

## Lecture 2A – Conductors and Insulators

*The Bohr atom. The energy barrier. Bound and free charges. Conduction. Dielectrics. The electric dipole. Effect of Boundaries. Polarisation. Ferroelectrics. Breakdown at sharp points.*

### The Bohr Atom

We need to investigate the behaviour of matter subjected to electric fields. This will aid in the understanding of conductors and insulators. We will use the Bohr model of the atom (Greek: *a* = not, *tomos* = cut. The first model of the atom was proposed by Democritus around 430 BCE, about one century prior to the time of Aristotle).

Matter modelled by the atom

John Dalton, in 1803, put forward the idea that the atomic nature of substances could be used to explain their chemical properties. In 1911, Rutherford performed experiments that revealed the structure of atoms. He proposed a model where a very small, positively charged nucleus which carries virtually all the mass of the atom is surrounded by a number of negatively charged electrons.

An atom made up of electric charges

Since there is charge separation, there must be a Coulomb force between the nucleus and electron. To account for the fact that electrons remain at large distances from the nucleus, despite the force of attraction, we assume that the electrons revolve in orbits (like the solar system). Since an electron moving is equivalent to a current, there must be a magnetic field. For the moment, we will consider only the electric field. Magnetic effects resulting from electron motion will be looked at later.

“Solar system” model of the atom

At the atomic level, protons and electrons obey Coulomb’s Law, but the internal effects of this are not always detected externally. Consider a solid piece of metal, which is made up of a lattice of nuclei and a “sea” of electrons. The distance between an electron and nucleus is approximately  $10^{-10}$  m. Coulomb’s Law gives  $E \approx 10^{12} \text{ Vm}^{-1}$ . Violent variations in  $E$  would be expected along the path of a wandering electron, but average out and large

Microscopic view of matter sees large fields

## 2A.2

Macroscopic view of matter does not see large fields

variations are not detected externally. It is clear that we have to delve inside the atom to explain some physical properties that are observed externally.

Bohr's model of the atom

The resulting problems with the Rutherford model (accelerating charge radiates energy – where does this energy come from?) are overcome by using Bohr's model. He proposed that there were certain stable orbits in which electrons could exist indefinitely.

Consider the hydrogen atom, because it is the easiest to analyse:

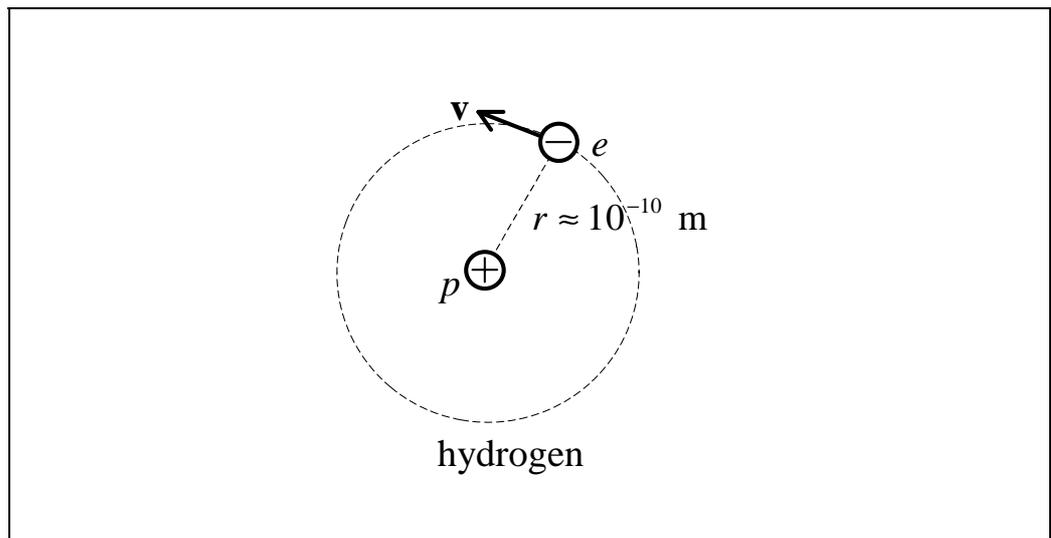


Figure 2A.1

Steps leading to the energy of an electron in Bohr's model of the atom

The electrostatic force of attraction between the charges:

$$F = \frac{e^2}{4\pi\epsilon_0 R^2} \quad (2A.1)$$

provides the centripetal force and, from Newton's second law, since the electron is orbiting with constant speed:

$$\frac{e^2}{4\pi\epsilon_0 R^2} = \frac{mv^2}{R} \quad (2A.2)$$

## 2A.3

Therefore, the kinetic energy of the electron at radius  $R$  is:

$$K = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\epsilon_0 R} \quad (2A.3) \quad \text{Kinetic energy of a bound electron}$$

The potential energy is defined with zero potential at infinity (analogy with gravitational potential energy defined with respect to ground level):

$$\begin{aligned} U &= \int_{\infty}^R \mathbf{F}_M \cdot d\mathbf{l} \\ &= \int_{\infty}^R -\mathbf{F}_E \cdot d\mathbf{l} \\ &= \int_{\infty}^R \frac{e^2}{4\pi\epsilon_0 r^2} dr \\ &= \frac{-e^2}{4\pi\epsilon_0 R} \end{aligned} \quad (2A.4) \quad \text{Potential energy of a bound electron}$$

so the total electron energy at radius  $R$  is:

$$E = U + K = \frac{-e^2}{8\pi\epsilon_0 R} \quad (2A.5) \quad \text{Total energy of a bound electron depends on } R$$

For a stable orbit, the radius determines the total energy of the electron (electron energy is always negative and more so at smaller orbits).

# 2A.4

## The Energy Barrier

Concept of energy barrier is introduced using a uniform field

Consider two equipotentials (*A* and *B*) in a uniform **E** field:

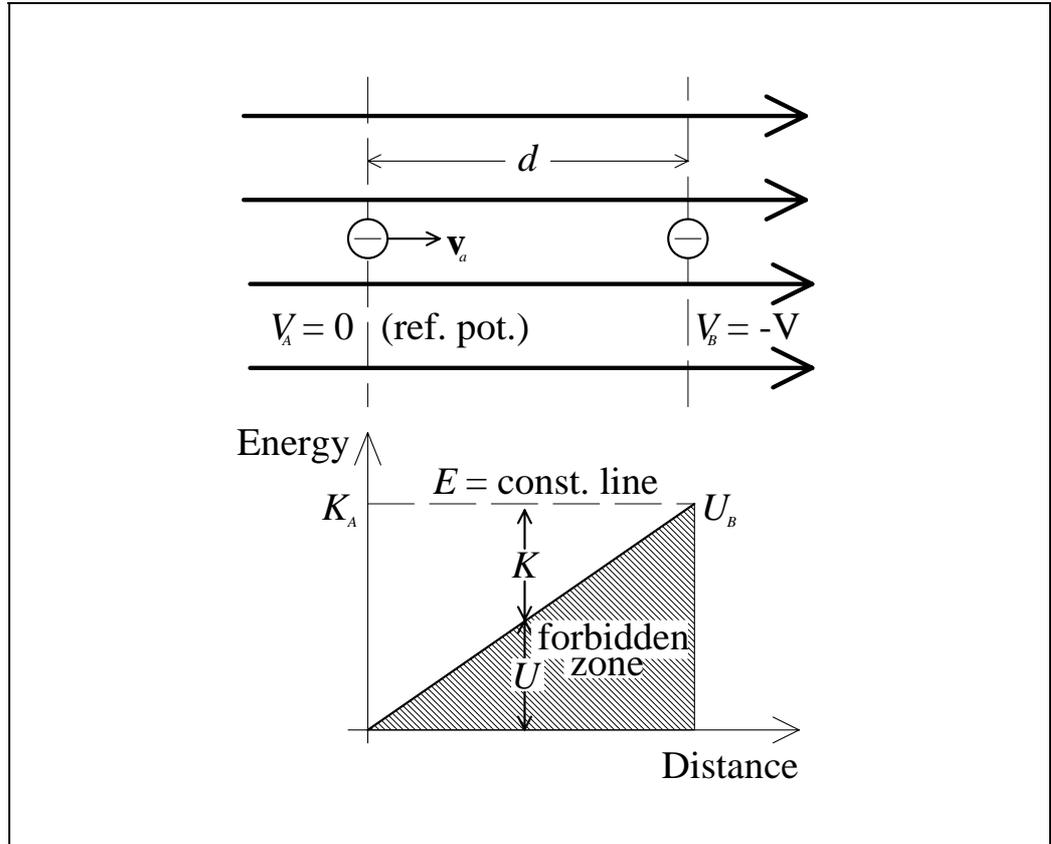


Figure 2A.2

As the electric field is conservative (no energy lost in the system), conservation of energy applies and:

The energy of a charge is constant because the electric field is conservative

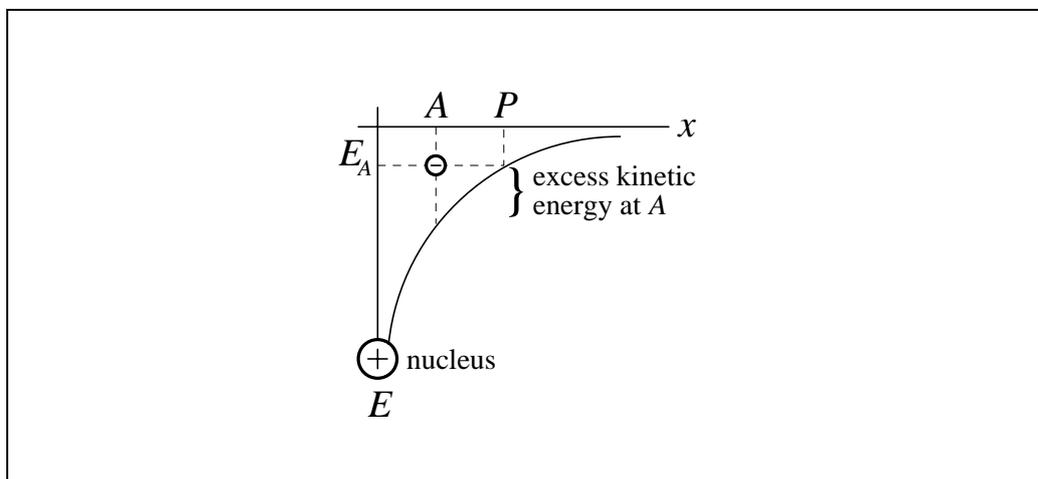
$$E_A = U_A + K_A = eV_A + \frac{1}{2} m_e v_a^2 = \text{const} \quad (2A.6)$$

All the energy at *A* is kinetic energy. As the electron travels towards *B*, some of this kinetic energy is converted into potential energy – the electron must be slowing down. At *B*, the electron has come to rest so  $K_B = 0$ . At this point, the energy of the electron is all in the form of potential energy. This potential energy has the “potential” to do work, for the electron reverses its motion and returns towards *A*. No matter what the energy is to start with, the electron can never get past the “energy barrier” caused by the electric field, i.e. the shaded area of Figure 2A.2 cannot be entered.

An energy barrier converts all energy to potential energy - it stops charges from moving

## Bound and Free Charges

We know from the Bohr model that if an electron is in a stable orbit, then its energy is a constant and given by Eq. (2A.5). We can graph the electron energy for this case as a function of distance  $x$  from the nucleus:



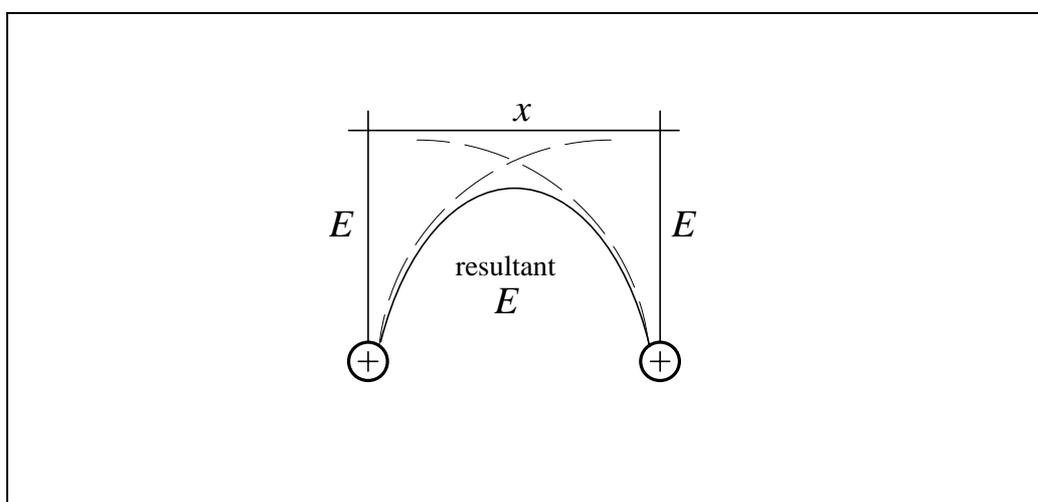
Bound electron energy graphed as a function of position from the nucleus

Figure 2A.3

Let's suppose an electron is at position  $A$  with a certain energy  $E_A$ . Is it in a stable orbit, or is it free? We know where it would be if it were in a stable orbit (right on the line of the graph), and we know that kinetic energy is positive. Therefore, if it lies above the graph, then it is free to move about. It can only move as far as point  $P$ , where all its energy is used in orbiting a nucleus. *Can a stable position exist below the graph?*

The graph shows an energy barrier for electrons - we can determine if an electron is free or bound from it

In the presence of two nuclei, the energy of a bound electron looks like:



The energy barrier applies to more than one nucleus

Figure 2A.4

## 2A.6

The energy barrier is used as a model of matter

If we consider the edge of a material, the energy for a bound electron has the form of:

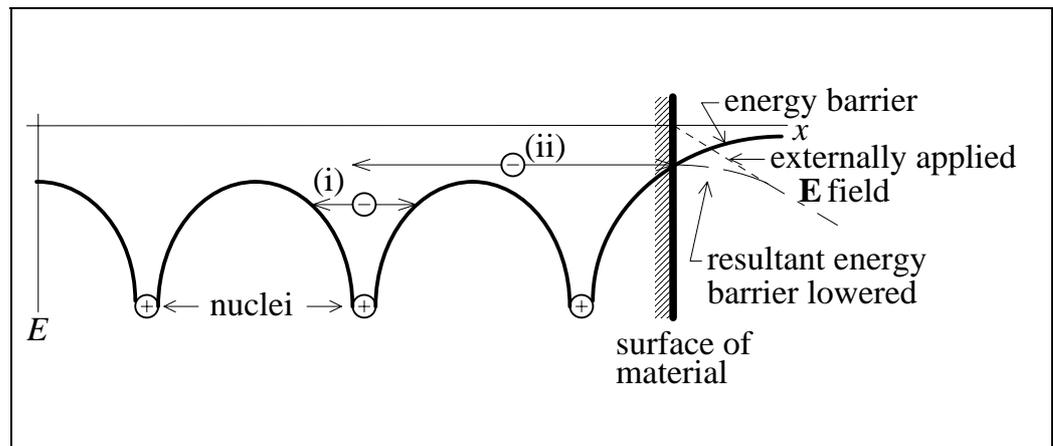


Figure 2A.5

This is our model for the inside of a metal. On a microscopic scale, the potential varies greatly near a nucleus. Electrons can be one of two types, depending on their energy:

Bound electrons are trapped by the energy barrier

(i) *Bound charge* – lower energy electron, trapped in a “potential well”. It cannot leave the parent atom (or atoms).

Free electrons have more energy which is used to overcome the energy barrier

(ii) *Free charge* – higher energy electron, above the top of the potential wells. They are free to move in the crystal lattice. They contribute to the current when an external voltage is applied.

## Escape from a Surface

If a “free electron” crosses the surface of the material, it comes up against an energy barrier. It “falls” back into the metal, unless the “escape energy” is greater than the “barrier energy”. An energy barrier exists at the surface of a material

An electron can escape from the surface in a number of ways:

The surface energy barrier can be overcome in a number of ways

- (i) We can lower the barrier energy by using an externally applied  $\mathbf{E}$  field. (Called the Schottky effect, or high field emission).
- (ii) We can increase the escape energy by heating the material. The increased thermal agitation means that some electrons will have enough energy to overcome the barrier.
- (iii) Photons can also free electrons (discovered by Einstein) – called the photoelectric effect.

# 2A.8

## Conduction

Free electron motion is normally random

The free electrons in a metal drift randomly in all directions and constitute thermal motion. The average drift velocity of all electrons is zero.

Free electrons move in response to an applied field

An applied voltage across a conductor gives rise to an  $E$  field within the conductor. After electrons have been accelerated by the field, they collide with the lattice, resulting in energy transfer to the lattice in the form of heat. This gives rise to an average drift velocity within the conductor, based upon Newton's second law and the concept of relaxation time (the mean free time between collisions). From this, Ohm's law can be derived.

Free electrons colliding with the lattice produce heat

Alternately, we could set up an experiment to measure the resistance of a specimen:

Ohm's Law can be obtained experimentally

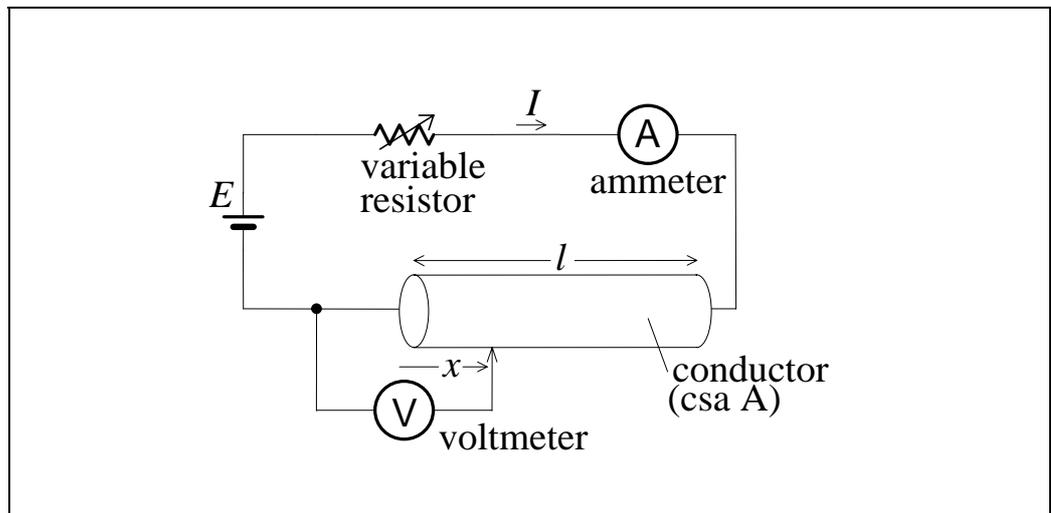


Figure 2A.6

If the applied field is varied (by changing the variable resistor), the voltage  $V_x$  along the conductor varies smoothly – no violent changes are observed. A linear  $V_x - I$  relationship results, with slope  $R = V_x/I$  being the conductor resistance.

Resistivity  $\rho$  is obtained from:

Resistivity defined

$$R = \frac{\rho l}{A}$$

(2A.7)

The variation of  $R$  with temperature can be investigated by placing a specimen in a calorimeter and repeating the above experiment at various temperatures:

Resistance varies with temperature

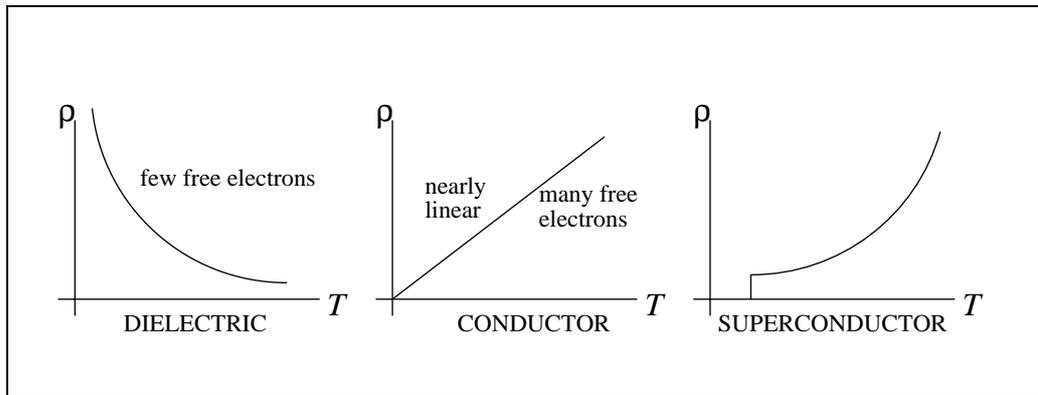


Figure 2A.7

For dielectrics (insulators), the resistivity decreases with increasing temperature because electrons are being freed (remember potential wells). For metallic conductors, the increase in resistivity is due to larger thermal vibrations of the lattice atoms. The density of conduction electrons is essentially independent of temperature. and varies with the type of material

For metallic conductors, the relationship between resistivity and temperature is nearly linear, so we can express resistivity as a function of temperature easily:

$$\rho = \rho_0 [1 + \alpha(T - T_0)]$$

(2A.8) For metal, resistivity varies approximately linearly

where we have defined the temperature coefficient:

$$\alpha = \frac{1}{\rho} \frac{\delta\rho}{\delta T}$$

(2A.9) Temperature coefficient defined

# 2A.10

## Dielectrics

Dielectrics defined -  
insulators

Dielectrics are materials that consist of atoms with their outermost electron shell almost completely filled. It is therefore relatively difficult to dislodge an electron from the shell – it is a bound electron. Dielectrics therefore have few electrons available for conduction and are classified as insulators.

## The Electric Dipole

Electric dipole  
defined

A dipole consists of equal positive and negative charges held a distance apart. For example, the displacement of the electron cloud from the nuclei in an atom (due to an applied field) is a dipole. Another example is polar molecules which have the centres of positive and negative charge permanently displaced due to chemical bonding.

and modelled

The electric dipole can be represented by:

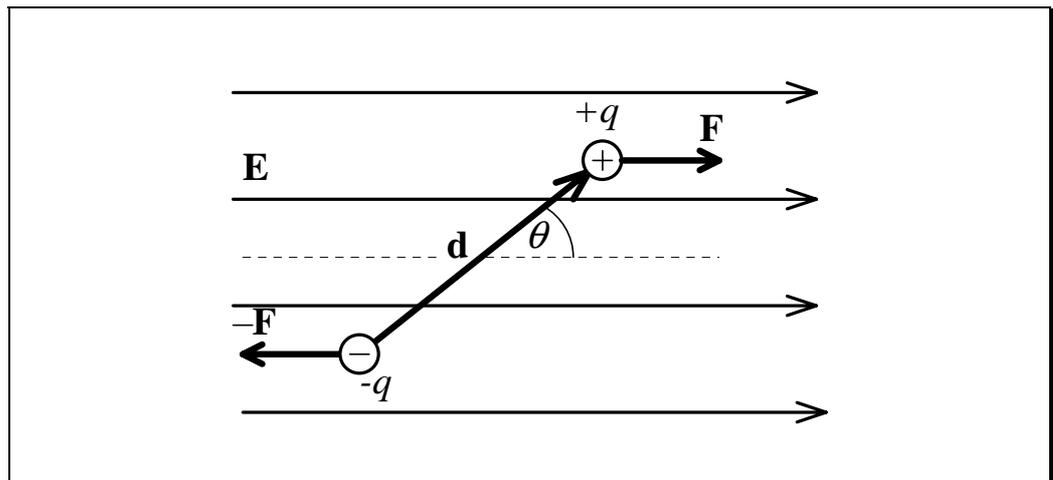


Figure 2A.8

A dipole tends to align with the applied  $\mathbf{E}$  field. Coulomb's Law gives:

$$\mathbf{F} = q\mathbf{E} \quad (2A.10)$$

Therefore, the torque experienced by a dipole when it rotates due to an applied field is:

$$T = 2 \frac{d}{2} F \sin \theta = qdE \sin \theta \quad (2A.11)$$

We define the electric dipole moment:

$$\mathbf{p} = q\mathbf{d}$$

(2A.12) Electric dipole moment defined

so that the torque on the dipole is given by:

$$\mathbf{T} = \mathbf{p} \times \mathbf{E}$$

(2A.13) The torque experienced by an electric dipole in an electric field

The  $\mathbf{d}$  (and therefore the  $\mathbf{p}$ ) points from negative to positive charge.

A dipole experiences no net force in a uniform field. It only experiences rotation.

## Effect of Boundaries

When an electric field goes from one material to another, what happens? Changes in material affect an electric field

Firstly consider what happens to a perpendicular  $\mathbf{D}$  field:

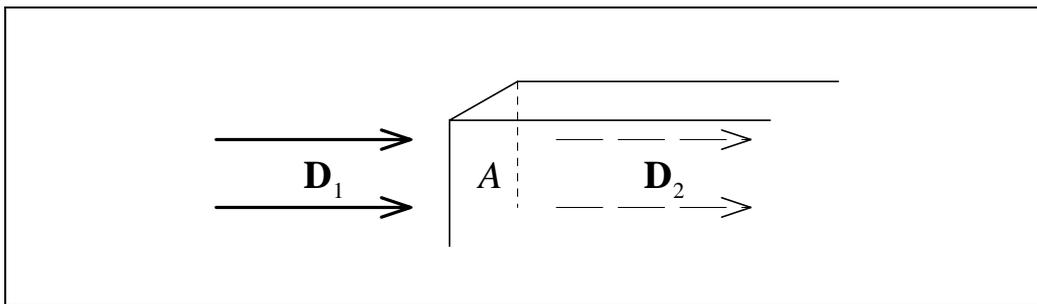


Figure 2A.9

There are no *free* sources of flux, since the material has no net charge. The flux outside and inside the material should be the same.

## 2A.12

We can then say:

A perpendicular  $\mathbf{D}$  field is not affected by a boundary

$$\begin{aligned} \psi_1 &= \psi_2 \\ D_1 A &= D_2 A \\ D_1 &= D_2 \end{aligned} \tag{2A.14}$$

Now consider what happens to a tangential  $\mathbf{E}$  field:

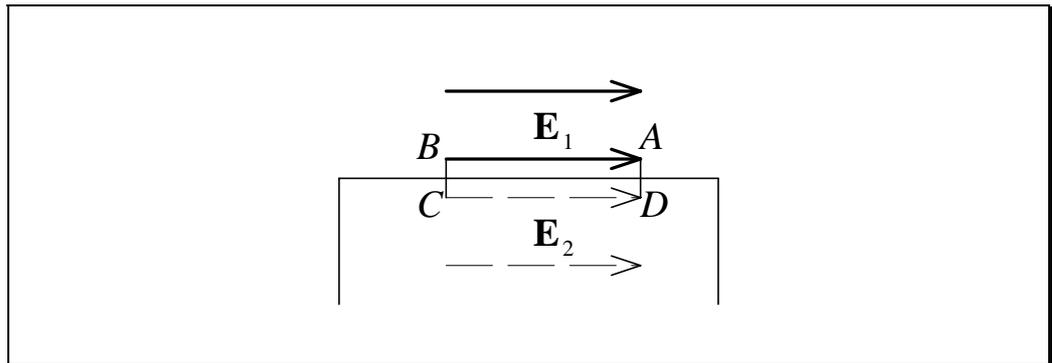


Figure 2A.10

The  $\mathbf{E}$  field is conservative, so that:

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = 0 \tag{2A.15}$$

Performing this integral around the path  $ABCD$  gives (the distances  $BC$  and  $AD$  are assumed to be very small):

$$\begin{aligned} \int_A^B \mathbf{E}_1 \cdot d\mathbf{l} + \int_C^D \mathbf{E}_2 \cdot d\mathbf{l} &= \\ \int_A^B E_1 dl - \int_C^D E_2 dl &= 0 \end{aligned} \tag{2A.16}$$

The integrals are easy to perform since the electric field is constant over the small distance  $AB$ . The result is:

A tangential  $\mathbf{E}$  field is not affected by a boundary

$$\begin{aligned} E_1 l - E_2 l &= 0 \\ E_1 &= E_2 \end{aligned} \tag{2A.17}$$

In general, a boundary doesn't affect the perpendicular component of  $\mathbf{D}$  and the tangential component of  $\mathbf{E}$ .

We can now consider what happens when an electric field enters another material at an angle: The effect of a boundary when the field is at an angle

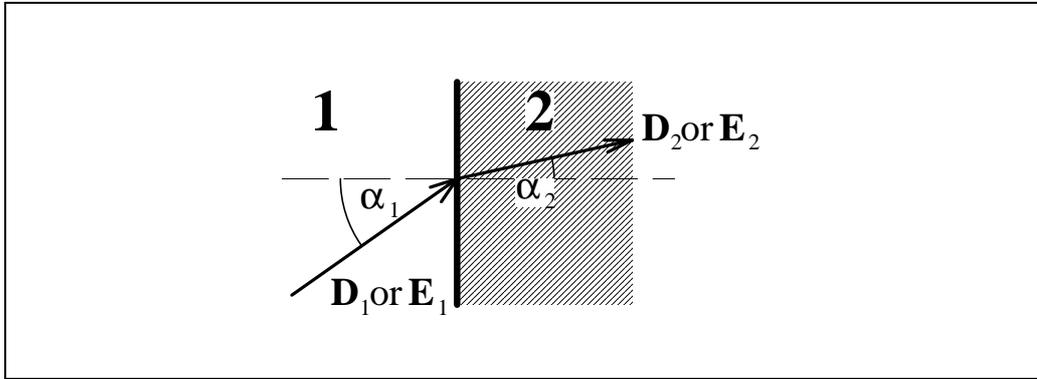


Figure 2A.11

We know from the previous results that:

$$E_1 \sin \alpha_1 = E_2 \sin \alpha_2 \quad (2A.18a)$$

$$D_1 \cos \alpha_1 = D_2 \cos \alpha_2 \quad (2A.18b)$$

$$D = \epsilon_r \epsilon_0 E$$

which leads to:

$$\frac{\epsilon_{r1}}{\epsilon_{r2}} = \frac{\tan \alpha_1}{\tan \alpha_2}$$

(2A.19) The direction and magnitude of the field changes

It is interesting to note that we can apply the same reasoning to current in a conductor. In that case we can simply replace  $D$  and  $\epsilon$  by  $J$  and  $\sigma$ .

# 2A.14

## Polarisation

A dielectric changes the permittivity

Why does the permittivity vary from that of free space for dielectrics?

Consider first a parallel-plated capacitor with an air dielectric:

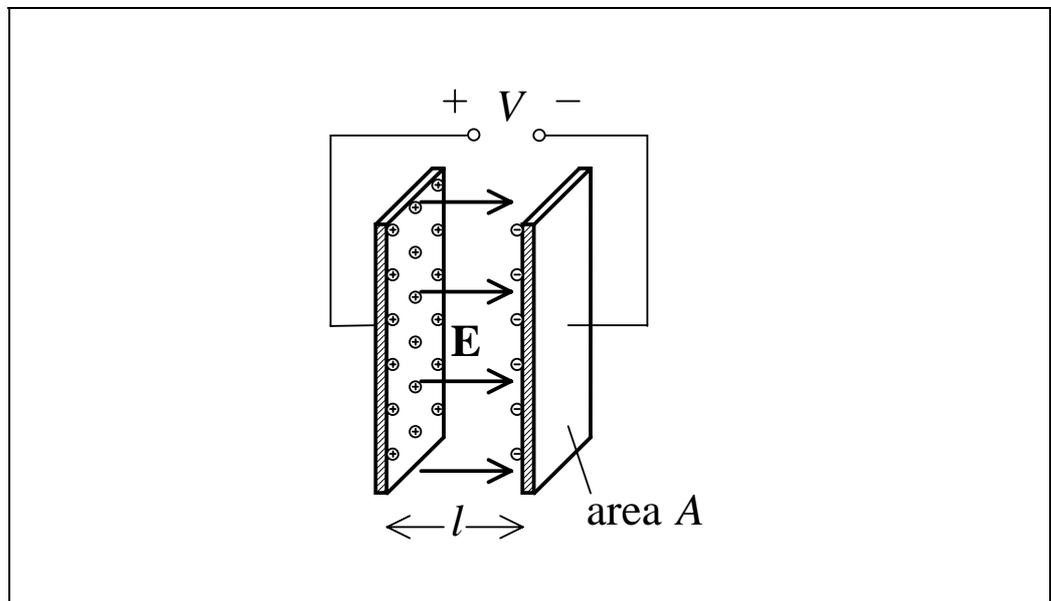


Figure 2A.12

The applied potential  $V$  forces positive and negative charges to flow to the metal plates till  $-\int_l \mathbf{E} \cdot d\mathbf{l} = V$ . As the field is uniform (the lines of force are parallel to each other), inside the capacitor  $\mathbf{E}$  and  $\mathbf{D}$  are constant. (This ignores fringing – a valid assumption for a small distance  $l$ ).

Therefore, by performing the above integral, we get:

The fields in air for a parallel plated capacitor

$$\begin{aligned} E_0 &= V/l \\ D_0 &= \epsilon_0 E_0 \end{aligned} \tag{2A.20}$$

This is valid only in between the capacitor plates.

We define the capacitance of the structure as:

Capacitance defined

$$C = \frac{\text{electric flux}}{\text{electric potential}} = \frac{\Psi}{V} = \frac{D_0 A}{E_0 l} = \frac{\epsilon_0 A}{l} \tag{2A.21}$$

What happens when we insert a slab of dielectric material between the metal plates? The molecules in the dielectric become polarised, and the electric dipoles will align in the direction of  $\mathbf{E}$ :

The effect of a dielectric in a parallel plated capacitor

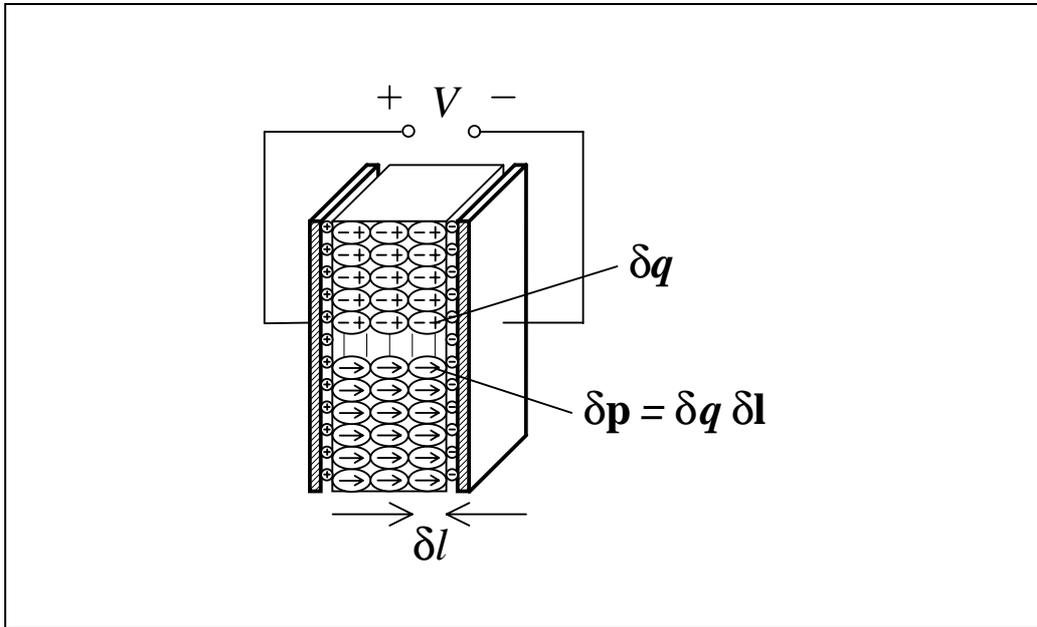


Figure 2A.13

In the material the charges of the dipoles will cancel, leaving (in effect) only the surface charges. The polarised slab is therefore equivalent to positive and negative surface charges which give rise to an internal field  $\mathbf{E}_i$ . The direction of this induced  $\mathbf{E}$  field opposes the applied field.

A polarised dielectric can be modelled using the surface charges

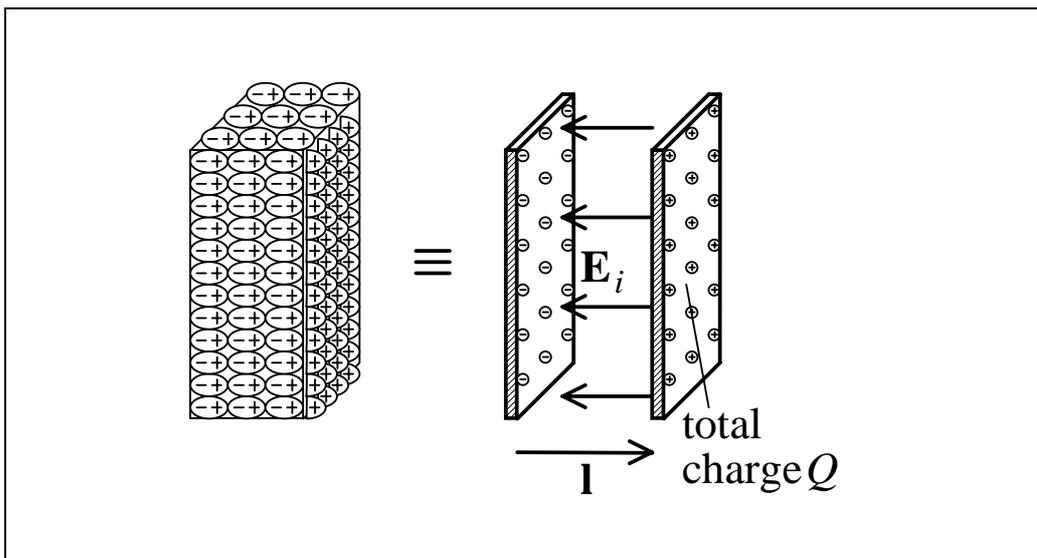


Figure 2A.14

## 2A.16

The surface charges look like one big dipole:

$$\mathbf{p} = Q\mathbf{l} \quad (2A.22)$$

where  $Q$  is the total charge on one surface,  $l$  is the distance between them.

We define polarisation as the dipole moment per unit volume:

Polarisation defined

$$\mathbf{P} = \frac{\mathbf{p}}{V} \quad (2A.23)$$

and in this case we have:

and applied to a dielectric in a parallel plate capacitor

$$\mathbf{P} = \frac{\mathbf{p}}{V} = \frac{Q\mathbf{l}}{Al} = \frac{Q}{A}\hat{\mathbf{l}} \quad (2A.24)$$

The induced electric flux density between the two charge layers is:

Polarisation gives rise to an induced field

$$\mathbf{D}_i = -\frac{Q}{A}\hat{\mathbf{l}} = -\mathbf{P} \quad (2A.25)$$

The applied voltage is constant which means the magnitude of the electric field inside the dielectric is fixed by the voltage source (see Eq. (2A.20)). To keep the electric field the same, more charges must be deposited onto the metal plates by the voltage, to compensate for the polarisation effect. The resultant magnitude of the flux density inside the slab is given by:

The relationship between  $\mathbf{D}$  and  $\mathbf{E}$  for a dielectric

$$\begin{aligned} D_d &= D_0 - D_i \\ &= \epsilon_0 E_0 + P \\ &= \epsilon_d E_0 \end{aligned} \quad (2A.26)$$

We define the dielectric permittivity:

$$\epsilon_d = \epsilon_0 + \frac{P}{E_0}$$

(2A.27) Permittivity defined in terms of polarisation

and relative permittivity:

$$\epsilon_r = \frac{\epsilon_d}{\epsilon_0}$$

(2A.28) Relative permittivity defined (again)

If the applied **E** reverses periodically, **P** tries to keep up with it – the dipoles “flip”. At very high frequencies,  $\epsilon$  decreases as the dipoles are unable to follow the applied **E**.

Frequency affects polarisation

# 2A.18

## Ferroelectrics

Ferroelectrics have domains of permanent polarisation

Some materials exhibit permanently polarised regions (called domains):

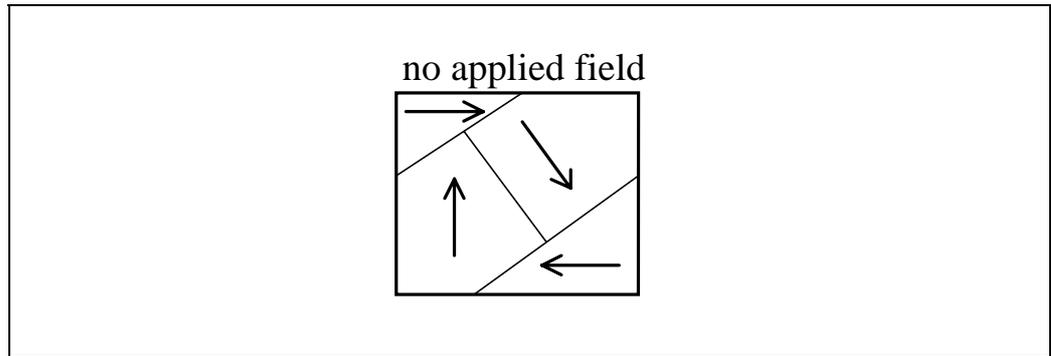


Figure 2A.15

An external field causes the domains to grow

An externally applied  $\mathbf{E}$  field causes the growth of domains in its direction:

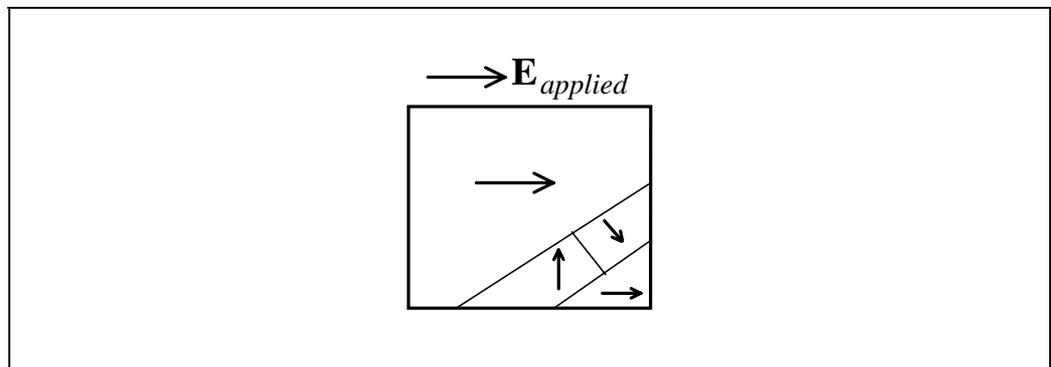


Figure 2A.16

A first look at hysteresis

The process is not reversible (hysteresis):

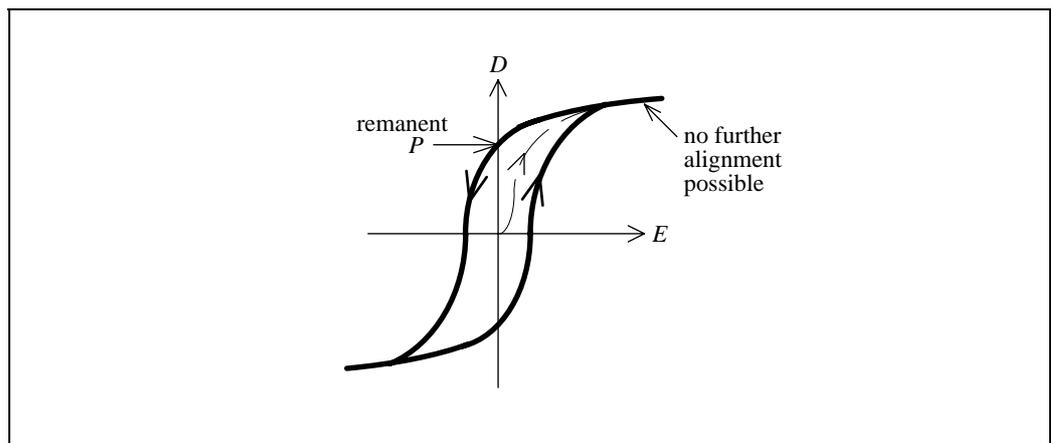


Figure 2A.17

When the applied  $\mathbf{E}$  field is reduced to zero, a remanent  $\mathbf{P}$  remains (we then have an electret – similar to a magnet).

## Breakdown at Sharp Points

Charges tend to accumulate at sharp points on a conducting body. The charge tries to spread itself out as much as possible over the surface of the conductor. A sharp point is a long way from most of the surface.

Sharp points accumulate free charge

Consider two conducting spheres connected by a perfect conductor (carrying no surface charge) and *assume that all the surface charge on the metal spheres is concentrated at their centre*. Also, the spheres are sufficiently far apart so that *proximity effects are negligible*.

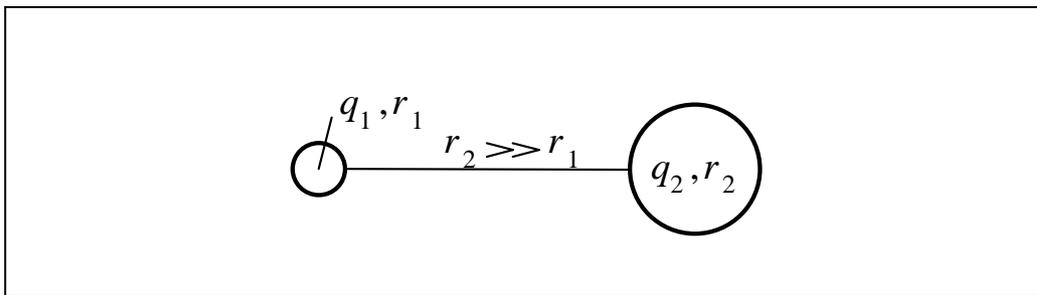


Figure 2A.18

The electric potential at the surface of sphere 1 is:

$$V_1 = -\int_{\infty}^{r_1} \mathbf{E}_1 \cdot d\mathbf{l} = -\int_{\infty}^{r_1} \frac{q_1}{4\pi\epsilon_0 r^2} dr = \frac{q_1}{4\pi\epsilon_0 r_1} \quad (2A.29)$$

Similarly:

$$V_2 = \frac{q_2}{4\pi\epsilon_0 r_2} \quad (2A.30)$$

As the surface of a conductor is an equipotential surface (*Why?*), all surfaces are at the same potential and  $V_1 = V_2$ . Then:

$$\frac{q_1}{r_1} = \frac{q_2}{r_2} \quad \text{or} \quad \frac{4\pi r_1^2 D_1}{r_1} = \frac{4\pi r_2^2 D_2}{r_2} \quad (2A.31)$$

## 2A.20

and since  $D = \epsilon_0 E$ , this becomes:

$$r_1 E_1 = r_2 E_2 \quad \text{or} \quad \frac{E_1}{E_2} = \frac{r_2}{r_1} \quad (2A.32)$$

Therefore, at the surface of the spheres:

Sharp points have large fields around them

$$E_1 \gg E_2 \quad \text{and} \quad D_1 \gg D_2 \quad (2A.33)$$

This analysis would apply to shapes such as:

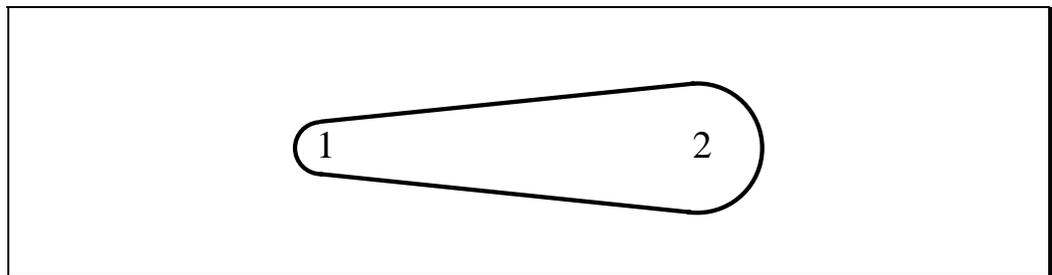


Figure 2A.19

In air (which is an insulator), molecules in the immediate vicinity of sharp points on the surface of a conductor will be under a high electric stress (a strong  $\mathbf{E}$  field of high intensity as well as a high electric potential). Free charges (electrons and ions) are accelerated and acquire enough kinetic energy to displace bound electrons by collision with neutral air molecules. This results in large numbers of free charges. The surrounding air becomes conducting and the metal surface discharges (i.e. the *insulation breaks down*). This is the principle behind the lightning rod.

The mechanism of insulator breakdown for air

The insulation breakdown may be in the form of:

Low current breakdown in the form of a corona

(i) *Corona discharge* – low current, high potential. Hissing noise and glow results (energy emitted due to collisions).

High current breakdown in the form of an arc

(ii) *Arc discharge* – complete breakdown in the air path between two charged objects kept at different potentials. A large current results. The potential is very low.

### Example – Parallel plate capacitor

It is useful to examine the various field quantities as a function of position inside a capacitor. Firstly consider what happens when a dielectric is introduced into a parallel plate capacitor when the voltage across the plates is kept constant:

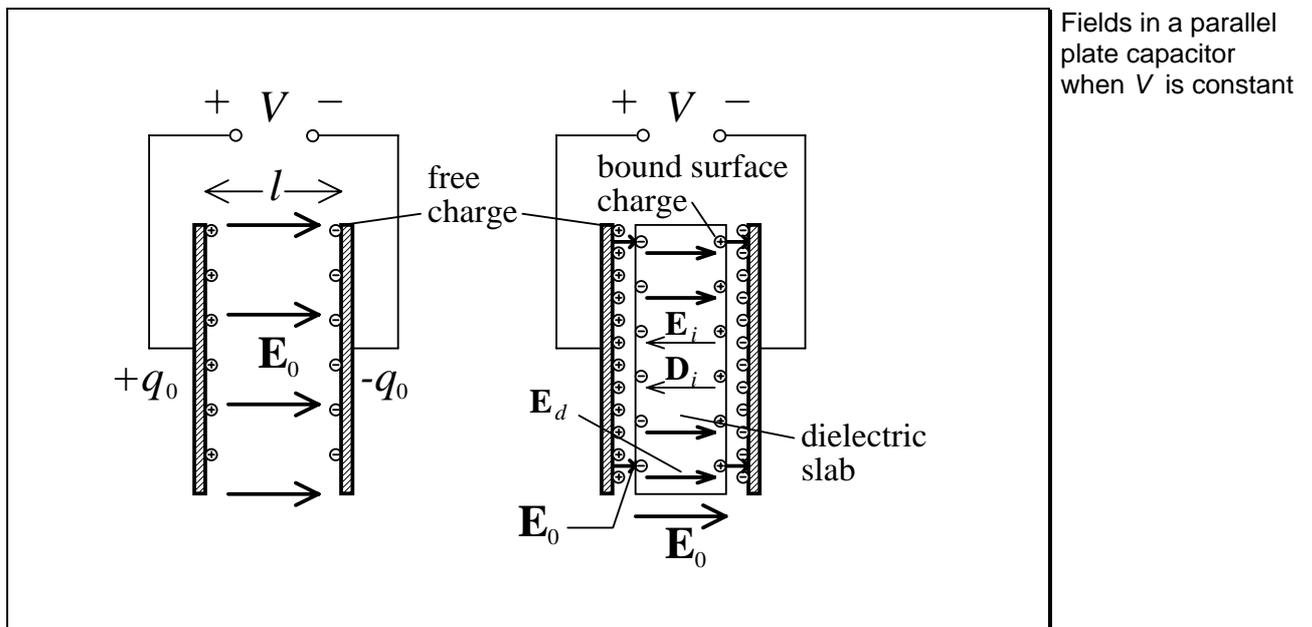


Figure 2A.20

With the dielectric inserted the system is equivalent to:

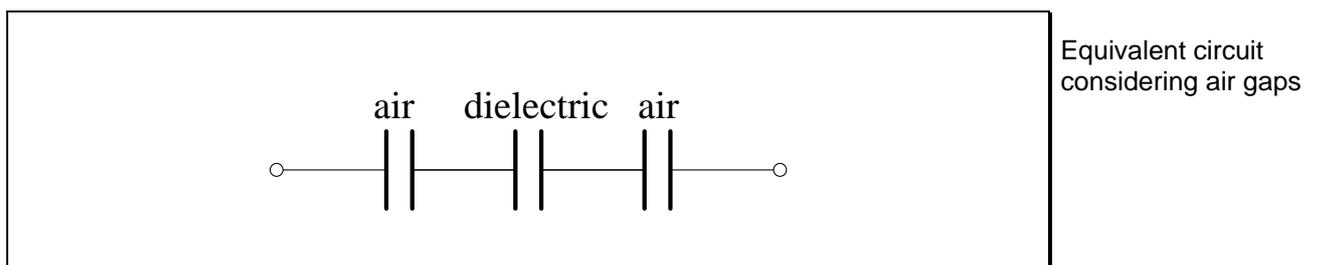


Figure 2A.21

The applied voltage  $V$  is held constant. Inside the dielectric, the “bound charge” rearranges itself for a short time as polarisation  $\mathbf{P}$  takes place.

## 2A.22

Inside the dielectric, the total electric field magnitude is:

$$E_d = E_0 - E_i \quad (P = -\epsilon_0 E_i) \quad (2A.34)$$

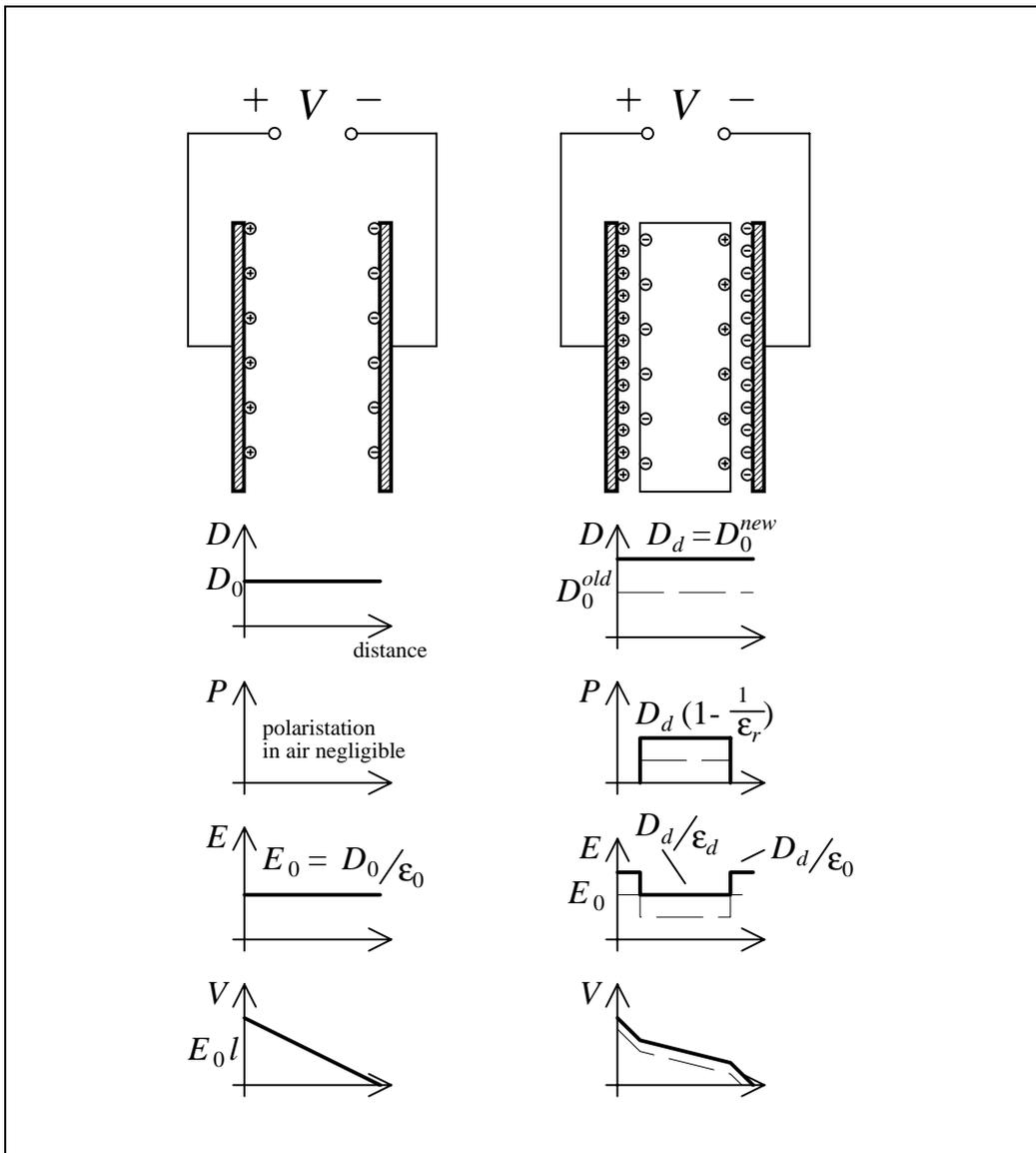
If the dielectric is isotropic (same in all directions), then:

$$D_d = D_0^{new} = \epsilon_0 E_0 + P = \epsilon_d E_0 \quad (2A.35)$$

Permittivity accounts for all polarisation effects

The permittivity  $\epsilon_d$ , which can be determined experimentally, accounts for all polarisation effects.

As  $V$  is constant, when the dielectric is introduced additional free charge is induced on the capacitor plates to balance the polarisation equivalent surface charge.



**Figure 2A.22**

The other possible way of introducing the dielectric is to keep  $Q$  constant instead of  $V$  (--- in Figure 2A.22). After charging the capacitor,  $V$  is removed so there is no way  $Q$  can change. When the dielectric is introduced,  $Q = CV$  applies and the potential decreases (also  $E_d < E_0$ ).

If  $Q$  is constant, the fields are different

# 2A.24

## Example – Air cavities in dielectrics and partial discharges

Consider a spherical air cavity inside a dielectric:

Field caused by an air cavity in a dielectric

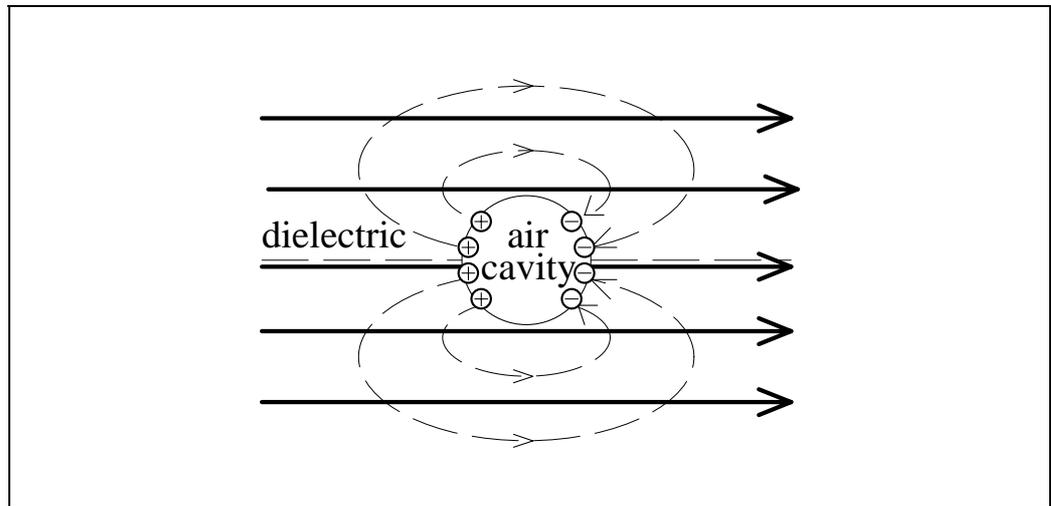


Figure 2A.23

The polarisation in air is negligible. The net charge at the boundary (caused by the dielectric's dipoles) acts as a spherical dipole with a field opposing the main field ( $\epsilon_d > \epsilon_0$ ) in the dielectric. Therefore:

$$D_d > D_0, \quad \epsilon_d E_d > \epsilon_0 E_0 \quad (2A.36)$$

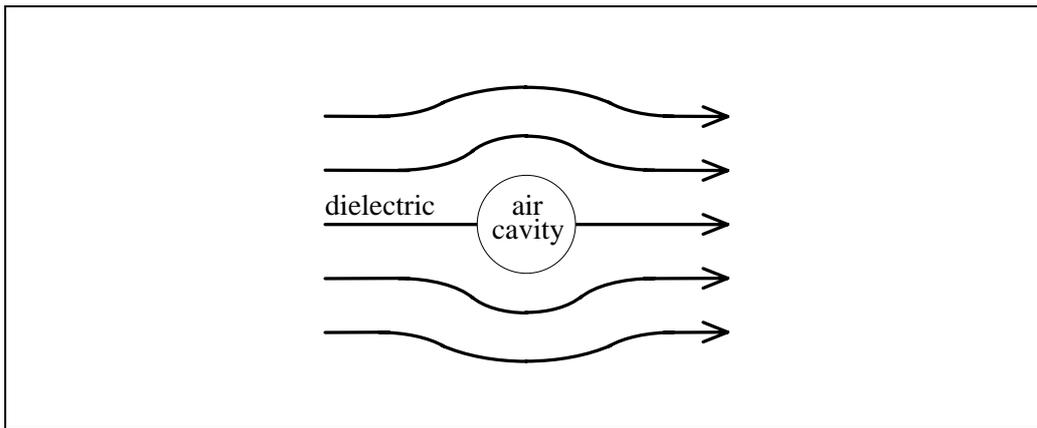
Also, charge concentration on the surface of the air cavity is large. Therefore:

The  $E$  field is larger in the air cavity than in the dielectric

$$E_0 > E_d \quad (2A.37)$$

and breakdown in the air may occur. (N.B. For a plane boundary  $\epsilon_d E_d = \epsilon_0 E_0$ , and since  $\epsilon_d \gg \epsilon_0$ , then  $E_0 \gg E_d$ ).

The resultant field looks like:

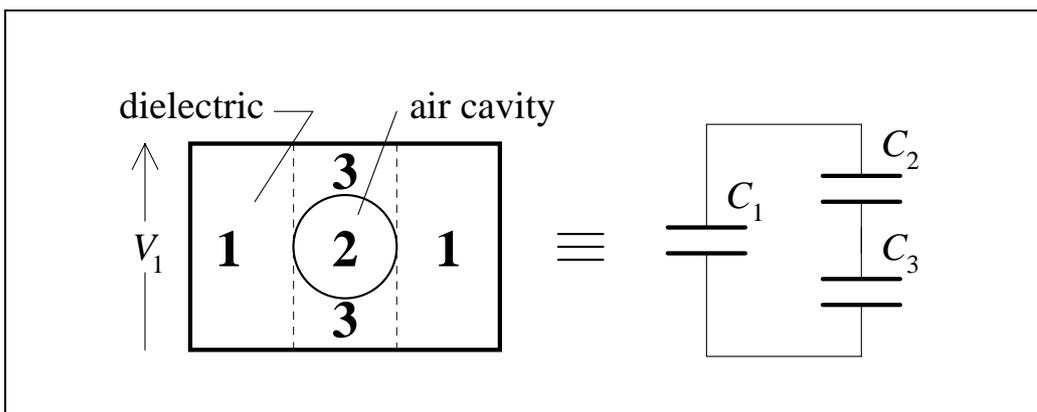


The resultant **E** field of a dielectric caused by an air cavity

**Figure 2A.24**

Draw the **E** and **D** fields in the air cavity.

The electrical equivalent circuit of an air cavity in a dielectric is shown below:



Electrical equivalent circuit of the system

**Figure 2A.25**

The electrical equivalent circuit gives:

$$V_1 = V_2 \frac{C_2 + C_3}{C_3} \quad (2A.38)$$

Internal voltages can be estimated by the external voltages

and we can estimate the internal discharge voltages from changes in  $V_1$ .

# 2A.26

A dielectric in air

If the dielectric sphere has a larger permittivity than the surrounding medium, then the fields are different:

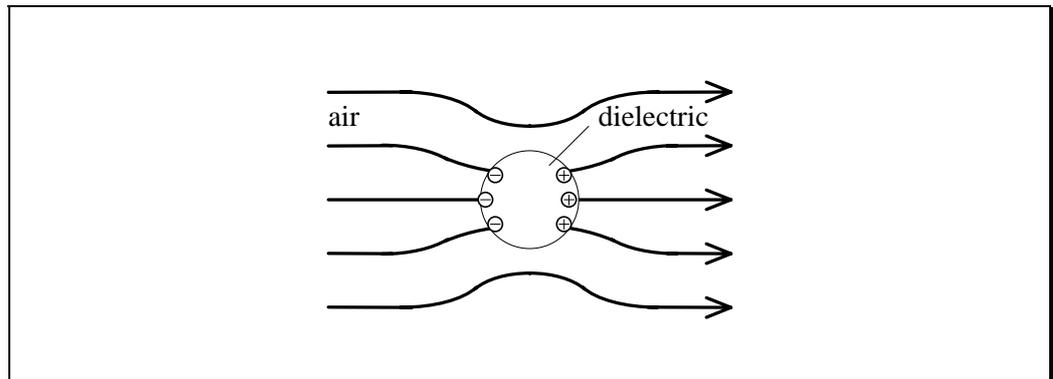


Figure 2A.26

Draw the fields in the dielectric.

Here, we have:

The  $\mathbf{E}$  field in the dielectric is smaller than in the air

$$D_0 < D_d, \quad E_0 > E_d \quad (2A.39)$$

How to measure  $\mathbf{E}$  and  $\mathbf{D}$  in a dielectric

To measure  $\mathbf{D}$  and  $\mathbf{E}$  inside a dielectric, we can use a probe that measures  $\delta V = E\delta l$ . To measure  $\mathbf{D}$  we use a disc shaped cavity, to measure  $\mathbf{E}$  we use a needle shaped cavity:

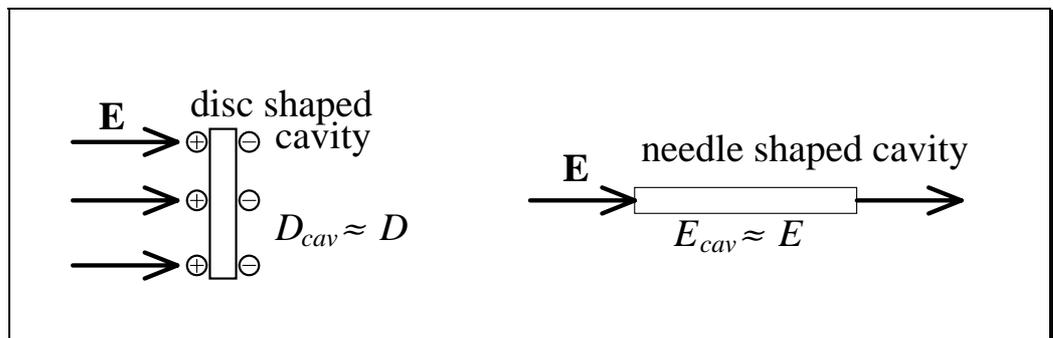


Figure 2A.27

### Example – Variable permittivity capacitor

A capacitor has a dielectric with an  $\epsilon_r$  that varies linearly with distance. Determine the capacitance per unit area. A capacitor with varying permittivity is modelled

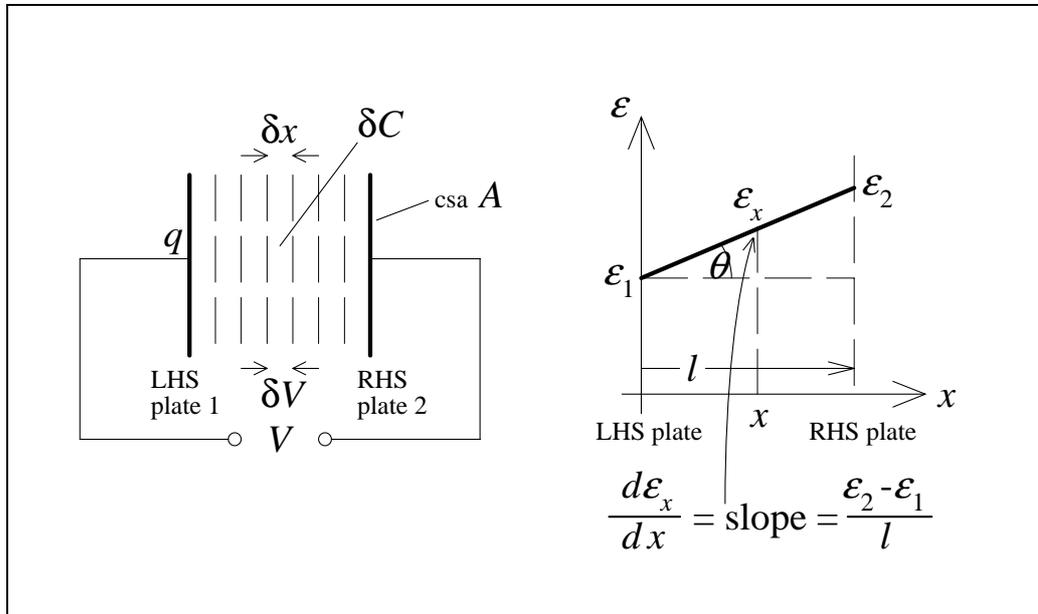


Figure 2A.28

We have:

$$V = \sum \delta V = q \sum \frac{1}{\delta C} \quad \text{where} \quad \delta C = \frac{\epsilon_x A}{\delta x} \quad (2A.40) \quad \text{as lots of capacitors in series}$$

Therefore:

$$\begin{aligned} V &= q \int_1^2 \frac{dx}{\epsilon_x A} = \frac{q}{A} \int_1^2 \frac{l}{\epsilon_2 - \epsilon_1} \frac{d\epsilon_x}{\epsilon_x} \\ &= \frac{ql}{A(\epsilon_2 - \epsilon_1)} [\ln \epsilon_x]_{\epsilon_1}^{\epsilon_2} = \frac{ql}{A(\epsilon_2 - \epsilon_1)} \ln \frac{\epsilon_2}{\epsilon_1} \end{aligned} \quad (2A.41)$$

Finally, we get the capacitance per unit area:

$$\frac{C}{A} = \frac{\epsilon_2 - \epsilon_1}{l \ln(\epsilon_2/\epsilon_1)} \text{ Fm}^{-2} \quad (2A.42)$$

# 2A.28

## Example – Electrostatic generator (Van de Graaff generator)

Consider a Van de Graaff generator, with a dome radius  $r = \frac{1}{3}$  m, and an equivalent current source  $I = 1 \mu\text{A}$  :

The Van de Graaff generator produces a large static voltage

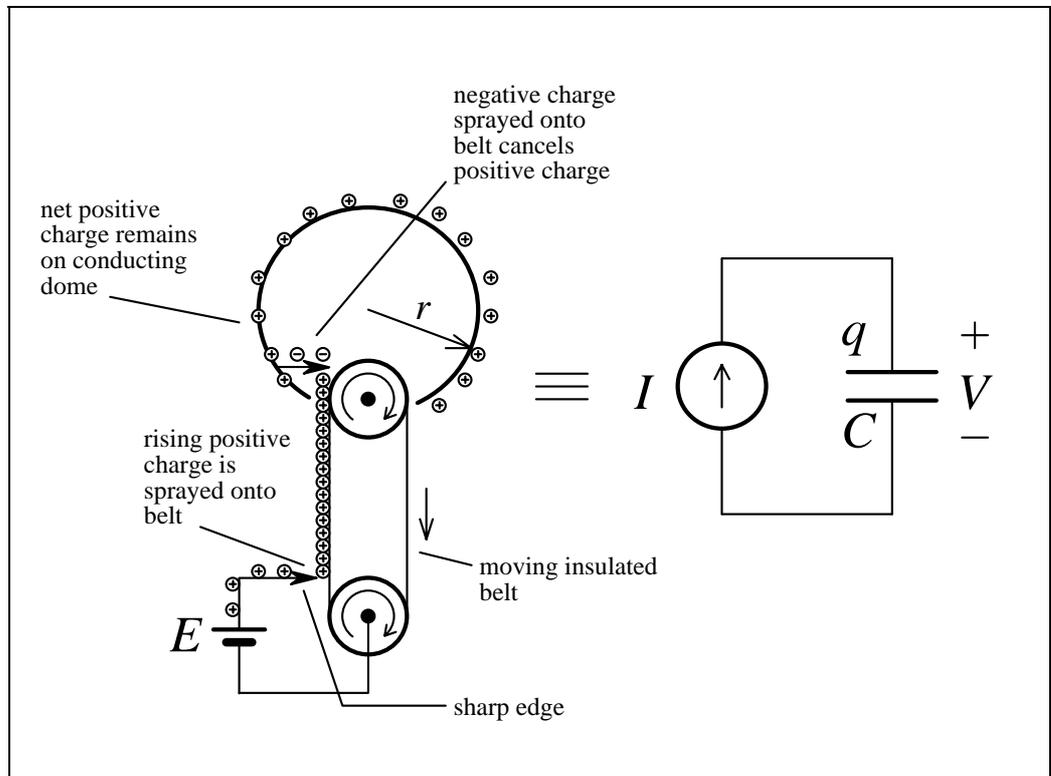


Figure 2A.29

If we approximate the dome by a sphere, then:

$$V = -\int_{\infty}^r \mathbf{E} \cdot d\mathbf{r} = -\int_{\infty}^r \frac{q}{4\pi\epsilon_0 r^2} dr = \frac{q}{4\pi\epsilon_0 r} \quad (2A.43)$$

If  $E_B$  = electric field breakdown strength ( $\approx 3 \times 10^6 \text{ Vm}^{-1}$  for air, plane-plane) then:

(i) maximum dome voltage is:  $V_{\max} = E_B r = 3 \times 10^6 \times \frac{1}{3} = 10^6 \text{ V}$

(ii) dome capacitance is:  $C = \frac{q}{V} = 4\pi\epsilon_0 r = 4\pi \times 8.85 \times 10^{-12} \times \frac{1}{3} \approx 37 \text{ pF}$

(iii) time to reach breakdown is:

$$i = C \frac{dv}{dt}$$

$$\therefore \text{change in voltage is } \delta V = \frac{I}{C} \delta t$$

$$t_B = \frac{V_{\max} C}{I} = \frac{10^6 \times 37 \times 10^{-12}}{1 \times 10^{-6}} = 37 \text{ s}$$

but eventually  
reaches breakdown

### Example – Lord Kelvin’s Water Dynamo

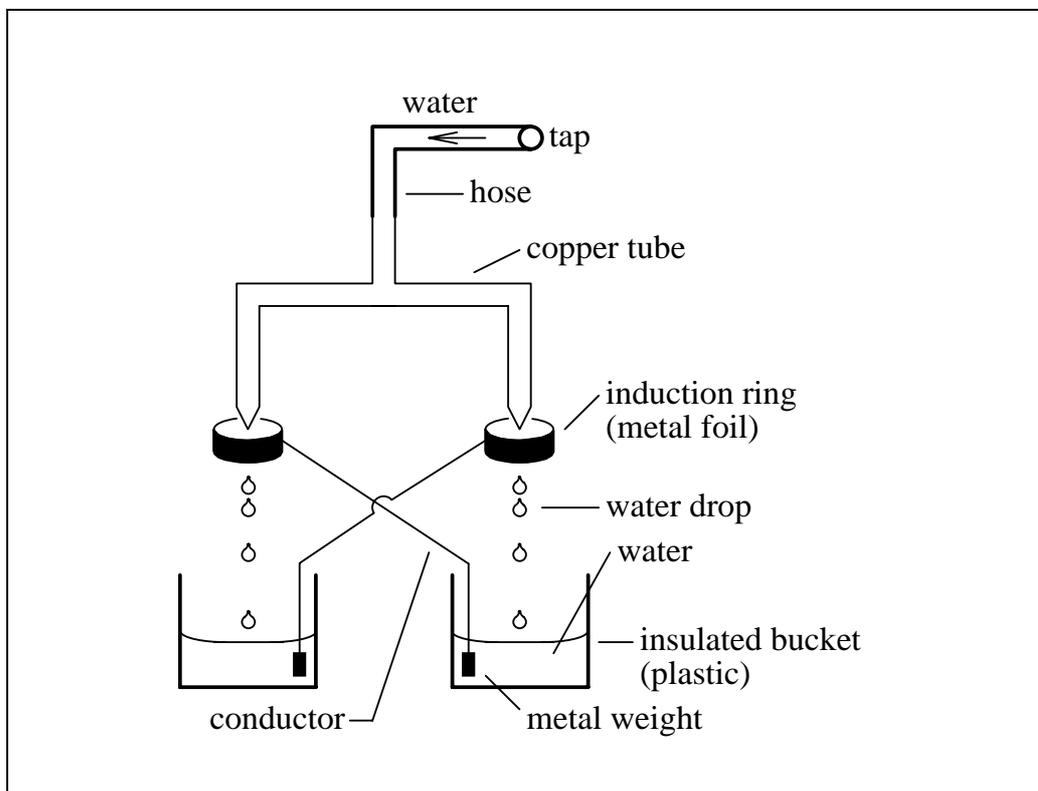


Figure 2A.30

# 2A.30

## Summary

- The Bohr model of the atom leads to the concept of the energy barrier, and of free and bound charges. This concept then leads to an understanding of conductors and insulators.
- Resistivity is an intrinsic property of a conductor, and it varies with temperature.
- An electric dipole consists of equal positive and negative charges held a distance apart. An electric dipole experiences no net force in a uniform field. It only experiences rotation.
- When an electric field goes from one material to another, a perpendicular  $\mathbf{D}$  field is not affected, nor is the tangential  $\mathbf{E}$  field.
- A dielectric placed in an electric field produces a polarisation, which acts to increase the relative permittivity – for a capacitor, the capacitance will increase due to the polarisation.
- Sharp points of metallic objects accumulate free charge and have large electric fields around them – which can lead to *insulation breakdown*.
- Air cavities in dielectrics can produce *partial discharge*.

## References

Plonus, Martin A.: *Applied Electromagnetics*, McGraw Hill Kogakusha, Ltd., Singapore, 1978.

Shadowitz, Albert: *The Electromagnetic Field*, Dover Publications, Inc., New York, 1975.