Lecture 1B – Electrodynamics

Magnets. Gauss' Law for magnetostatics. Law of Biot-Savart. Magnetic field near a long, thin conductor. Magnetic scalar potential. Ampère's Law. Reluctance, magnetomotive force and magnetic circuits. The axial field of a current loop. The axial field of a solenoid. The Lorentz force. Electromotive force (emf). Flux linkage. Faraday's Law. Inductance.

Revision

Coulomb's law

Experimental law. Applies to electrostatics.

Potential Difference

Defined as work done per unit charge in moving charge through an electric field. Equipotentials join all points in a field that have equal potential. The static electric field is a conservative field – no net work is done if you push a charge around some path and finish back at its starting point. *Why?* This is analogous with a gravitational field (on Earth we would have to disregard the friction caused by the atmosphere).

Conservative electric field

Flux and Flux Density

Flux streams from the source to permeate all of space – it is an imaginary concept. Its message is: "You are now in a force field". It explains how we can have an action-at-a-distance. How does a charge know where another charge is? Flux provides this information. Imagine a picture of the flux. It looks like a picture using lines of force. We say that a tube of flux is the flux in between these lines. *Draw a picture of a charge free region of space with an electric field. (Use lines of force to represent a field). Draw a tube of flux and perform Gauss' Law at each end.*

Flux "explains" action-at-a-distance

Flux tube defined

Gauss' Law

Can be derived from experimental laws. Easy to apply to cases where geometry has symmetry. Relates flux density to field strength.

Magnets

A brief history of magnetostatics

In 1600, William Gilbert (physician to England's Queen Elizabeth I) investigated the attraction between magnets and the electrostatic effects observed when certain objects were rubbed. His work, *De Magnete*, was mainly qualitative and provided little understanding of the nature of the phenomena.

Magnetism is similar in some ways (but not others) to electrostatics

The study of magnetism began as the study of mechanical attraction of some objects to certain other objects – not too much different to that of electrostatics. Coulomb showed that there was an inverse square law that applied to the force of attraction between two magnets, like electrostatics. The analogy stops there, because there has been no demonstration of the existence of isolated magnetic poles. *Consider what happens when we try to cut a magnet in half to isolate the two poles.*

Demo

Two magnets. Show attraction and repulsion. Show that steel is attracted. Show that it behaves just like an extension of the magnets. Show attraction and repulsion. Remove one piece of steel. Explain? Use one magnet. Sprinkle iron filings. Use compass needle. Filings align along the lines of force, and give a picture of the magnetic field.

Gauss' Law for Magnetostatics

An analogous consideration applies to the magnetic field, but in this case there are no isolated sources of the magnetic field so we get:

$$\boxed{\oint_A \mathbf{B} \cdot d\mathbf{A} = 0} \tag{1B.1}$$

Gauss' Law applied to magnetics

and:

$$\boxed{\mathbf{B} = \mu\mathbf{H}} \tag{1B.2}$$

Magnetic flux density is related to magnetic field intensity

The magnetic constant of the medium is the permeability:

$$\mu = \text{permeability of the medium} \tag{1B.3a}$$

$$\mu_0 = \text{permeability of free space} = 4\pi \times 10^{-7}\ \text{Hm}^{-1} \tag{1B.3b}$$

Permeability as a magnetic property of the medium

and **H** is the magnetic field strength. We also similarly define:

$$\mu_r = \mu / \mu_0 \tag{1B.4}$$

Relative permeability defined

Law of Biot-Savart

Magnetic force is caused by moving charge

In 1819, Hans Christian Oersted found that a wire carrying electric current produces a force similar to the magnetic force.

Demo

Set up the CT and conductor. Show how a horseshoe steel piece is attracted. Sprinkle iron filings to show that the magnetic field is circular - concentric circles of flux. Demonstrate shielding with pieces of steel and aluminium.

The magnetic field can be produced by (*is?*) a moving electric field.

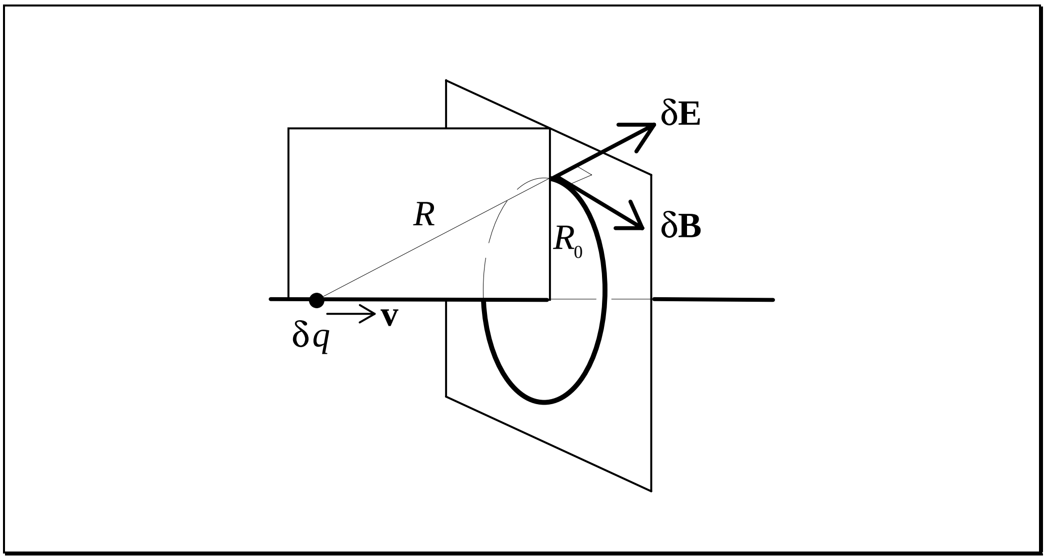


Figure 1B.1

With the situation shown above, experiment shows that the magnetic flux density is:

Experimental law relating **B** and **E**

$$\delta\mathbf{B} = \frac{1}{c^2}\left(\mathbf{v}\times\delta\mathbf{E}\right) \tag{1B.5}$$

That is, **B** is perpendicular to both **v** and **E**. In free space, the speed of light is:

$$c = 2.99793\times 10^8 \text{ ms}^{-1} \tag{1B.6}$$

But, when considering the charge δq, we also have:

$$\mathbf{v} = \frac{\delta \mathbf{l}}{\delta t}, \quad \delta \mathbf{E} = \frac{\delta q}{4\pi\varepsilon_0 R^2}\hat{\mathbf{R}},$$
$$i = \frac{\delta q}{\delta t}, \quad \mathbf{B} = \mu_0 \mathbf{H} \tag{1B.7}$$

so that the magnetic field strength due to a *current element* $i\delta\mathbf{l}$ in free space is:

Current element defined

$$\delta \mathbf{H} = \frac{1}{\mu_0 c^2}\left(\frac{\delta \mathbf{l}}{\delta t}\right) \times \left(\frac{i\delta t}{4\pi\varepsilon_0 R^2}\hat{\mathbf{R}}\right) \tag{1B.8}$$

In free space, it turns out that:

The relationship between electric and magnetic constants

$$\mu_0 = \frac{1}{\varepsilon_0 c^2} \tag{1B.9}$$

which gives us:

The magnetic field expressed in terms of current

$$\delta \mathbf{H} = \frac{i}{4\pi R^2}\delta \mathbf{l} \times \hat{\mathbf{R}} \tag{1B.10}$$

But current only exists in a closed circuit, so the total magnetic field strength is obtained by adding up all the small contributions of each current element in the circuit C:

The Law of Biot-Savart

$$\boxed{\mathbf{H} = \oint_C \frac{i}{4\pi R^2}\left(d\mathbf{l} \times \hat{\mathbf{R}}\right)} \tag{1B.11}$$

This is the Law of Biot-Savart.

Magnetic Field Near a Long, Thin Conductor

We can apply the Law of Biot-Savart to various conductor arrangements to obtain simple expressions for the magnetic field.

H due to a current in a length of long, thin conductor *XY* can be determined this way:

The Law of Biot-Savart applied to a long, thin conductor

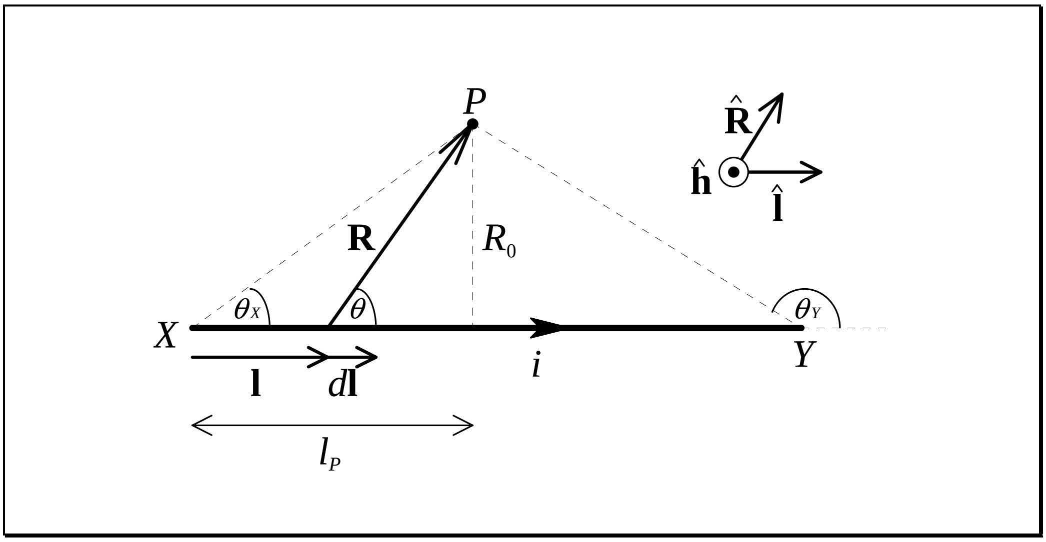


Figure 1B.2

Reducing the integrand to a single variable

$$\mathbf{H}_P = \int_X^Y \frac{i}{4\pi R^2}\left(d\mathbf{l}\times\hat{\mathbf{R}}\right)$$

$$l_P - l = R_0 \cot\theta \qquad \frac{dl}{d\theta} = R_0 \mathrm{cosec}^2\theta$$

$$R = R_0 \mathrm{cosec}\,\theta \qquad d\mathbf{l}\times\hat{\mathbf{R}} = dl\sin\theta\hat{\mathbf{h}}$$

H at a point near a long, thin conductor

$$\begin{aligned} H_P &= \int_X^Y \frac{i}{4\pi R^2}\sin\theta dl \\ &= \frac{iR_0}{4\pi R_0^2}\int_{\theta_X}^{\theta_Y}\frac{\mathrm{cosec}^2\theta}{\mathrm{cosec}^2\theta}\sin\theta d\theta \\ &= \frac{i}{4\pi R_0}\left(\cos\theta_X - \cos\theta_Y\right) \end{aligned} \tag{1B.12}$$

If $XY = \infty$ then $\theta_X = 0,\ \theta_Y = \pi$ and:

H at a point near an infinitely long, thin conductor

$$\mathbf{H}_P = \frac{i}{2\pi R_0}\hat{\mathbf{h}} \tag{1B.13}$$

Magnetic Scalar Potential

To further draw the analogy between the electric and magnetic field, we might suppose there exists a magnetic potential similar to V in electrostatics. Therefore, we define the *magnetic scalar potential* to be:

Magnetic scalar potential defined

$$U_{BA} = -\int_A^B \mathbf{H} \cdot d\mathbf{l} \quad \text{A} \tag{1B.14}$$

So far, this definition is based upon an analogy with the electric field – it is a purely mathematical relation that requires a physical interpretation.

Ampère's Law

Consider an infinitely long conductor:

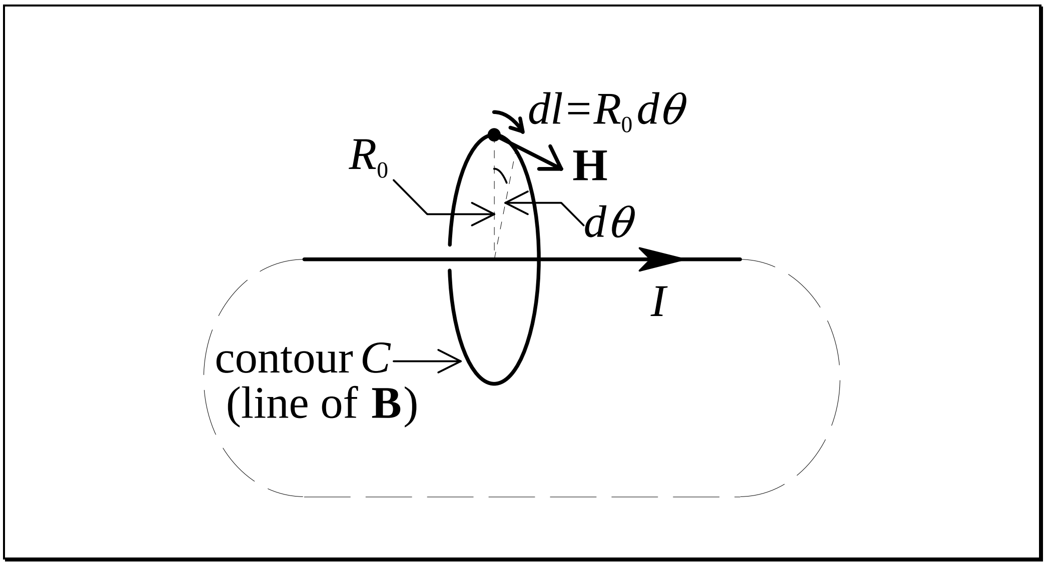


Figure 1B.3

Ampère's Law can be derived from the Law of Biot-Savart

If we add up the magnetic scalar potentials around an arbitrary path (contour) that surrounds the current, what do we get? To make the integration in Eq. (1B.14) easier, we choose the arbitrary path to be one that follows a line of **H** so that the dot product is easy to evaluate. *What does the dot product come to in this case?* To get the value of H at each point along the contour, we use Eq. (1B.13) which was derived using the Law of Biot-Savart.

The sum of the magnetic scalar potentials is then:

$$\sum U = \oint_C \mathbf{H} \cdot d\mathbf{l} = \int_0^{2\pi} \frac{i}{2\pi R_0} R_0 d\theta = i \qquad (1B.15)$$

It turns out that this is the answer we always get, no matter which path C we take. It also turns out that this equation is true for other conductor arrangements. It is so general that we call it Ampère's Law:

Ampère's Law defined

$$\boxed{\oint_C \mathbf{H} \cdot d\mathbf{l} = i} \qquad (1B.16)$$

Describe Ampère's Law in words.

Ampère's Law always applies

Ampère's Law is easy to apply to cases where geometry has symmetry. "Ampère's Law always applies, but *you* may not always be able to apply it." *Demonstrate ease of field calculation with an infinitely long, straight conductor.*

Reluctance, Magnetomotive Force and Magnetic Circuits

Demo

Use a solenoid. Pour iron filings. Equivalent to a magnet. Insert steel rod. Even better. Field looks uniform inside. Maybe if it was infinite? Infinite solenoid = toroid. Pour iron filings. Use solenoid with steel core. Build magnetic circuit. We can direct the flux – like directing current in a wire.

Making an analogy between electric and magnetic circuits

We can use material with a high permeability to direct the magnetic flux along a certain path. This is analogous to using a wire to conduct a current. By experiment, we have seen that the flux density appears uniform inside air cored solenoids and toroids – maybe it is uniform inside other material as well. Even if the flux density isn't uniform, this may be a good approximation to make.

Imagine a *magnetic circuit* that is made up of a high permeability material (such as iron), an air gap, and a winding. For such a circuit, the flux path is well defined, and Ampère's Law reduces to a simple form:

Magnetic circuits are used where the flux path is well defined

$$i = \oint_C \mathbf{H} \cdot d\mathbf{l} = \int_C \frac{B}{\mu} dl = \frac{B}{\mu} l = \frac{l}{\mu A} \phi = \mathcal{R} \phi \tag{1B.17}$$

We call $\mathcal{R}$ the *reluctance* of the magnetic circuit (similar to the concept of resistance in an electrical circuit):

$$\mathcal{R} = \frac{l}{\mu A} \tag{1B.18}$$

Reluctance defined

To make the analogy with electric circuits complete, we introduce the *magnetomotive force* (or mmf for short):

$$\mathcal{F} = Ni \tag{1B.19}$$

Magnetomotive force (mmf) defined

where N is the number of conductors that carry the current i (e.g. a solenoid). Now Ampère's Law reduces to something similar to Ohm's Law:

$$\boxed{\mathcal{F} = \mathcal{R} \phi} \tag{1B.20}$$

Ampère's Law for magnetic circuits

The Axial Field of a Current Loop

We are now in a position to develop some useful formula for the magnetic field intensity caused by some current arrangements. These will be used later on. Consider first a simple circular current loop:

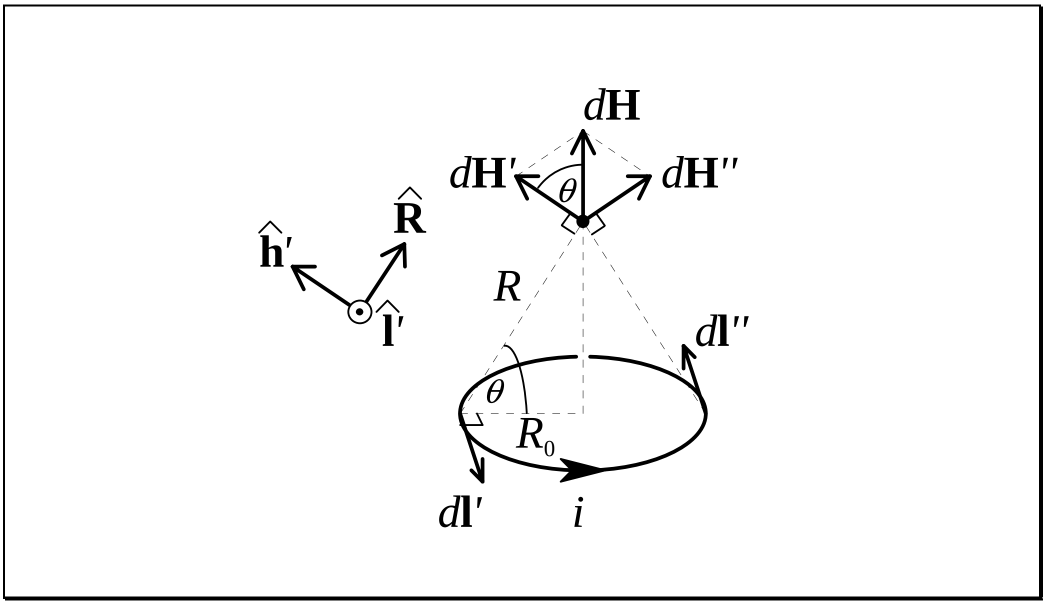


Figure 1B.4

We want to find the field at any point on the axis. As can be seen by the figure, we can use symmetry to give the resultant field due to two opposite current elements:

$$dH = 2dH'\cos\theta \tag{1B.21}$$

By the Law of Biot-Savart, and noting that the angle between $d\mathbf{l}'$ and $\hat{\mathbf{R}}$ is always 90°, we have:

$$dH = \frac{2i}{4\pi R^2}dl'\cos\theta \tag{1B.22}$$

Adding up all the contributions of the current elements as we go around current loop, we get:

$$H=\int_0^{\pi R_0}\frac{2i\cos\theta}{4\pi R^2}dl'=\left[\frac{i(R_0/R)l'}{2\pi R^2}\right]_0^{\pi R_0}$$

$$\mathbf{H}=\frac{iR_0^2}{2R^3}\hat{\mathbf{h}}=\frac{iA}{2\pi R^3}\hat{\mathbf{h}} \tag{1B.23}$$

The Axial Field of a Solenoid

We can immediately use the previous result to derive the axial field of a circular solenoid. Let the solenoid have N turns, axial length l, radius R_0, diameter $d=2R_0$ and current I. To approximate the windings of the solenoid (which form a spiral), we can say that since they are so close together, they look like many independent loops.

A solenoid can be approximated by a number of current loops

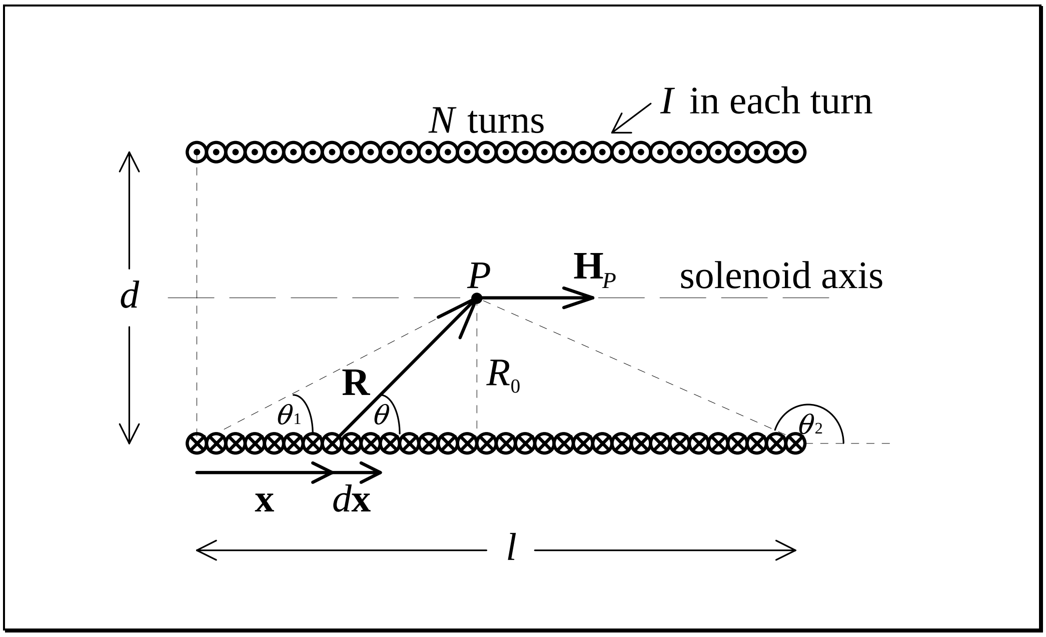


Figure 1B.5

In length dx there are $N/l\,dx$ turns.

We first find the contribution to the total field that is due to one of the current loops:

$$x_P - x = R_0 \cot\theta \qquad \frac{dx}{d\theta} = R_0 \mathrm{cosec}^2\theta$$
$$R = R_0 \mathrm{cosec}\,\theta \tag{1B.24}$$

Application of Eq. (1B.23) gives:

$$d\mathbf{H} = \frac{Ni}{l} dx \frac{R_0^{\,2}}{2R^3}\hat{\mathbf{h}} = \frac{Ni}{2l}\sin\theta d\theta\,\hat{\mathbf{h}}$$

H on the axis of a solenoid

$$\mathbf{H}_P = \frac{Ni}{2l}\int_{\theta_1}^{\theta_2}\sin\theta d\theta\,\hat{\mathbf{h}}$$
$$= \frac{Ni}{2l}\left(\cos\theta_1 - \cos\theta_2\right)\hat{\mathbf{h}} \tag{1B.25}$$

Since:

$$\cos\theta_1 = \frac{l}{\sqrt{d^2 + l^2}}\,,\quad \cos\theta_2 = \frac{-l}{\sqrt{d^2+l^2}} \tag{1B.26}$$

then:

H on the axis of a real solenoid

$$\boxed{\mathbf{H}_P = \frac{Ni}{\sqrt{d^2+l^2}}\hat{\mathbf{h}}} \tag{1B.27}$$

If $l >> 2R_0$ (or $\cos\theta_1 \approx 1$, $\cos\theta_2 \approx -1$), then:

H on the axis of an infinite solenoid

$$\boxed{\mathbf{H}_P = \frac{Ni}{l}\hat{\mathbf{h}}} \tag{1B.28}$$

The Lorentz Force

Demo

The turns of a "loose" solenoid experience attraction when carrying a current. Two parallel conductors experience attraction when carrying current in the same direction, repulsion when currents are in opposite direction.

A charge moving through a magnetic field (caused by, for example, a current or a permanent magnet) will experience a force.

A moving charge will interact with **B**

Consider the case of the two wires attracting each other. The wires need not be parallel to experience a force:

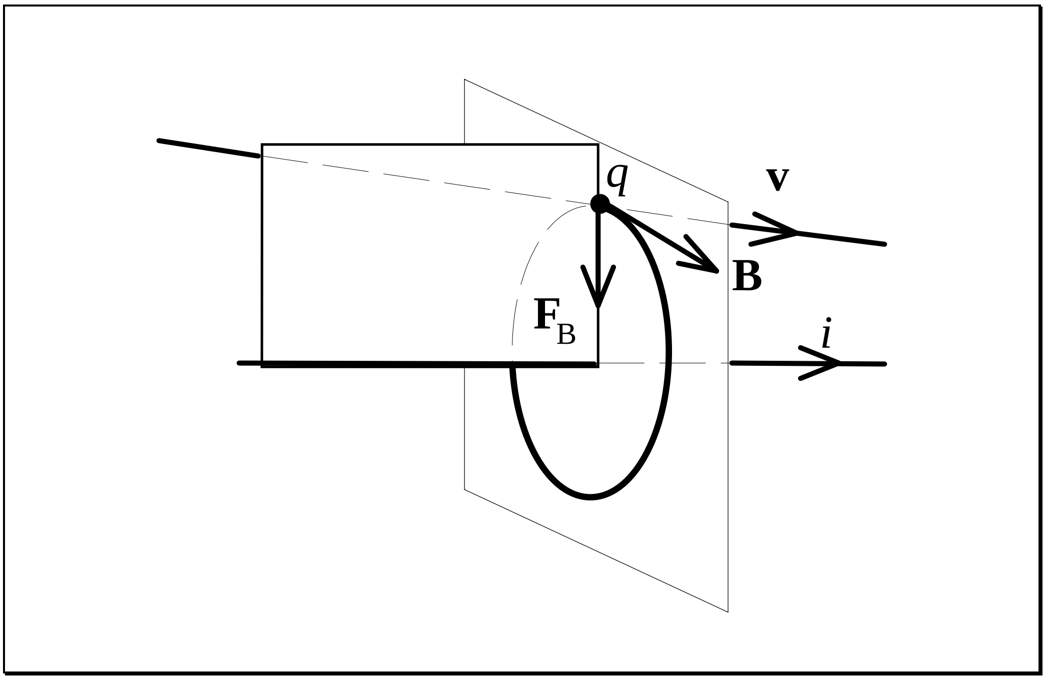


Figure 1B.6

From experiments, the magnetic force is given by:

$$\mathbf{F}_B = q(\mathbf{v} \times \mathbf{B}) \tag{1B.29}$$

The force on a charge moving through **B**

The total (electric and magnetic) force experienced by the charge q is given by the Lorentz Force Law:

$$\boxed{\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})} \tag{1B.30}$$

Lorentz Force Law

Electromotive Force (emf)

Consider a conductor (with lots of free electrons) moving in a magnetic field.

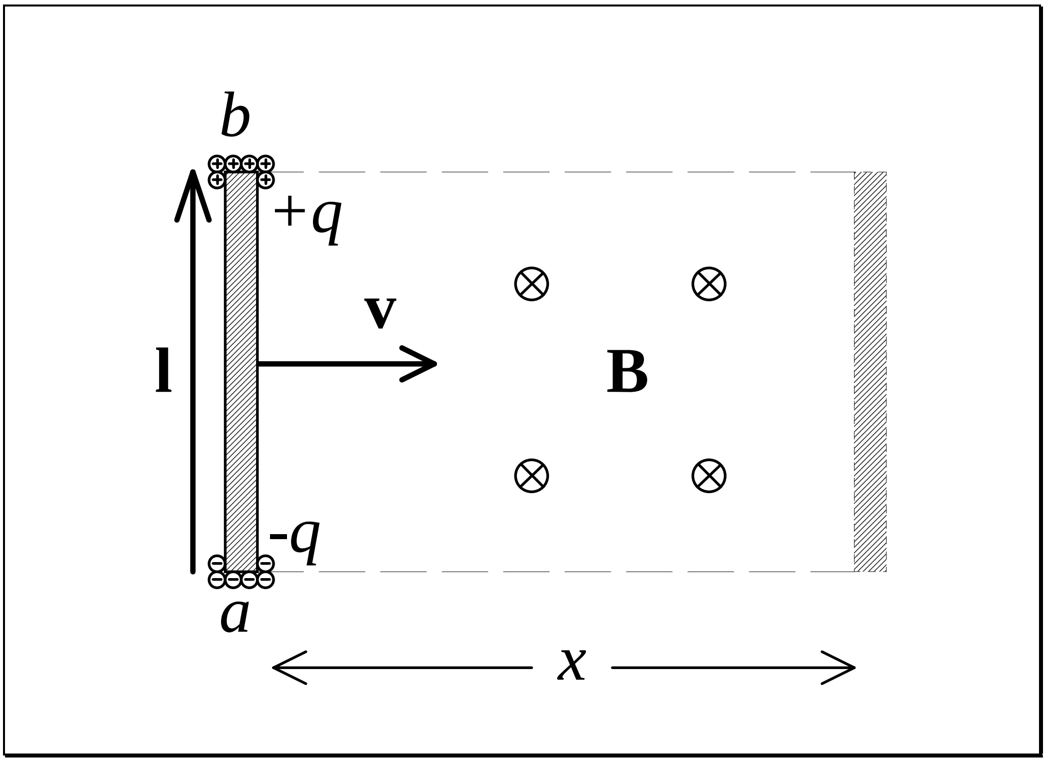


Figure 1B.7

A conductor moving in a magnetic field has a voltage impressed across it

The Lorentz force applies and acts on each electron. The electrons experience a force, and will move in the direction of the force. *Show that the electrons experience a force downwards*. Charge separation results. An electrostatic force (a Coulomb force) now also exists between the charges at the ends of the rod. An *equilibrium* or *steady state* will be reached when:

$$\mathbf{F}_E + \mathbf{F}_B = \mathbf{0} \tag{1B.31}$$

When this occurs, the electrons will cease moving, since there is no force on them.

At equilibrium, the voltage can be deduced using electrostatics

Since we have a static situation in the vertical direction as far as the charges are concerned (they are not moving vertically), we can use our electrostatic knowledge to calculate the work per unit charge involved in moving a charge along the conductor.

Define electromotive force (emf), or more properly, voltage:

$$V_{ba} = -\int_a^b \mathbf{E} \cdot d\mathbf{l} = \int_a^b (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} = vBl \qquad \text{(1B.32)}$$

The voltage developed across a straight conductor in a uniform field

Voltage and potential difference, although similar concepts, are different things. Voltage can cause a steady current, potential difference cannot.

For example, suppose we have a capacitor that has charge on its plates, and hence a potential difference between them. If we connect a resistor across the plates, the charge will flow through the resistor and the capacitor will discharge. There is no mechanism to restore charge on the capacitor, so the current will be short-lived and decay to zero.

Voltage is different to potential difference

With the moving conductor, we can connect a resistor across ends *a* and *b*, and charge will flow through the resistor. As soon as charge escapes from the ends in this manner, Eq. (1B.31) does not apply, and electrons will feel a net force again. Thus, there is a mechanism operating that restores the lost charge. This arrangement will therefore support a steady current. The moving conductor is like a pump – it can push electrons through a resistor.

A voltage requires an energy source

This is the principle of the generator.

Now consider the same arrangement, but used in a different way. If there is a current in the conductor which is immersed in the magnetic field, then the Lorentz force law applies in a different way. The force is on the moving charges which flow through the conductor:

A current in a magnetic field experiences a force

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) = it\left(\frac{1}{t}\mathbf{l} \times \mathbf{B}\right) = i(\mathbf{l} \times \mathbf{B}) \qquad \text{(1B.33)}$$

which is just another form of the Lorentz Force Law

The force on the charges is translated to a force on the rod, causing it to move.

This is the principle of the motor.

Flux Linkage

Consider a loop of wire immersed in a magnetic field:

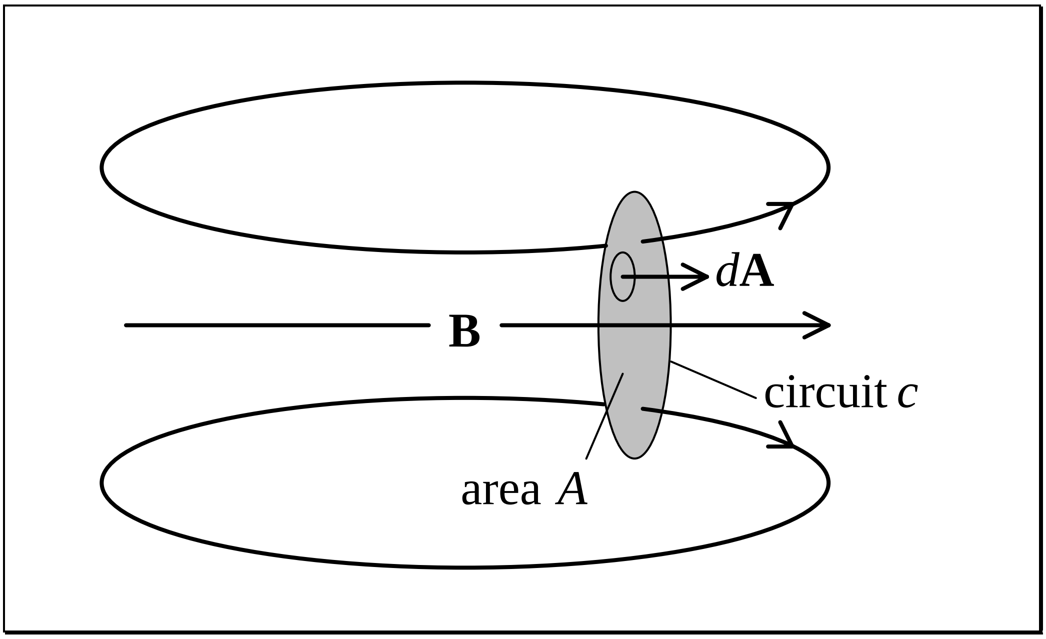


Figure 1B.8

Flux linkage explained,

A flux line (which is a closed loop) that passes through the circuit (which is a closed loop), forms a "link", like two links in a chain. To calculate the total amount of flux linking the circuit, we use:

how we calculate it,

$$\boxed{\phi = \int_A \mathbf{B} \cdot d\mathbf{A}} \tag{1B.34}$$

This is analogous to the calculation of electric flux and current seen previously.

If there are N turns in a solenoid (which is approximately N individual loops), then the flux that links with the circuit is the sum of all the flux linking each turn. We define this flux linkage to be:

and generalised to circuits where flux can link more than once

$$\begin{aligned} \lambda &= \phi_1 + \phi_2 + \phi_3 + \ldots + \phi_N \\ \phi_{av} &= \frac{\phi_1 + \phi_2 + \phi_3 + \ldots + \phi_N}{N} = \frac{\lambda}{N} \\ \lambda &= N\phi_{av} \end{aligned} \tag{1B.35}$$

Faraday's Law

Demo

Connect a meter to a coil and move a magnet nearby. The meter deflects with each movement of the magnet. It is the Lorentz Force law in action. Spin a magnet from the end of a string to demonstrate a simple generator.

We have seen that the magnetic field of a solenoid looks like that of a permanent magnet. We can repeat the experiment with a solenoid. It still works.

In 1840, Michael Faraday discovered a simple relationship to describe the phenomenon of induced voltage. When the magnetic flux linking a circuit changes, a voltage is induced in the circuit. Faraday's Law says the induced voltage is equal to the rate of change of magnetic flux:

Induced voltage is caused by a rate of change of flux linkage

$$\boxed{e = -\frac{d\lambda}{dt}} \tag{1B.36}$$

Faraday's Law

It can be derived from the Lorentz force, but it was discovered experimentally.

Using partial differentiation and the chain rule, we can also write Faraday's Law as two components – a *transformer* voltage and a *motional* voltage:

$$e = -\frac{d\lambda}{dt} = -\frac{\partial\lambda}{\partial t} - \frac{\partial\lambda}{\partial x}\frac{dx}{dt} \tag{1B.37}$$

Transformer voltage and motional voltage

Lenz's Law can be used to give the induced voltage a polarity in a circuit diagram. The polarity of the voltage is such as to oppose the *change* causing it.

Lenz's Law

In reality, Faraday's Law is enough to determine the polarity of the induced voltage, but it depends on the way in which we define the path of integration (refer back to Eq. (1B.32) to see this). For our purposes, Eq. (1B.36), without the minus sign, will give us the *magnitude* of the voltage, and we will use Lenz's Law to give us the *polarity* of the voltage.

Field version of Faraday's Law determines voltage polarity, but Lenz's Law is easier

An electrical circuit can show magnetic effects - the inductor

If the circuit is closed, then there will be a current. The electrical equivalent circuit of our loop of wire is:

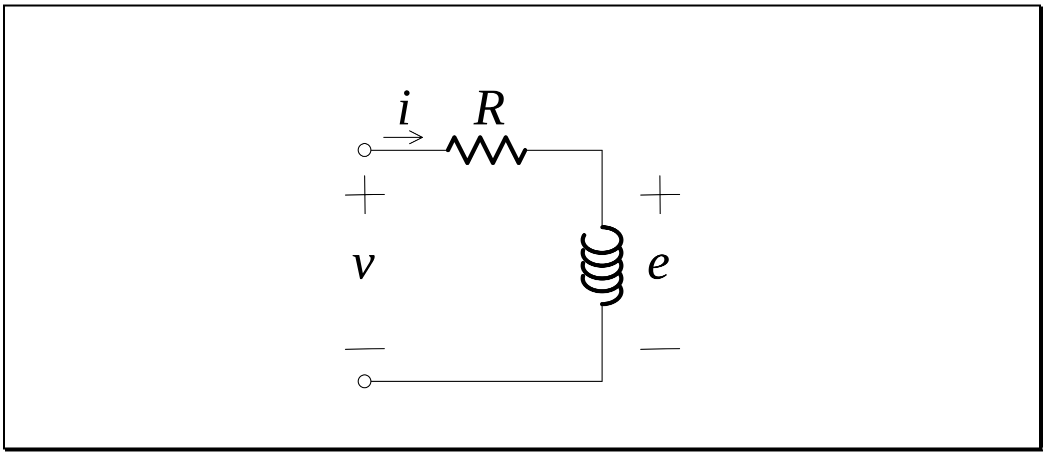


Figure 1B.9

In this equivalent circuit, R represents the resistance of the wire, and e is the induced voltage caused by the changing magnetic flux linkage. KVL gives:

$$v = Ri + e \tag{1B.38}$$

Inductance

Self Inductance

For linear media, flux linkage is proportional to current

Imagine loops of wire wrapped around a toroid with a core material that has a constant permeability (such as plastic or wood). The amount of flux linking the circuit will then be directly proportional to the current in the loop:

$$\begin{aligned}
\lambda &= N\phi = NBA = N\mu HA \\
&= N\mu \frac{Ni}{l} A = \frac{N^2}{l/\mu A} i \\
&= \frac{N^2}{\mathcal{R}} i \\
&= Li
\end{aligned} \tag{1B.39}$$

We call the constant of proportionality the inductance. Its value depends only on geometric factors (lengths, areas, etc) and the material (μ), but is normally determined by:

Inductance defined

$$\boxed{L = \frac{\lambda}{i}} \tag{1B.40}$$

Show the analogy with capacitance.

Mutual Inductance

When flux links with another circuit, we need the concept of mutual inductance

We have seen that a changing magnetic field causes an induced voltage. If a coil is producing the magnetic field, as it did above, then the flux linkage depends directly upon the coil's current – this is summarised in Eq. (1B.40). What if the magnetic field linking a circuit is not caused by itself? We then have to define "mutual" inductance.

Consider two coils wound in the same direction:

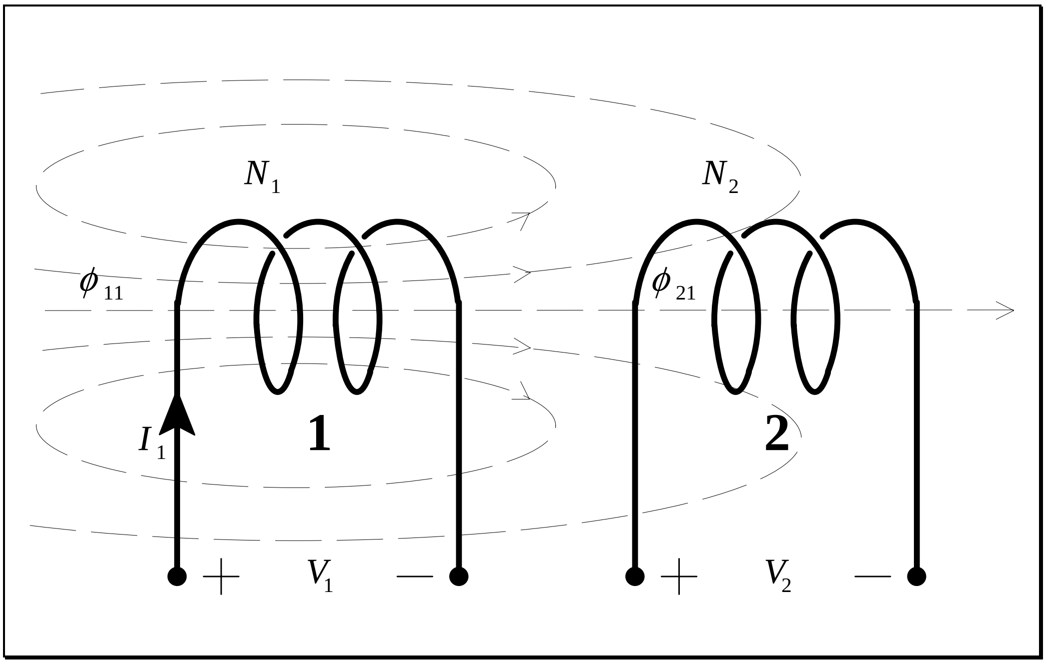


Figure 1B.10

Some of the flux produced by Coil 1 links with Coil 2. We define:

Flux linkage is generalised further

$$\lambda_{11} = N_1\phi_{11} = \text{flux linking 1 due to 1} \tag{1B.41a}$$

$$\lambda_{21} = N_2\phi_{21} = \text{flux linking 2 due to 1} \tag{1B.12b}$$

We can now define mutual inductance:

Mutual inductance defined

$$\boxed{L_{21} = \frac{\lambda_{21}}{i_1}} \tag{1B.42}$$

An emf can be caused by another circuit

What if Coil 1 is excited in such a way as to produce a changing magnetic field? (A sinusoidal voltage could do the job.) Coil 2 is an open circuit, but it has been immersed in the field of Coil 1. Magnetic flux produced by Coil 1 will link with Coil 2. An *induced voltage* will be produced across its terminals due to Faraday's Law:

$$v_2 = e_2 = -\frac{d\lambda_{21}}{dt} \tag{1B.43}$$

If the voltage across Coil 2 has the polarity shown in the diagram, is ϕ_{21} *increasing or decreasing?*

Summary

- Gauss' Law for magnetostatics is: $\oint_A \mathbf{B} \cdot d\mathbf{A} = 0$.

- The magnetic flux density is related to the magnetic field intensity by: $\mathbf{B} = \mu\mathbf{H}$.

- The Law of Biot-Savart relates magnetic field intensity to the current elements that cause it by: $\mathbf{H} = \oint_C \frac{i}{4\pi R^2}\left(d\mathbf{l} \times \hat{\mathbf{R}}\right)$.

- Ampère's Law relates magnetic field intensity around a conductor to the enclosed current by: $\oint_C \mathbf{H} \cdot d\mathbf{l} = i$.

- Ampère's Law for a magnetic circuit states: $\mathcal{F} = \mathcal{R}\phi$.

- A charge moving through electric and magnetic fields will experience a Lorentz force: $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$.

- A flux tube streaming through a circuit creates a flux linkage which is given by: $\phi = \int_A \mathbf{B} \cdot d\mathbf{A}$.

- When the magnetic flux linking a circuit changes, a voltage is induced in the circuit, given by Faraday's Law: $e = -\frac{d\lambda}{dt}$.

- The amount of self-generated flux linking a circuit is directly proportional to the current in the circuit. The constant of proportionality is called the self inductance, and is given by: $L = \frac{\lambda}{i}$.

- The amount of externally-generated flux linking a circuit is directly proportional to the current that generates the flux. The constant of proportionality is called the mutual inductance, and is given by: $L_{21} = \frac{\lambda_{21}}{i_1}$.

- A changing magnetic flux generated by one circuit can produce a voltage in another. This is called an induced voltage.

References

Plonus, Martin A.: *Applied Electromagnetics*, McGraw Hill Kogakusha, Ltd., Singapore, 1978.

Shadowitz, Albert: *The Electromagnetic Field*, Dover Publications, Inc., New York, 1975.

Shamos, Morris H. (Ed.): *Great Experiments in Physics - Firsthand Accounts from Galileo to Einstein*, Dover Publication, Inc., New York, 1959.

Problems

1.

Calculate the field intensity **H** and the flux density **B** at the centre of a current loop, radius 50 mm, carrying current $I = 5$ A when the loop is wound on a core made of:

(a) air

(b) aluminium

(c) iron $(\mu_r = 10000)$

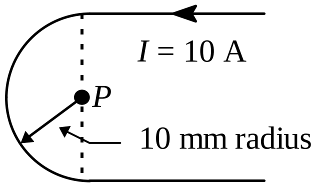
2.

A plane square loop of wire, side l, carries clockwise current I.

Show that at the centre $\mathbf{H} = 2\sqrt{2}I/\pi l$ (down).

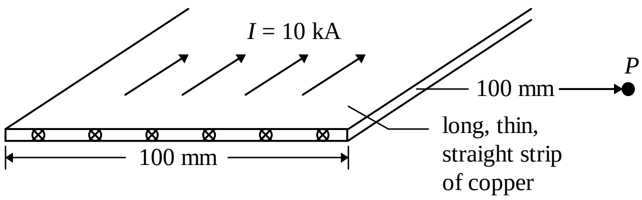
3.

Determine the magnetic flux density at point P.



4. [Current sheet]

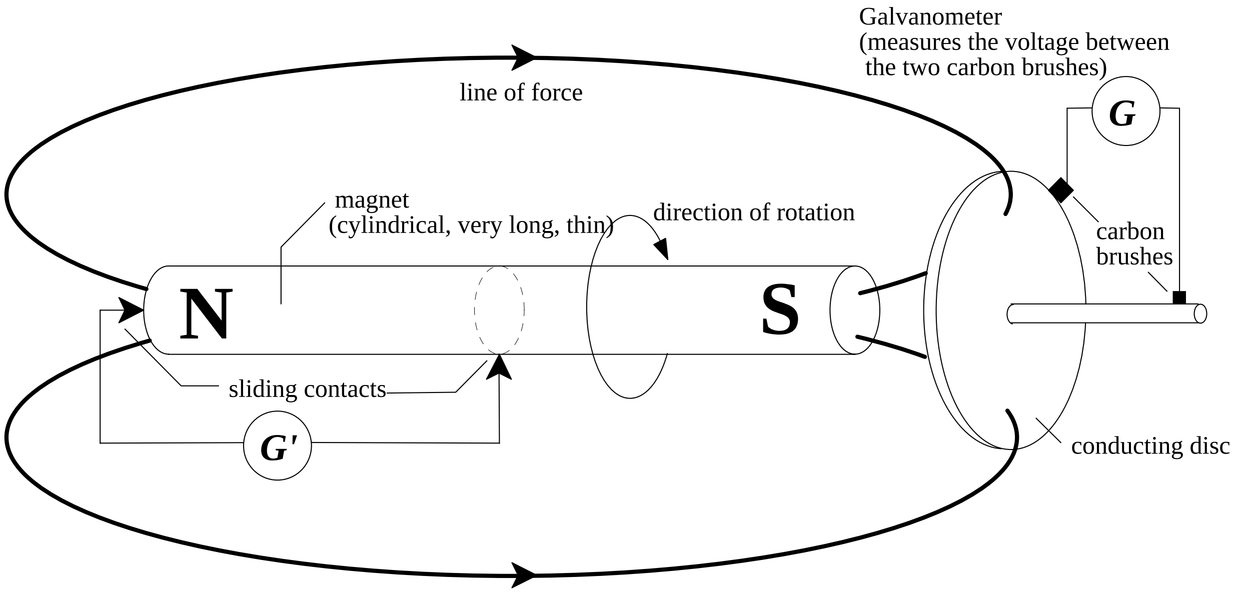
Calculate the flux density at a point P, 100 mm from an edge, and on the same plane of the strip.



Assume: Current is uniformly distributed.
There are no magnetic materials in the vicinity.

1B.24

5. [Faraday's Disc Experiment]



Is there an emf induced in the galvanometers *G* and *G'*?

Is Lenz's Law obeyed?

In the following cases:

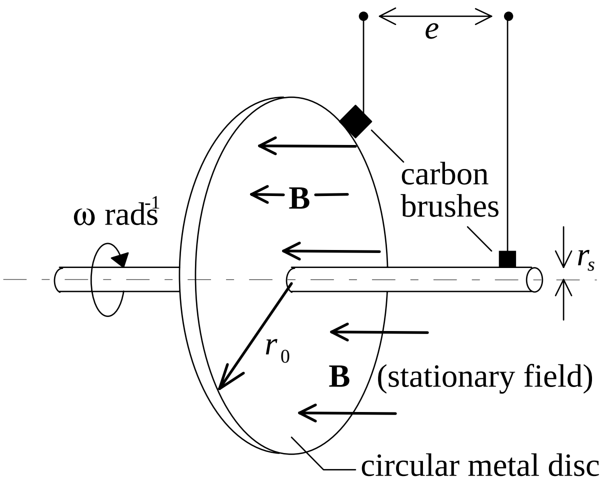
(a) Disc only rotates.

(b) Magnet only rotates.

(c) Both rotate in same direction.

6.

The uniform magnetic field, of density B Tesla is perpendicular to the plane of the disc.



(a) Make a sketch showing the magnetic forces on the free electrons in the disc, the charge distribution on the disc and the electrostatic forces on the free electrons.

(b) Derive an expression for the voltage e between the carbon brushes on the outside rim and on the surface of the shaft.

7.

A 132 kV, 500 A (RMS), single phase transmission line has two conductors, each 20 mm in diameter and 1.5 m apart. The span between supporting poles is 200 m.

Determine the average force acting on the conductors, over one span, during "short circuit" conditions if the short circuit current = 12 x normal current (ignore line sag).

8.

Two parallel circuits of an overhead power line consist of four conductors carried at the corners of a square.

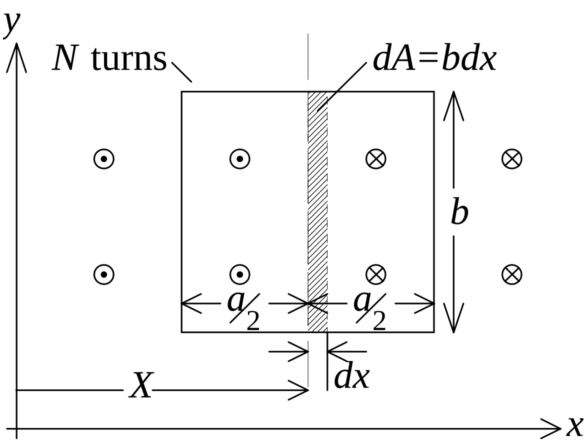
Find the flux per unit length of circuit, in webers/km, linking one of the circuits when there is a current of 1 A in the other circuit.

9.

Use the Lorentz Force Law to determine the force experienced by a 10 m long conductor carrying current I = 100 A (S to N) due to the Earth's magnetic field ($B = 5\times10^{-5}$ T, angle of dip 60°).

10. [Coil moving in magnetic field]

Consider a small rectangular coil (N turns into paper) wound on a non-magnetic former and in a magnetic field of density B (perpendicular to paper).



(a) Calculate the flux linking the coil if $B = \hat{B}\sin\alpha x$ where α = constant.

(NB. Use strip dA, $\therefore\ d\phi = BdA$ and $\phi = \int_{X-a/2}^{X+a/2} \hat{B}\sin\alpha x\ bdx$)

(b) Calculate the emf e induced in the coil when it moves in the x-direction with speed v.

(c) Determine the value of a which maximises e determined in (b).

(d) Calculate the induced emf e if $B = \hat{B}\sin(\omega t - \alpha x)$ and the coil moves in the x-direction with velocity v (ω = constant), and determine e when $v = \omega/\alpha$.