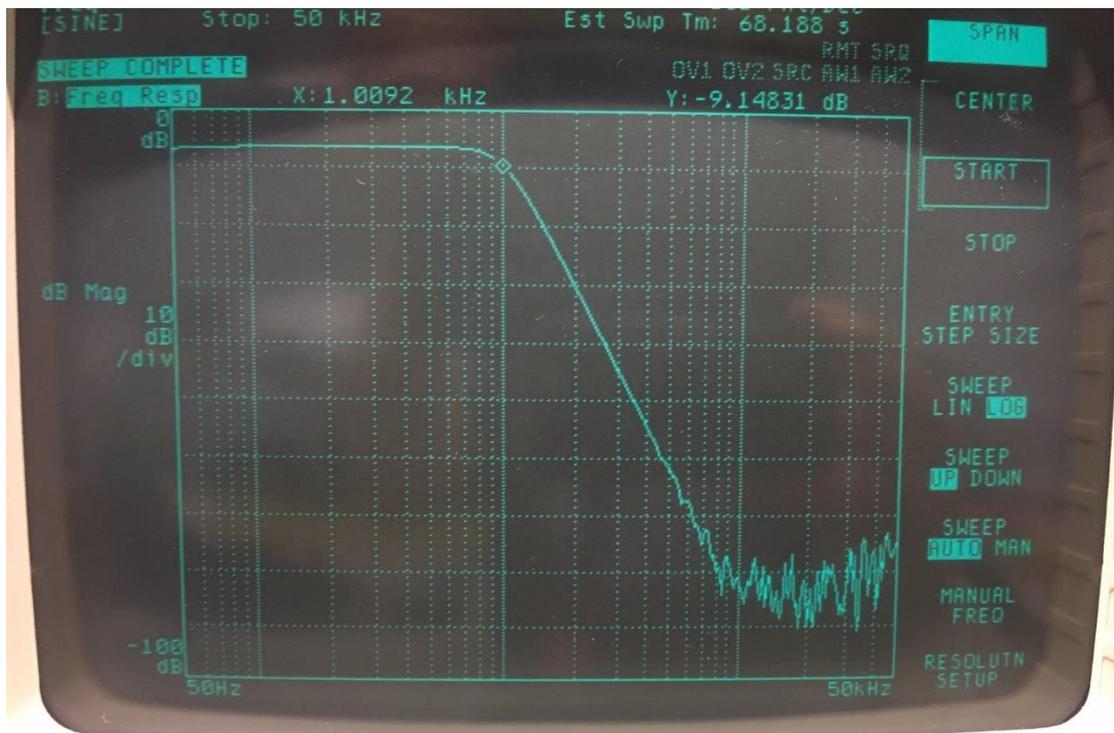
8.24

A photo of the PCA is shown below:



The power supply is laid out on the left-hand side of the board. The filter is on the right-hand side, and you can see the ladder structure of the direct realisation in the layout of the components around the quad op-amp – it is laid out in a very similar manner to the schematic diagram (you can see the FDNR “chain” of components down each side of the quad op-amp).

A graph of the filter's magnitude response, obtained using a Dynamic Signal Analyzer, is shown below:



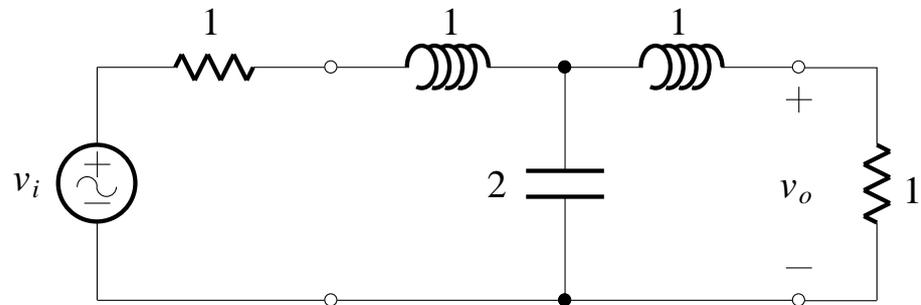
Note a few features of the characteristic:

- In the passband, there is a gain of -6 dB (because it matches the doubly-terminated lossless ladder). The response is flat, since it is a Butterworth response.
- At the cutoff frequency of 1 kHz, the response is -3 dB down (≈ -9.1 dB).
- The roll-off in the transition region is -80 dB/decade, representing “4-pole roll-off”, because it is a 4th-order filter.
- The “noise floor” of the instrument is around -80 dB, as you can see from the response above around 10 kHz. In this region the instrument can’t distinguish the tiny signal from the noise.

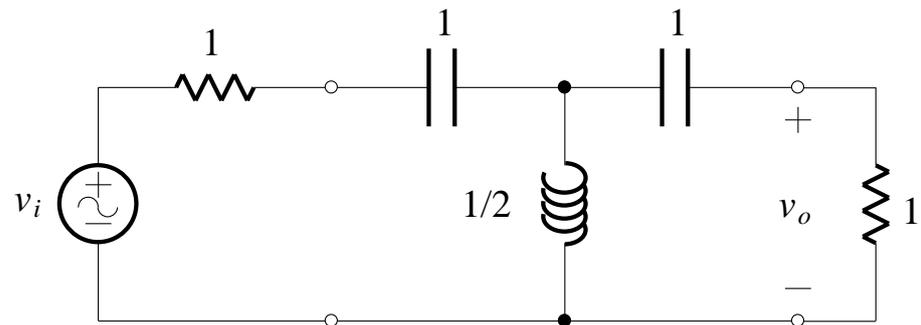
The use of simulated elements is a very practical realisation technique.

EXAMPLE 8.4 Simulated Element Highpass Filter

We require a third-order highpass Butterworth filter with a half-power frequency of 10 Hz. The prototype with element values determined from Table 8.1 is shown below:



Making use of the lowpass to highpass frequency transformation for elements as given in Table 8.2, we obtain the highpass circuit shown below:

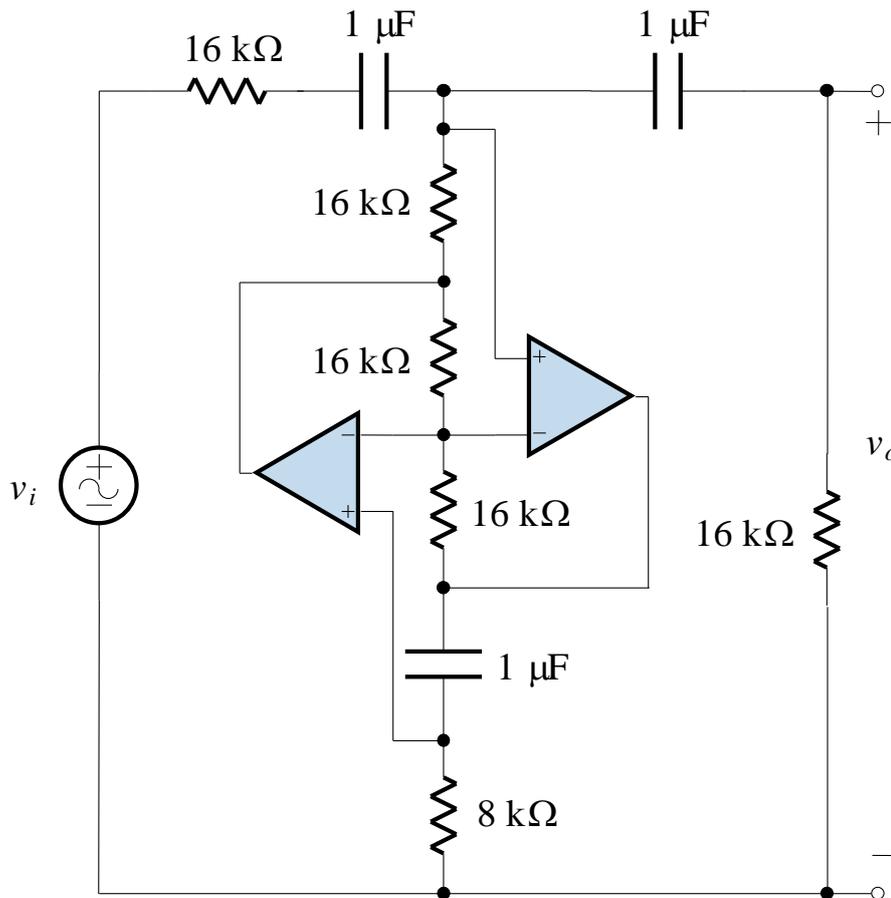


If we were to frequency scale this circuit, we would end up with large inductors and capacitors. A better implementation would be to use a simulated inductor. We will use the circuit in Figure 8.7, with all element values unity except for R_5 . Using Eq. (8.28), we have $R_5 = L = \frac{1}{2}$.

We frequency scale to meet the specification and also choose to use $1\mu\text{F}$ capacitors in the implementation:

$$k_f = 2\pi \times 10 \quad \text{and} \quad k_m = \frac{1}{k_f \times 1 \times 10^{-6}}$$

The circuit realisation is thus:



Simulation of a doubly terminated lossless ladder filter using a simulated inductor

When we simulate a ladder filter with op-amps, we can adjust the gain to any desired level with the addition of a gain stage. In the examples above, since the circuits are direct simulations of a doubly terminated ladder circuit, the passband gain is only 1/2.

We only considered GIC circuits that simulated an element with one end connected to the common. Consequently, we have only considered lowpass and highpass filters. When we perform a lowpass to bandpass transformation, we get inductors in both series and parallel branches of the ladder. To simulate a “floating” inductance (one not connected to the common) we need to use special GIC circuits.

Simulation of a doubly terminated lossless ladder bandpass filter is more difficult

8.4 References

Van Valkenburg, M. E.: *Analog Filter Design*, Holt-Saunders, Tokyo, 1982.

Exercises

1.

We would like to eliminate interference from a television set. Using a spectrum analyser, it is found that reception will be improved if the following *attenuations* can be obtained by a filter:

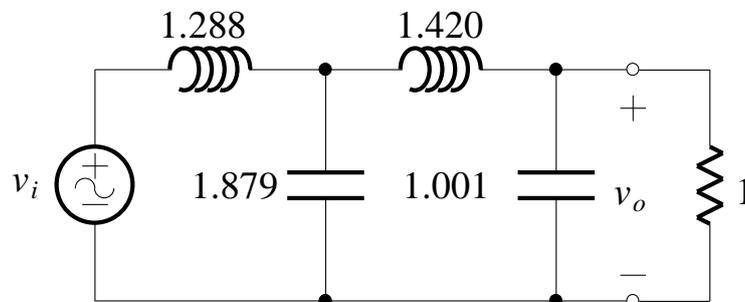
f , MHz	K , dB
48	20
60	2
66	20

Design a passive filter to put on the front end of a television receiver to match a $50\ \Omega$ antenna feed.

Hint: Assume a 6 dB attenuation across all frequencies is allowable; due to the terminations of a doubly terminated lossless ladder.

2.

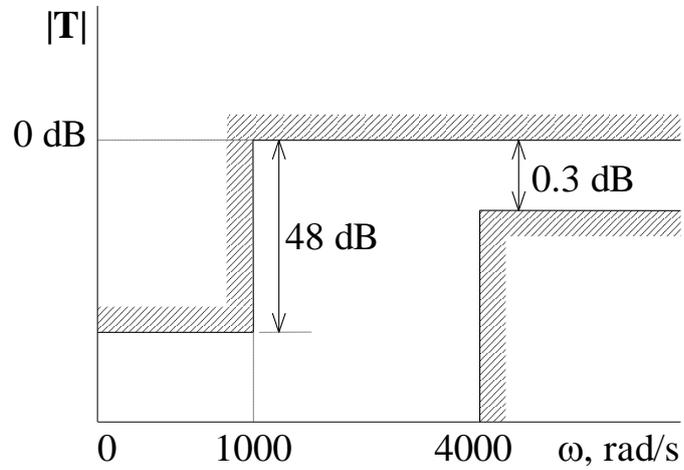
For a particular filtering problem, the prototype shown below is chosen:



This is a singly terminated lowpass Chebyshev filter having a ripple width of 0.9 dB and a bandwidth of 1 rad/s. For the final design, the filter is to be frequency-scaled by a factor of 1000 and magnitude-scaled to give practical element sizes. Find a realisation of the filter without using inductors.

3.

The specifications for a highpass filter are shown below and it is further specified that the response be Butterworth. Find a filter realisation without using inductors.



9 Specialised Filters

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Introduction

The conventional op-amp, together with resistors and capacitors, are used in the vast majority of active filters. However, op-amp limitations preclude the use of these filters at high frequencies, and convenient voltage or current control schemes for externally adjusting the filter characteristics do not exist.

There are many specialised filter circuits

There are many other types of filter circuits – all have been designed to try and improve filter characteristics, performance, or flexibility. We will review just a few of the most common ones.

9.1 Leapfrog Simulation of Ladders

The leapfrog method of simulating ladders was developed in the era of analog computers. Any system can be represented by an electrical *analog*. These electrical analogs of the real system simply solve the equations describing the system. Voltages (or currents) in the electrical circuit are the analog of variables in the real system.

Leapfrog filters simulate the equations describing a doubly terminated lossless ladder filter using analog circuits

If we write the equations describing the series and parallel parts of an n th order ladder circuit, then we have n equations describing the filter. These equations can be solved, just like any other set of equations, using an analog circuit composed of multipliers, adders, integrators etc. The signal flow graph of such a system has feedback paths that remind us of the children's game *leapfrog* - hence the name leapfrog simulation.

Leapfrog filters are competitive with the doubly terminated lossless ladder circuit (or a simulation of it using FDNRs), since they simulate the describing differential equations of such a ladder.

9.2 Switched-Capacitor Filters

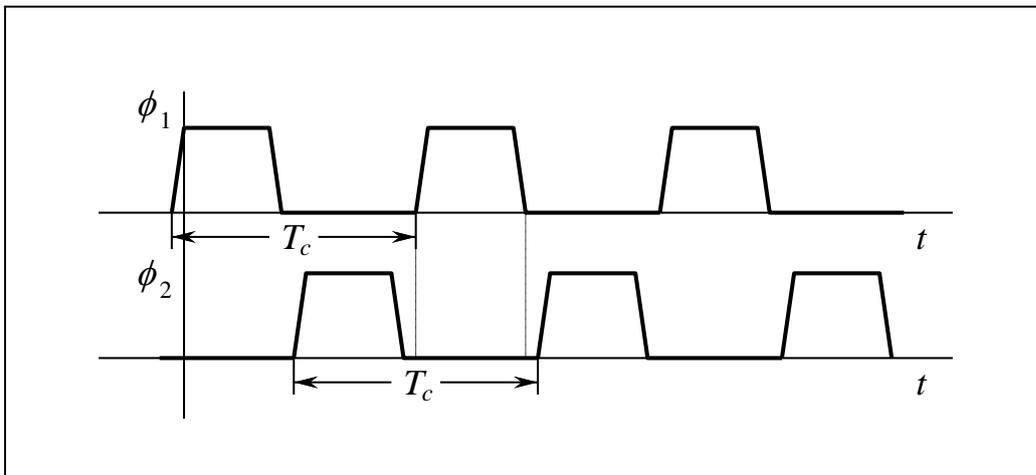
Switched-capacitor filters came about in the late 1970's, when it was required to implement voice-frequency filters using MOS technology. The key to this technology is the fact that we can simulate resistors using switches and capacitors.

Switched capacitor filters simulate resistors

An enhancement type MOSFET can be operated as a switch if the gate voltage is driven above and below the threshold voltage for the device. When this is the case, we will refer to the MOSFET as a MOS switch. The MOS switch is not ideal of course, but it is adequate for switched-capacitor circuits.

The switches in a switched capacitor filter are actually MOSFETs

There are two voltage waveforms used to activate the MOS switches in a switched capacitor filter. They are shown below:



A two-phase clock is used to switch the MOSFETs

Figure 9.1

The signals ϕ_1 and ϕ_2 are applied at the gate of a MOS switch, turning it on or off. The waveform is generated by a *two-phase* clock which ensures that ϕ_1 and ϕ_2 have the same frequency and are nonoverlapping. The quantity $f_c = 1/T_c$ is known as the *clock frequency* and has a typical value of 100 kHz.

The clock frequency is important in switched-capacitor filters

9.4

Consider the following circuit:

A switched-capacitor

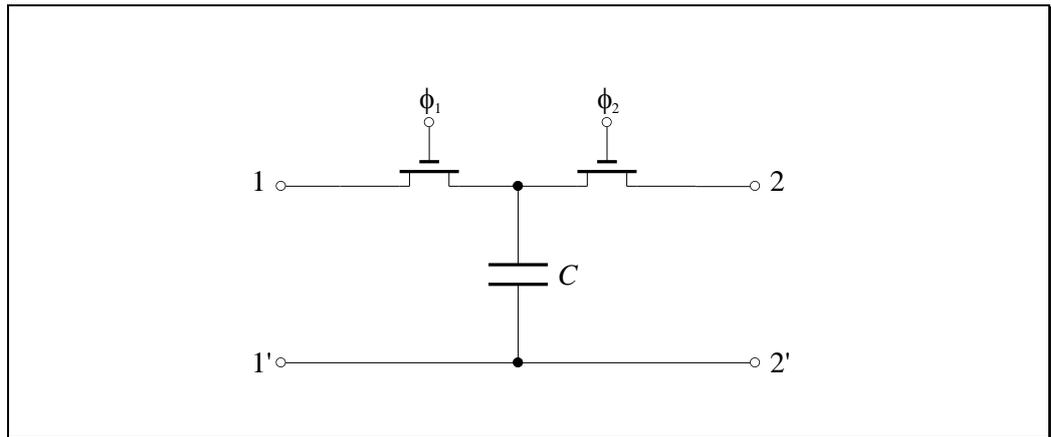


Figure 9.2

If the two-phase clock drives the two MOS switches shown in the circuit, then there will never be a direct connection between terminals 1 and 2, since one of the two switches will always be open.

This switching action is equivalent to a single-pole double-throw (SPDT) switch:

is equivalent to a SPDT switch

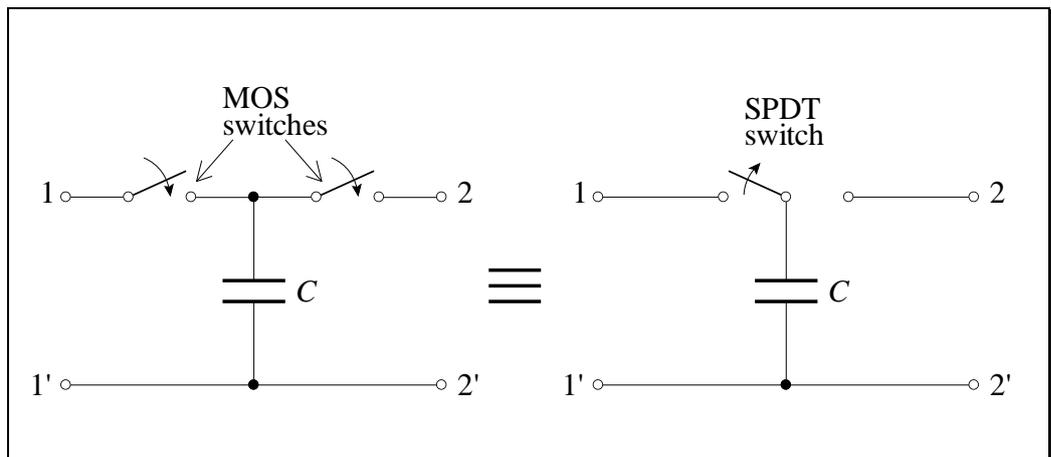


Figure 9.3

The capacitor with a periodic SPDT switch is one of the basic building blocks in switched-capacitor circuits. To see its function, consider an input voltage v_i that is time varying as shown below:

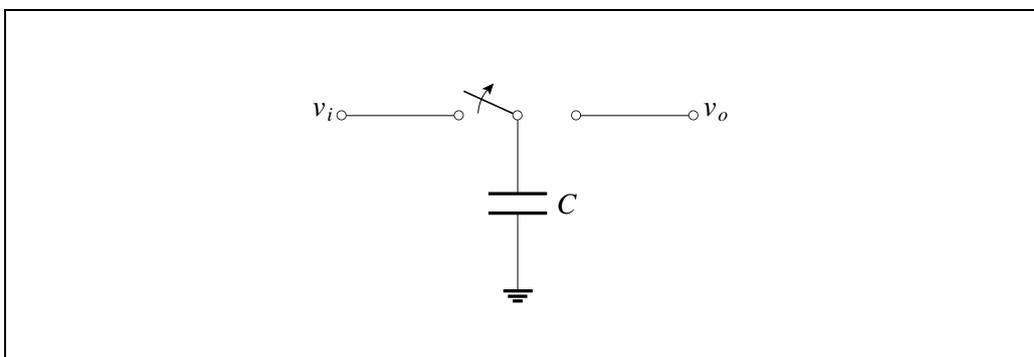


Figure 9.4

We will assume that the switch is connected to v_i at time $t = 0$. The “on” resistance of a MOSFET is about $10 \text{ k}\Omega$, and a typical value of C is 1 pF (the capacitors are implemented on the same piece of silicon as the MOSFETs, so they have very small values). The time for the capacitor to charge up to the voltage v_i is very small (in the nanosecond range). Compared to the variations in v_i (in the microseconds range), the capacitor essentially sees a constant voltage.

The approximation used when analysing switched-capacitor circuits

If the switch is now connected to v_o , then the charge transferred from the input to the output of the circuit is:

$$\Delta q = C(v_i - v_o) \quad (9.1)$$

and this will be accomplished in one cycle of the clock, T_c . The average current passing from input to output will therefore be:

$$i = \frac{\Delta q}{\Delta t} = \frac{C(v_i - v_o)}{T_c} = \frac{1}{R_c}(v_i - v_o) \quad (9.2)$$

9.6

The switched capacitor in Figure 9.4 is therefore equivalent to a resistor of magnitude:

The equivalent resistance of a switched-capacitor

$$R_C = \frac{T_c}{C} = \frac{1}{f_c C} \tag{9.3}$$

This is shown below:

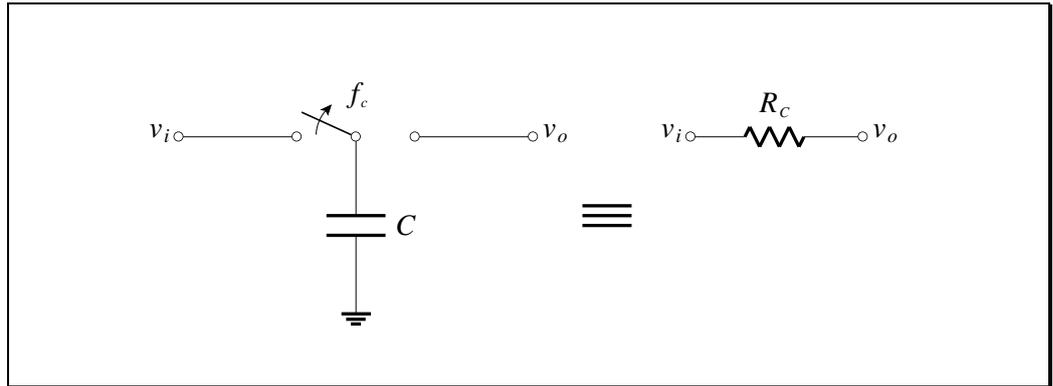
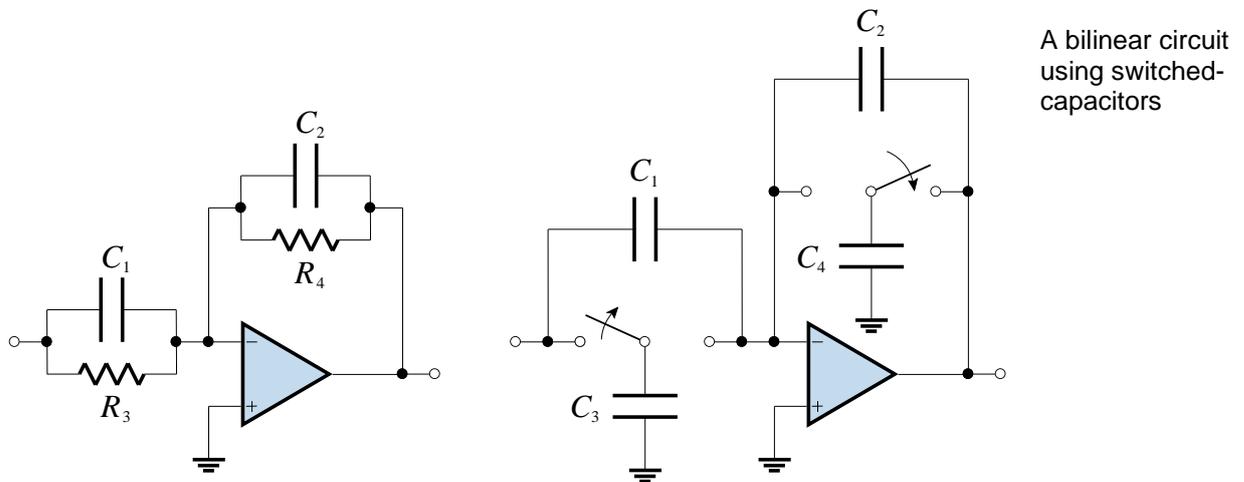


Figure 9.5

There are other arrangements of switches and capacitors to perform other functions, such as inversion and subtraction.

EXAMPLE 9.1 Switched Capacitor Bilinear Filter

A bilinear filter section with the switched capacitor equivalent is shown below:



Show that the transfer function for the circuit with resistors is:

$$\mathbf{T}(s) = -\frac{C_1 s + 1/R_3 C_1}{C_2 s + 1/R_4 C_2}$$

The corresponding equation for the switched-capacitor case is:

$$\mathbf{T}(s) = -\frac{C_1 s + f_c C_3 / C_1}{C_2 s + f_c C_4 / C_2}$$

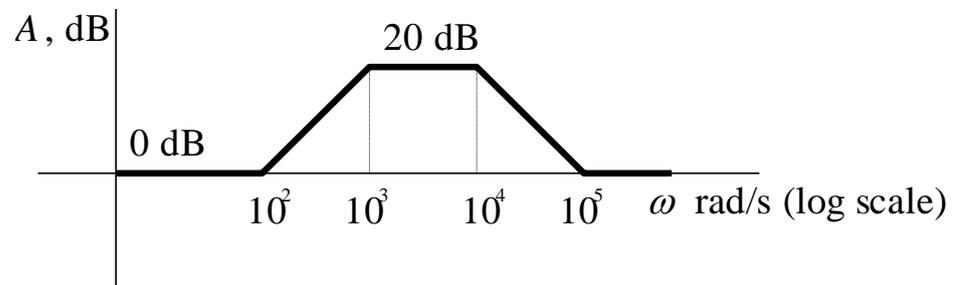
The bilinear transfer function when using switched-capacitors

It is significant that this equation only involves capacitance ratios. In MOS technology, capacitance ratios can be formed with great accuracy, and this is one of the advantages of a switched-capacitor realisation. Also, the clock frequency can be used to tune the circuit.

Switched-capacitor filters can be made very accurate

9.8

Suppose that the filter specifications we wish to realise are indicated by the asymptotic Bode plot given below, which is seen to be a band-enhancement filter:



The transfer function, in terms of a cascade of bilinear transfer functions, is:

$$\mathbf{T}(s) = \mathbf{T}_1(s)\mathbf{T}_2(s) = \frac{s + 10^2}{s + 10^3} \times \frac{s + 10^5}{s + 10^4}$$

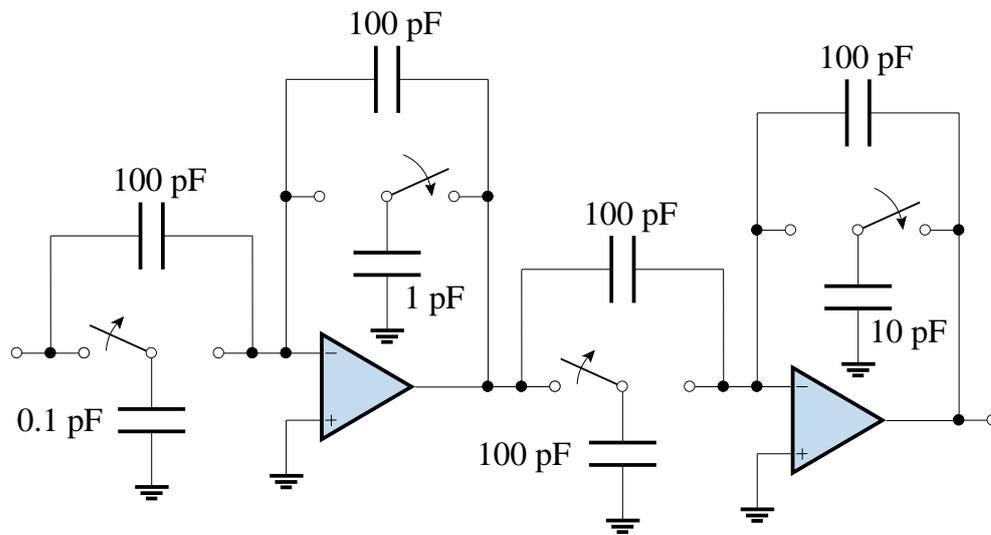
We equate the coefficients of these factors with those of the previous equation. We also select the clock frequency as $f_c = 100$ kHz, well above the highest passband frequency. For convenience, we select $C_1 = C_2 = 100$ pF for both sections so that $C_1/C_2 = 1$. For the first section we have:

$$C_3 = 0.1 \text{ pF} \quad \text{and} \quad C_4 = 1 \text{ pF}$$

while for the second section:

$$C_3 = 100 \text{ pF} \quad \text{and} \quad C_4 = 10 \text{ pF}$$

The final filter design is shown below:



9.3 Operational Transconductance Amplifiers

An operational transconductance amplifier (OTA) is a variation of a traditional op-amp. The main difference between an op-amp and an OTA is that, while an op-amp outputs a voltage that is proportional to the difference in voltage between its two inputs, the OTA outputs a *current* that is proportional to the difference in voltage between its two inputs.

The symbol for an OTA is shown below, and appears as though the output stage of an op-amp has been “chopped off” (many op-amps are internally constructed from an OTA plus voltage buffer):

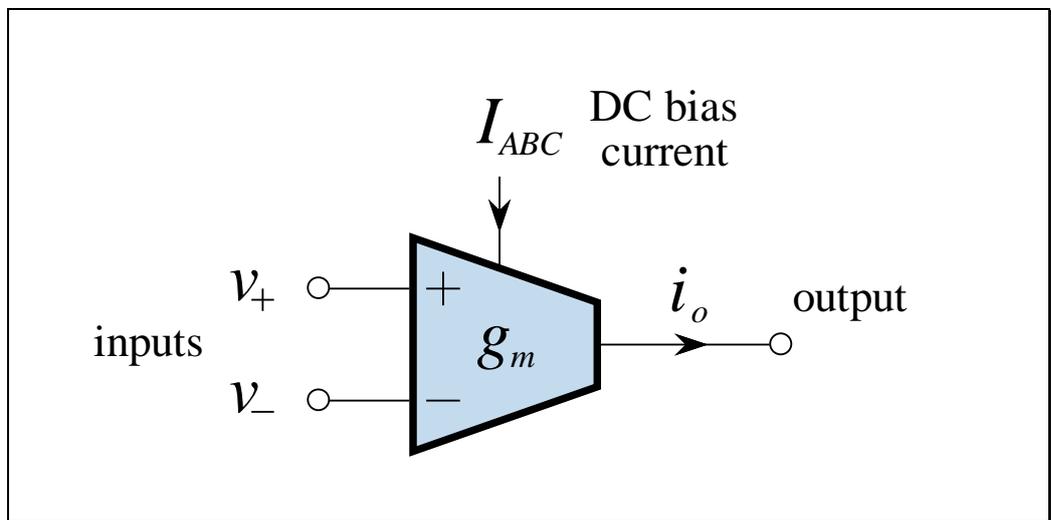


Figure 9.6

Like the conventional op-amp, the OTA has two differential input terminals, a high input impedance, and a high CMRR. Unlike the conventional op-amp, the OTA has a bias-current input terminal (labelled I_{ABC}), a high output impedance, and no fixed open-loop voltage gain.

For an OTA, voltage is the input variable and current is the output variable; therefore, the ratio of output current to input voltage is its gain:

$$g_m = \frac{i_o}{v_{in}} \quad (9.4)$$

In an OTA, the transconductance, g_m , is assumed to be proportional to the bias current, I_{ABC} :

$$g_m = hI_{ABC} \quad (9.5)$$

The proportionality constant, h , is dependent upon temperature, device geometry, and the fabrication process. The output current is therefore controlled by the input voltage and the bias current:

$$i_o = g_m v_{in} = hI_{ABC} v_{in} \quad (9.6)$$

Current control of the transconductance can be directly obtained with control of I_{ABC} . Since techniques abound for creating a current proportional to a given voltage (such as a voltage-to-current converter using a conventional op-amp), voltage control of the OTA gain can be attained through the I_{ABC} input.

A small-signal model of the OTA is:

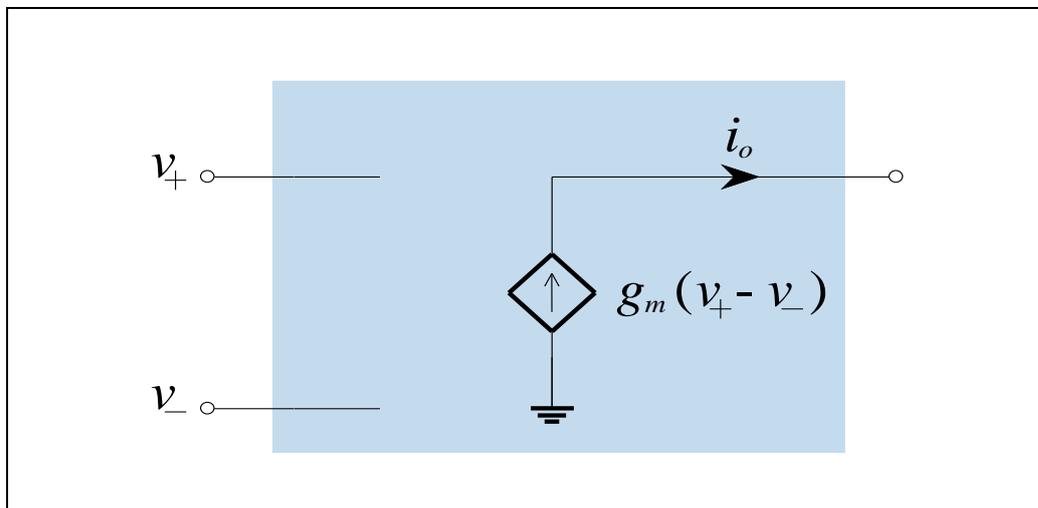


Figure 9.7

In this model both R_{in} and R_{out} are assumed to be infinite. Since we generally require a low-impedance output (like an op-amp), circuits built with OTAs will generally have a voltage buffer on their output. An alternative to an op-amp buffer is the [Darlington pair](#), which is included in the same package in commercial OTAs.

9.12

OTAs are generally operated *open-loop* (in contrast to op-amps) – instead of employing feedback to reduce the sensitivity of a circuit’s performance to device parameters, the transconductance is treated as a design parameter, in much the same way as resistors and capacitors are treated in op-amp based circuits.

The OTA is used in circuits that require a voltage-controlled parameter

The most important uses of the OTA are in micropower amplifier and oscillator applications, and in applications in which important parameters are variable via an external voltage. An OTA can be easily made to act as a voltage-controlled amplifier (VCA), voltage-controlled resistor (VCR), voltage-controlled filter (VCF), or voltage controlled oscillator (VCO), etc.

What makes the OTA so attractive in active filters is the ability to form filter circuits with voltage-variable control (via the I_{ABC} input) over a number of key performance parameters of the filter. The controlled parameter could be the mid-band gain of the circuit or the location of the cutoff frequency.

A simple example of a first-order lowpass filter is shown below:

A variable cutoff frequency first-order lowpass filter

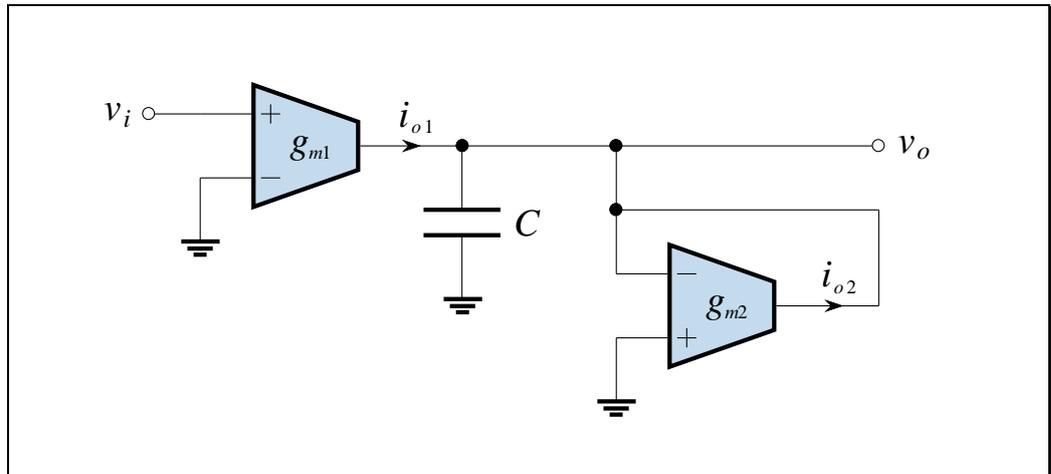


Figure 9.8

The output current of the first OTA is just $i_{o1} = g_{m1}v_i$. The output current of the second OTA is $i_{o2} = -g_{m2}v_o$. Notice that the second OTA has the “+” and “-” terminals swapped compared to the first OTA. Recall that the OTAs are generally operated open-loop, and interchanging the “+” and “-” terminals of the OTA will result only in changing the sign of the g_m coefficient in any equation derived for the original circuit.

Since inputs of OTAs are high impedance, the only path for the two OTA currents is down through the capacitor. The output voltage is given by KCL at the capacitor:

$$\begin{aligned}
 \mathbf{I}_{o1} + \mathbf{I}_{o2} &= \mathbf{sC}\mathbf{V}_o \\
 g_{m1}\mathbf{V}_i - g_{m2}\mathbf{V}_o &= \mathbf{sC}\mathbf{V}_o \\
 (g_{m2} + \mathbf{sC})\mathbf{V}_o &= g_{m1}\mathbf{V}_i \\
 \mathbf{V}_o &= \frac{g_{m1}}{g_{m2} + \mathbf{sC}}\mathbf{V}_i
 \end{aligned} \tag{9.7}$$

Thus, the transfer function for this circuit is:

$$\mathbf{T}(\mathbf{s}) = \frac{g_{m1}}{g_{m2} + \mathbf{sC}} \tag{9.8}$$

The circuit therefore has a DC gain given by g_{m1}/g_{m2} and a cutoff frequency:

$$\omega_0 = \frac{g_{m2}}{C} \tag{9.9}$$

Both the gain and cutoff frequency can therefore be controlled (although only the cutoff frequency can be controlled independently in this circuit).

The figure below shows a second-order filter circuit with three voltage inputs:

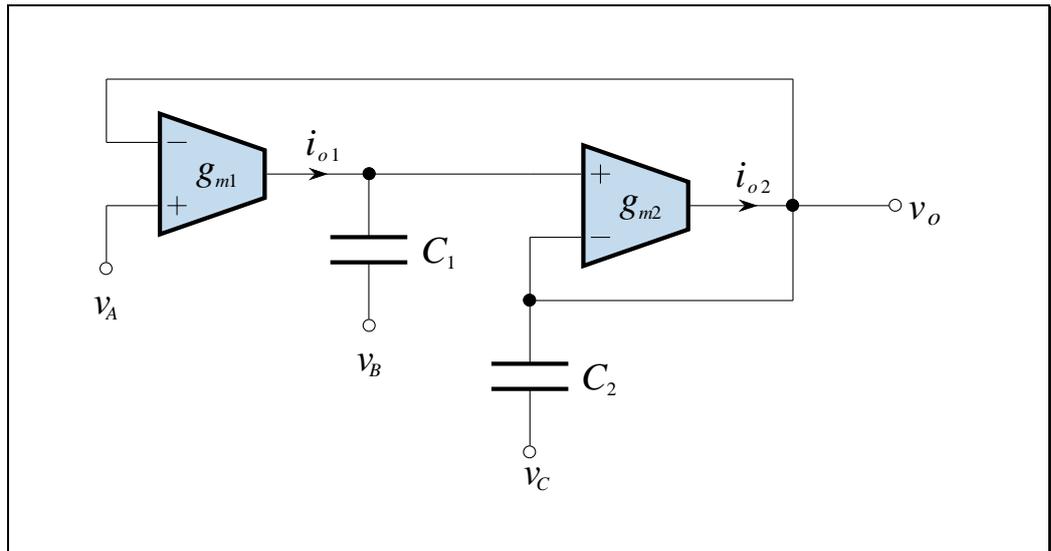


Figure 9.9

The derivation of the general relationship between the output voltage and the three input voltages is obtained by repeated application of the OTA gain equation and the impedance relationship for a capacitor:

$$\begin{aligned}
 \mathbf{I}_{o1} &= g_{m1}(\mathbf{V}_A - \mathbf{V}_o) \\
 \mathbf{V}_{C1} &= \frac{\mathbf{I}_{o1}}{sC_1} + \mathbf{V}_B \\
 \mathbf{I}_{o2} &= g_{m2}(\mathbf{V}_{C1} - \mathbf{V}_o) \\
 \mathbf{V}_o &= \frac{\mathbf{I}_{o2}}{sC_2} + \mathbf{V}_C
 \end{aligned} \tag{9.10}$$

Upon substituting into the last equation, we get:

$$\mathbf{V}_o = \frac{g_{m2}}{sC_2} \left(\frac{g_{m1}(\mathbf{V}_A - \mathbf{V}_o)}{sC_1} + \mathbf{V}_B - \mathbf{V}_o \right) + \mathbf{V}_C \tag{9.11}$$

Bringing all terms in V_o together and simplifying, we obtain:

$$V_o = \frac{s^2 C_1 C_2 V_C + s C_1 g_{m2} V_B + g_{m1} g_{m2} V_A}{s^2 C_1 C_2 + s C_1 g_{m2} + g_{m1} g_{m2}} \quad (9.12)$$

Depending on which of the three terminals are set to 0 V common, one can realize a lowpass, highpass, bandpass or notch filter. Each of these filters has a critical or centre frequency which can be set by varying the transconductance of the two OTAs. These filters are called adjustable frequency constant- Q_0 filters because they preserve the value of Q_0 while the critical frequencies are shifted.

If $g_{m1} = g_{m2} = g_m$ then we have:

$$\omega_0 = \frac{g_m}{\sqrt{C_1 C_2}} \quad (9.13)$$

and:

$$Q_0 = \sqrt{\frac{C_2}{C_1}} \quad (9.14)$$

Thus the Q_0 is determined by the capacitor ratio, which can be accurately maintained in monolithic chip designs.

General second-order circuits using OTAs exist that implement the general biquad circuit. In these, tuning the ω_0 and Q_0 for both the poles and zeros to any desired value is possible.

There was a time when OTAs were widely used in radio and microwave systems, but digital signal processing has taken over in these applications. Consequently, OTA integrated circuits are disappearing from the market. There are only two OTA ICs in production in 2020: the TI LM13700 and the On Semi NE5517.

9.4 IC Filters

Don't forget about ready-made filter ICs

There are many special purpose and universal filters available in integrated circuit form. For example, 8th-order switched capacitor Butterworth filters can be built with a single IC, a few capacitors and a clock. When designing a filter, we should always consider the IC option. Sometimes we cannot achieve the desired accuracy, stability, or PCB real estate requirements using IC filters, in which case we resort to a custom-designed filter.

Also, we cannot use IC filters for high frequency or high power applications. Switched capacitor filters also let through a remnant of the clock signal, and they suffer from phase jitter (but they are good enough for voice applications like telephones, where they are mostly used).

An example of a commercially available universal filter is the UAF42 from Texas Instruments. This uses the “state-variable” biquad topology rather than the Tow-Thomas. It requires just four external resistors to set the filter parameters, and makes available lowpass, highpass and bandpass outputs:

The UAF42 universal filter from Texas Instruments

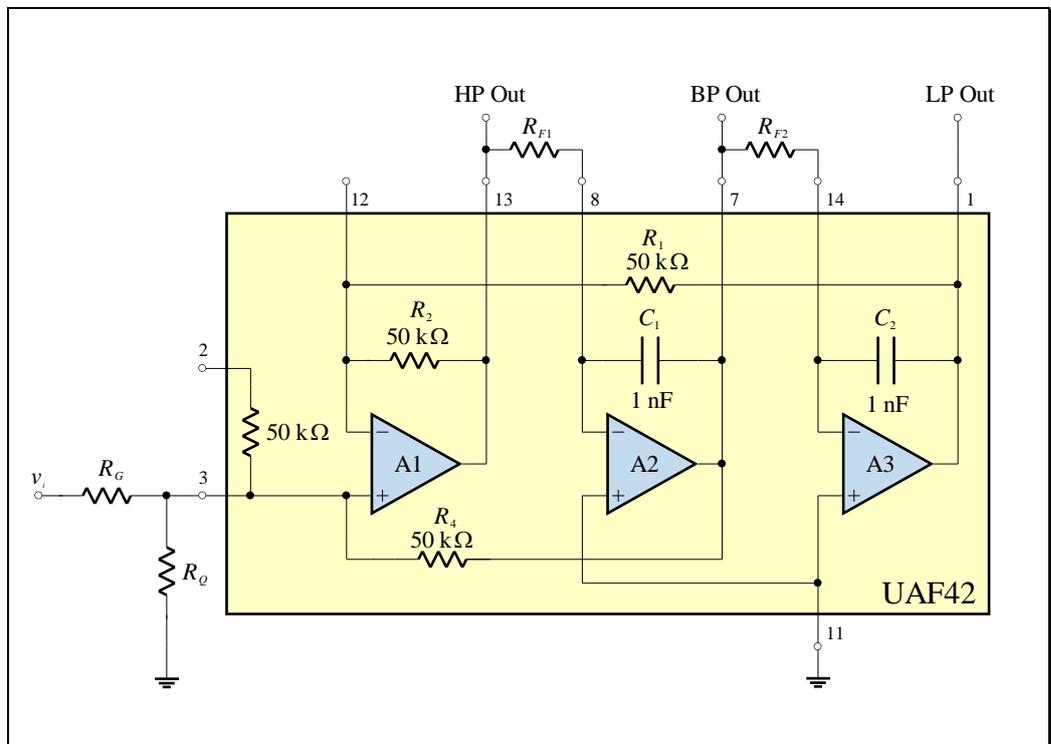


Figure 9.10

9.5 Digital Filters

A digital filter used in analog applications is a slight misnomer. As you may know from other subjects, “digital” systems actually require analog filters to ensure their success. Digital filters use analog filters

Digital filtering involves sampling, quantising and coding of the input analog signal (using an analog to digital converter, or ADC for short). Once we have converted voltages to mere numbers, we are free to do any processing on them that we desire. The processed numbers then have to be converted back into analog form (using a digital to analog converter, or DAC).

The components of a digital filter are shown below:

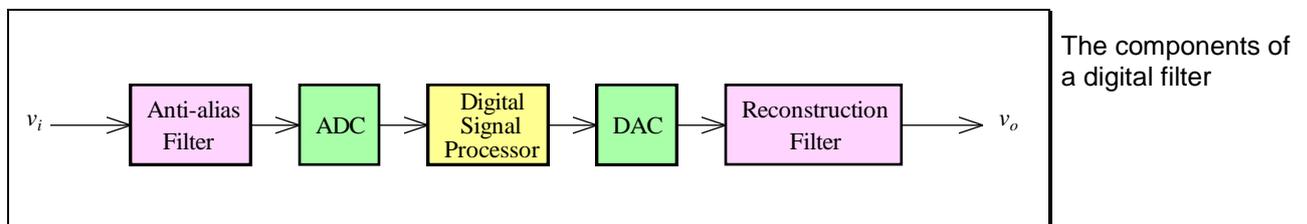


Figure 9.11

The digital signal processor can be custom built digital circuitry, or it can be a general purpose computer. There are many advantages of digitally processing analog signals: Digital filter advantages

1. A digital filter may be just a small part of a larger system, so it makes sense to implement it in software rather than hardware.
2. The cost of digital implementation is often considerably lower than that of its analog counterpart (and it is falling all the time).
3. The accuracy of a digital filter is dependent only on the processor word length, the quantising error in the ADC and the sampling rate.
4. Digital filters are generally unaffected by such factors as component accuracy, temperature stability, long-term drift, etc. that affect analog filter circuits.

9.18

5. Many circuit restrictions imposed by physical limitations of analog devices can be circumvented in a digital processor.
6. Filters of high order can be realised directly and easily.
7. Digital filters can be modified easily by changing the algorithm of the computer.
8. Digital filters can be designed that are always stable.
9. Filter responses can be made which always have linear phase (constant delay), regardless of the magnitude response.

Digital filter
disadvantages

Some disadvantages are:

1. Processor speed limits the frequency range over which digital filters can be used (although this limit is continuously being pushed back with ever faster processors).
2. Analog filters (and signal conditioning) are still necessary to convert the analog signal to digital form and back again.

9.6 Implementation Issues of Active Filters

Throughout the subject, we have always assumed that the op-amps were ideal. The theory behind sensitivity gave us an insight into the effects of passive components on a filter's response.

We will take another look at the op-amp to see what effect it may have in “the real-world”. The way we will do this is to be aware of some of the non-ideal properties of op-amps and then devise an easy way to take account of them in a design. The op-amp is not ideal

9.6.1 Gain Bandwidth Product (GB)

General purpose op-amps have internal “compensating” capacitors for reasons of stability. This causes the gain to decline with one-pole rolloff above a certain frequency. The gain bandwidth product is approximately equal to the “unity gain bandwidth” and we will use the terms interchangeably.

For example, the output of a simple inverting amplifier, using resistors, will exhibit one-pole rolloff at high frequencies. Instead of providing gain, it has actually introduced another pole into the circuit. This leads to a great deal of complexity when evaluating the overall transfer function of a filter. An op-amp will introduce a pole into the circuit

Consequently, there is *no such thing* as an active highpass filter, since the GB product will ultimately produce a lowpass characteristic, making a wideband bandpass filter instead. It is your responsibility to choose an op-amp with a frequency limit well above the bandwidth of interest.

9.6.2 Slew Rate

We have been dealing with filters in the frequency domain, but the time domain response is just as important. The most important time-domain factor that specifies op-amp performance is slew rate.

Slew rate is a measure of how well an op-amp's output can follow the input. It is normally measured in $V/\mu s$. A high slew rate is preferred, which means the output can rise or fall quickly and so follow a varying input.

9.6.3 Settling Time

The settling time is the time it takes an op-amp's output to reach a final value. In the process of the output "settling", it may undergo "ringing". This is important when designing filters that have their inputs switched between two or more signals (a multiplexed input).

9.6.4 Saturation

Ensure that op-amps do not saturate

Mathematically, signals can be quite large in the middle of a cascade designed filter, especially for high Q circuits. In reality, op-amps saturate close to their supply rail voltages. We must ensure that we do not saturate *any* op-amp in a cascade design. If large gain is required, it must be spread throughout the cascade in a careful manner, or placed at the end.

9.6.5 Noise

Like any component, op-amps introduce noise into the system. Just the reverse of saturation, we must also ensure that signals do not "get buried" in noise by being attenuated through the cascades of a filter. Related to noise is the issue of DC supply rail decoupling. We must ensure that DC rails are AC commons.

9.6.6 Design Verification

Get around all these problems using computer simulation

The way to get around all these problems is easy. All designs so far have been "paper" designs. The best test we can perform on a paper design is simulation using a computer package such as Altium Designer, OrCAD or LTSpice. At this point the filter can be adjusted (better op-amp, tighter tolerance components, etc.) to meet specifications in both the time-domain and frequency-domain.

9.6.7 Design Tips

- Increasing the order of a filter may mean specifications are easily met theoretically. It is tempting, if using a quad op-amp package for example, to increase the order of the filter to utilise spare op-amps. Be careful though – high-order cascade designs have sensitivity problems, so actually increasing the order too much can degrade real-life performance.
- As a rule of thumb, you need to ensure that the open-loop gain, A_{OL} , of the op-amp is 40 dB above the peak response required of the individual filter section. This means the unity-gain bandwidth should be:

$$f_T > 100 \cdot Gain \cdot f_c \quad (9.15)$$

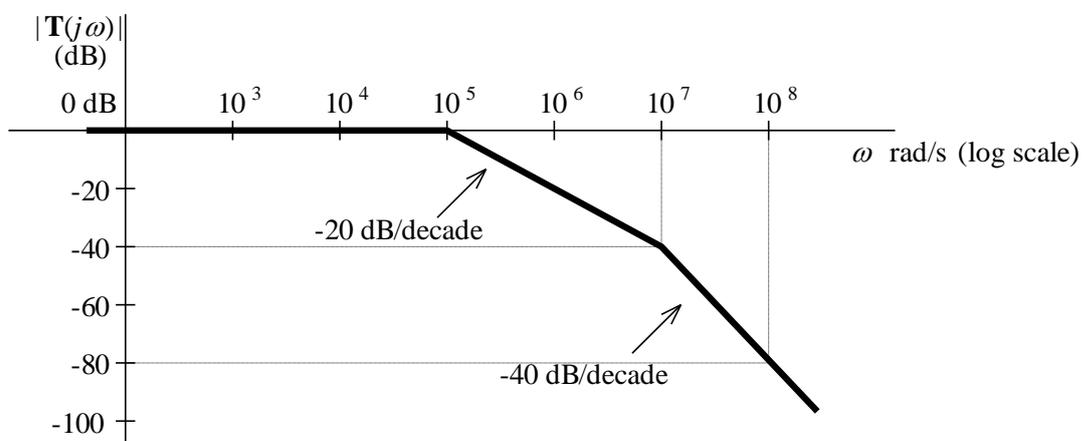
- Make sure you understand sensitivity – a quick hand analysis or reference book will tell you which circuits have the least amount of sensitivity.
- Always choose component tolerances using Monte Carlo analysis. Choose the highest tolerance your design can cope with – the components are generally cheaper. For through-hole technology, metal film 1% resistors are usual in analog filters. Through-hole capacitors are normally around 5% or 10% although some applications may require high stability 1% polystyrene capacitors. For surface-mount technology, thin-film 0.5% resistors are common and relatively cheap. If the application justifies it, thin-film resistors are available with tolerances as small as 0.01%, although they are expensive! Surface-mount capacitors in analog applications would normally be 1% or 5% tolerance, and made of ceramic class 1 dielectric.
- Don't ignore phase response. Some applications may require distortionless filters in which case the Bessel-Thomson response is the best, followed by the Butterworth. Phase response can be corrected with allpass filters, but it complicates design and realisation.

9.7 References

Van Valkenburg, M. E.: *Analog Filter Design*, Holt-Saunders, Tokyo, 1982.

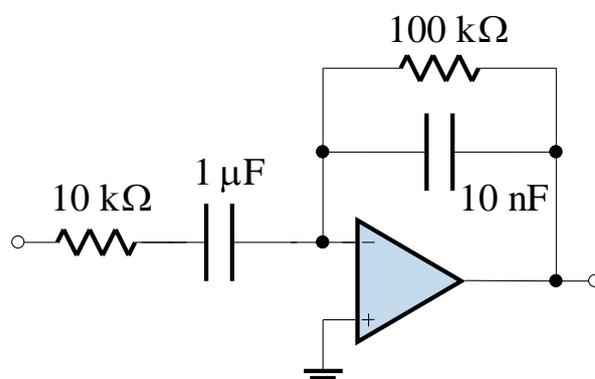
Answers

1.1



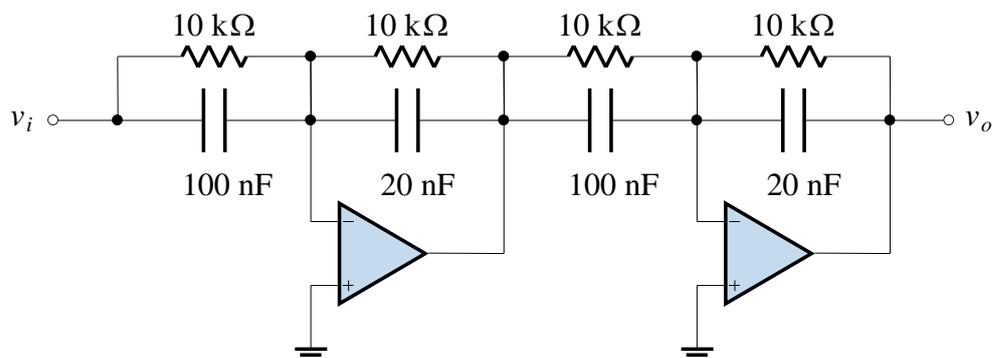
1.2

One possible solution is:



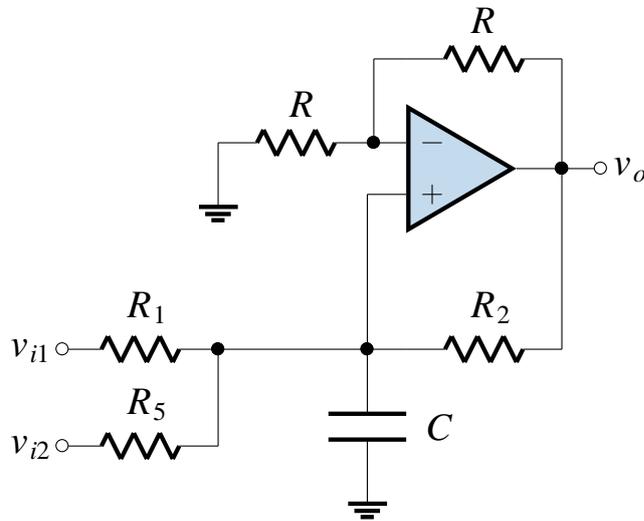
1.3

One possible solution is:



2.1

(a) If we analyse the circuit:



then the output voltage phasor is found to be:

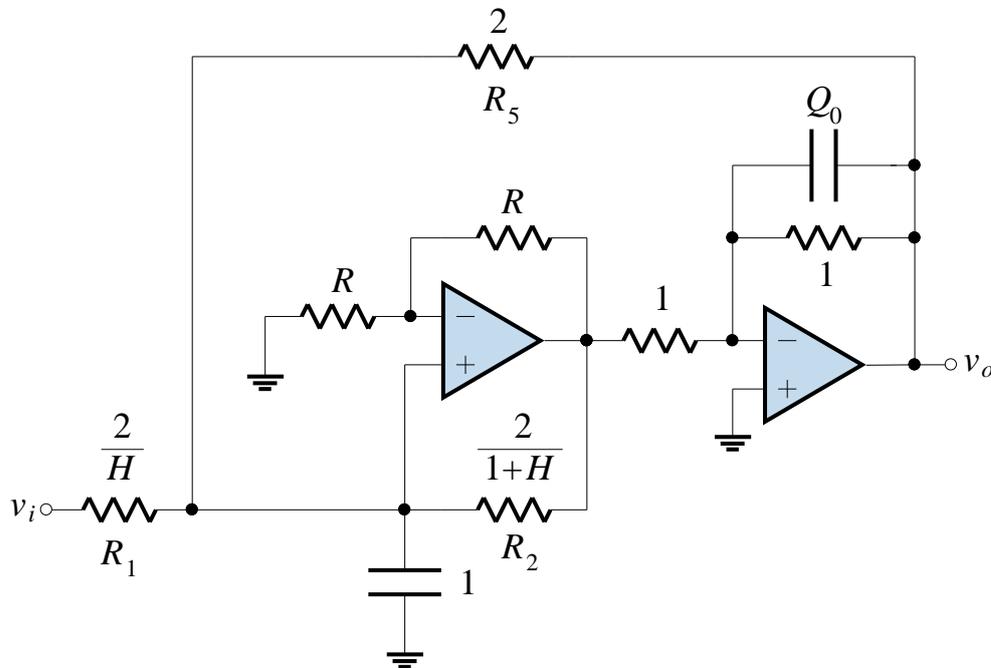
$$\mathbf{V}_o = \frac{2R_2(R_5\mathbf{V}_{i1} + R_1\mathbf{V}_{i2})}{R_1R_2R_5Cs + R_1R_2 + R_2R_5 - R_1R_5}$$

Note that we must have $R_2 = R_1 \parallel R_5$ for the circuit to be a pure integrator.

If $R_1 = R_5 = 2R$ then $R_2 = R$ and we have:

$$\mathbf{V}_o = \frac{2\mathbf{V}_{i1}}{R_1Cs} + \frac{2\mathbf{V}_{i2}}{R_5Cs} = \frac{1}{RCs}\mathbf{V}_{i1} + \frac{1}{RCs}\mathbf{V}_{i2}$$

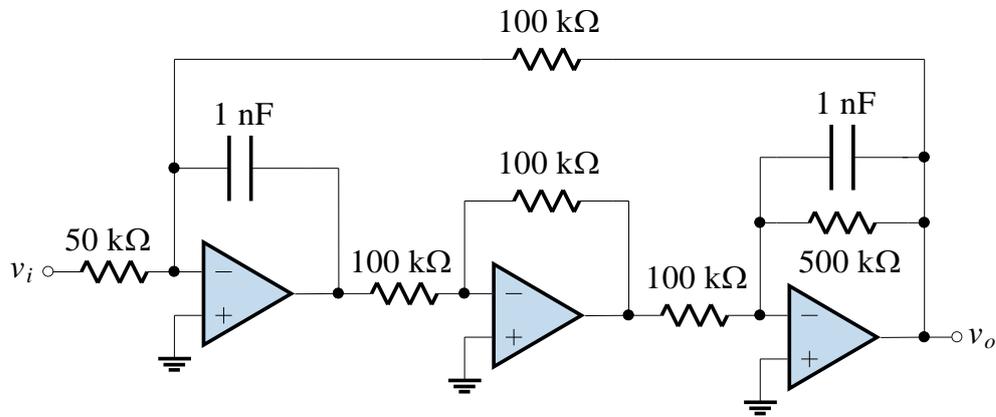
- (b) The circuit is a noninverting integrator, so that it accomplishes the same objective as cascading an inverting integrator and an inverter. A normalized version of the biquad with the noninverting integrator is shown below:



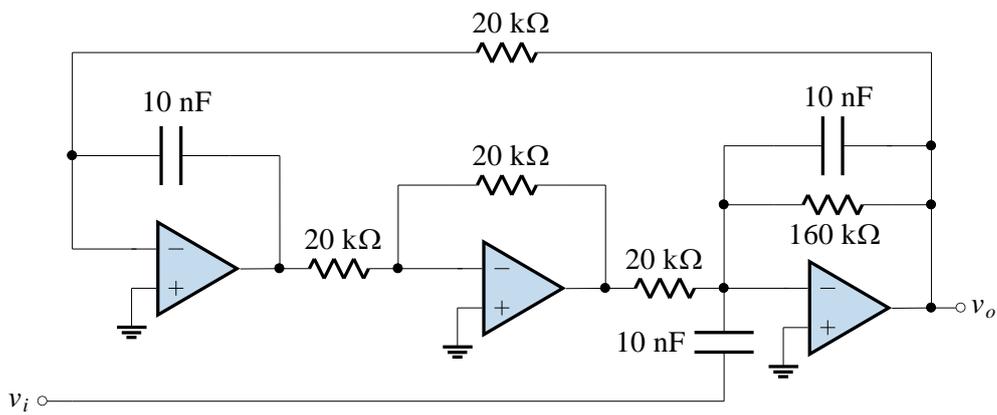
Note that we must have $R_2 = R_1 \parallel R_5$ for the noninverting integrator to be a pure integrator, and if $H=1$ then $R_1 = R_5 = 2$ and $R_2 = 1$.

2.2

(a) One possible solution is:



(b) One possible solution is:



2.3

(a) $n = 4$

(b) $\omega_0 = 1230\text{ rads}^{-1}$

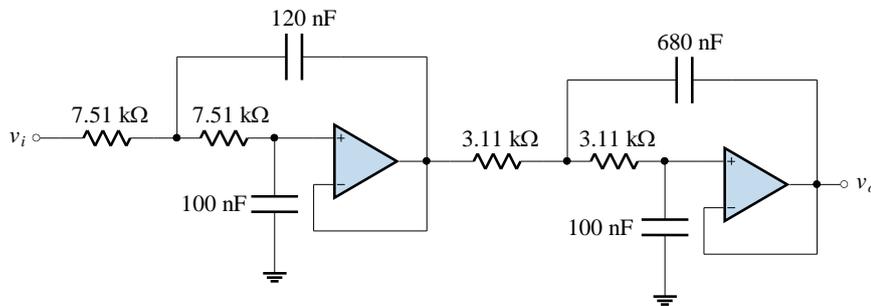
(c) $s = -1137 \pm j470.8, -470.8 \pm j1137, Q_{01} = 0.5412, Q_{02} = 1.307$

(d) $-0.7579\text{ dB}, -36.33\text{ dB}$

2.4

$$(a) \mathbf{T}(s) = \frac{1}{s^2 + 1/Q_0 s + 1}$$

(b) One possible solution is:

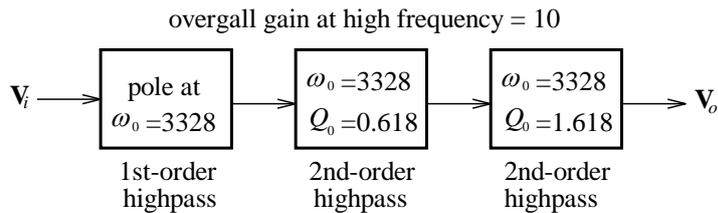


3.1

$$S = bw \frac{s}{s^2 + \omega_0^2}$$

It is interesting to note that this is the reciprocal of the frequency transformation from the lowpass to the bandpass case.

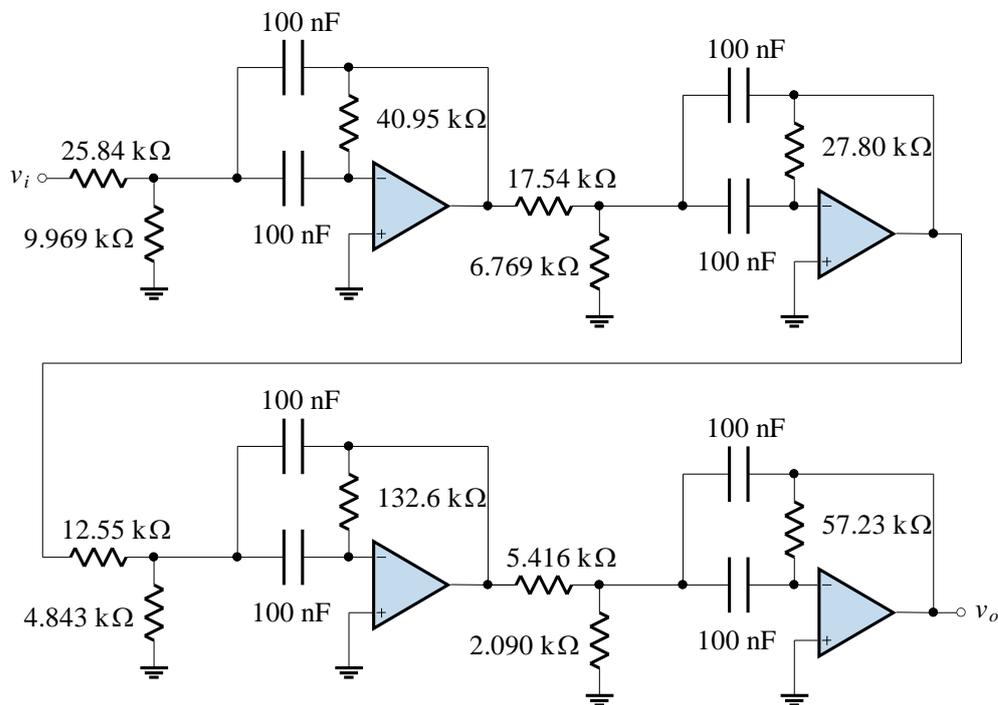
3.2



3.3

The specification is tightened so that we have equal K_s and $\omega_3 = 333.3 \text{ rads}^{-1}$.

One possible solution is:



3.4

$$R_a = \frac{1}{K}, R_b = \frac{1}{1-K}$$

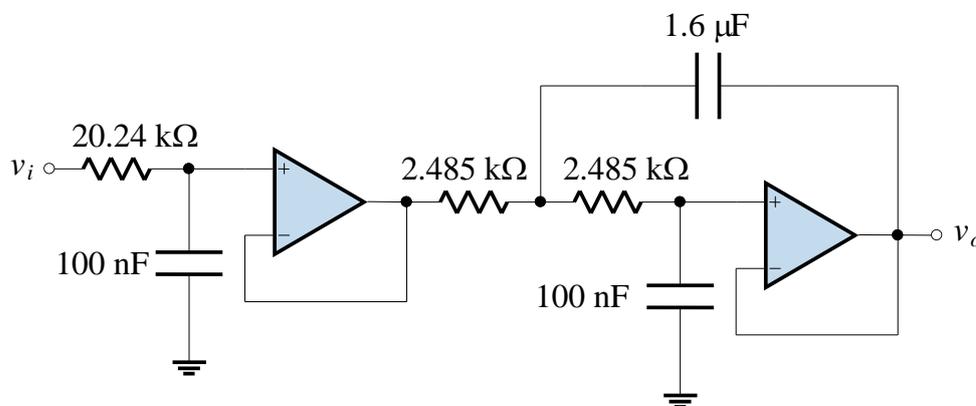
5.1

$$S_{R_1}^{a_0} = -\frac{1}{2}, S_{R_2}^{a_0} = -\frac{1}{2}, S_{C_1}^{a_0} = -\frac{1}{2}, S_{C_2}^{a_0} = -\frac{1}{2}$$

$$S_{R_1}^{Q_0} = 0, S_{R_2}^{Q_0} = 0, S_{C_1}^{Q_0} = \frac{1}{2}, S_{C_2}^{Q_0} = -\frac{1}{2}$$

5.2

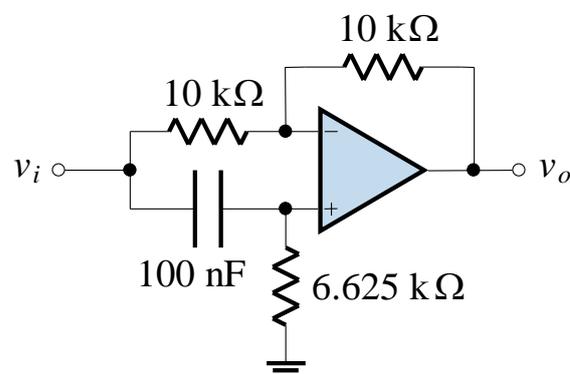
One possible solution to Exercise 2.3, implemented with a Chebyshev response, is:



Note that $n=3$ for this filter, as opposed to $n=4$ for the Butterworth filter.

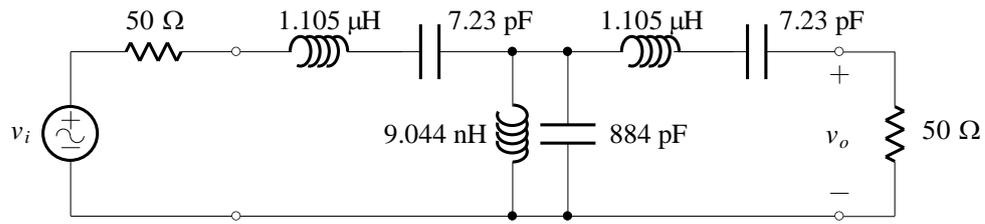
5.3

One possible solution is:



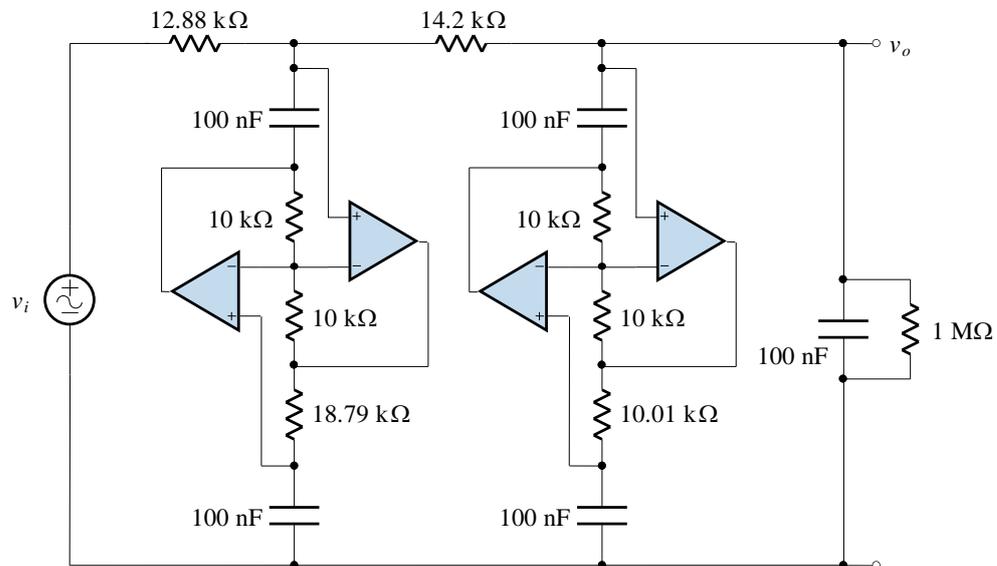
A.8

8.1



8.2

One possible solution is:



8.3

One possible solution is:

