Lecture 7B – The Transformer

Magnetising branch. Voltage, flux and current waveforms. Phasor diagram. Losses and efficiency. Measurement of transformer parameters. Current and voltage excitation. 3rd harmonics.

Magnetising Branch

Our electrical equivalent circuit for a transformer derived previously was:

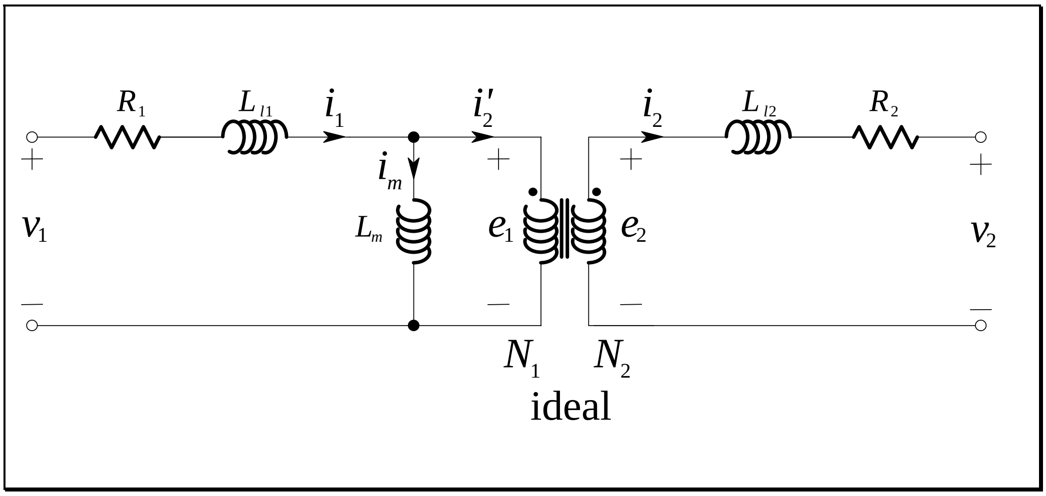


Figure 7B.1

This model assumed that the magnetizing inductance L_m was linear. If we do not put a load on the secondary of the transformer, then i_2 will be zero. From the mmf balance equation, this also means that i_2' will be zero – equivalent to an open circuit.

Therefore, on open circuit, our model reduces to that of an iron cored inductor:

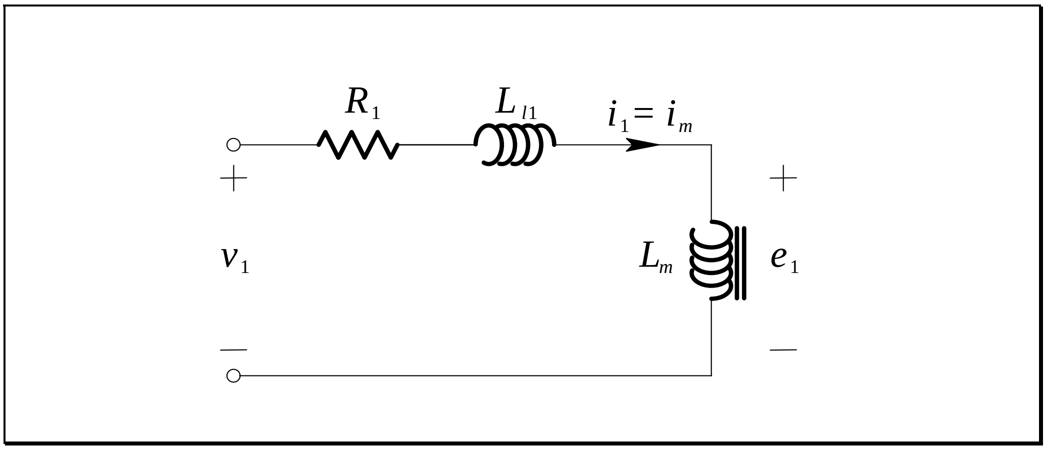


Figure 7B.2

We now want to take into account the *B*-*H* loop of the ferromagnetic core, instead of assuming that it is linear:

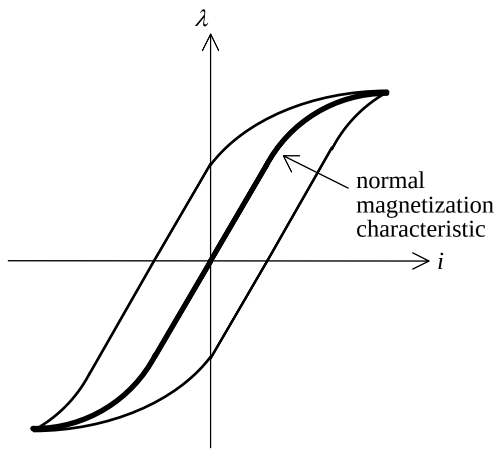


Figure 7B.3

We have seen that in traversing the *B*-*H* loop, we lose energy. To electrically model the *B*-*H* loop, we first reduce the area of the loop to zero, so that it has no losses. The *B*-*H* "loop" then reduces to the normal magnetization characteristic. Since the core is iron, eddy currents will be induced that will also contribute to the loss. To take into account all the losses, which we call *core losses*, we add a resistor in parallel to the magnetizing inductance. The total current, termed the *exciting current*, i_e, is therefore composed of a core loss component, i_c, and a magnetizing current, i_m:

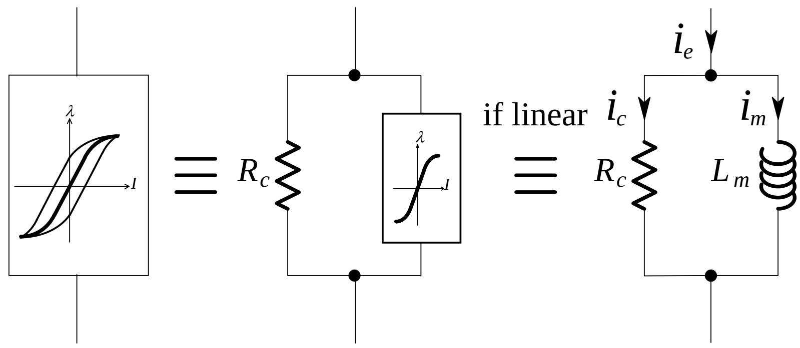


Figure 7B.4

If the transformer is operated over the linear region of the normal magnetization characteristic, then our equivalent circuit is all linear and represented by:

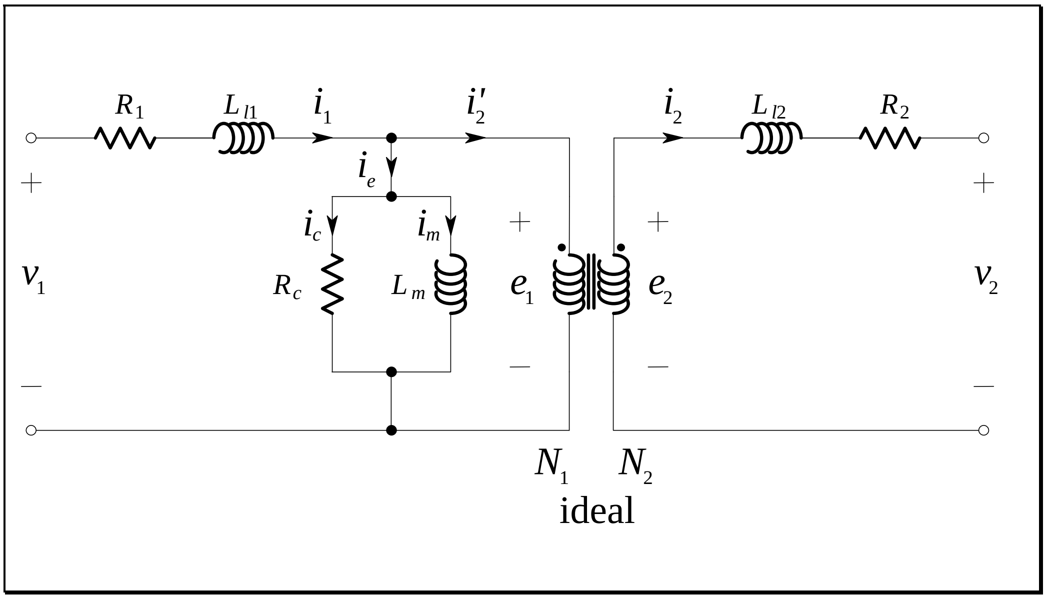


Figure 7B.5

Voltage, Flux and Current Waveforms

Assume that the winding resistance and leakage inductance are small enough to be ignored. Then the source appears directly across the ideal transformer and magnetizing branch. Also, ignore the core loss. If we assume AC sinusoidal excitation, then we can use impedances and phasors in our electrical *frequency-domain* model of the transformer:

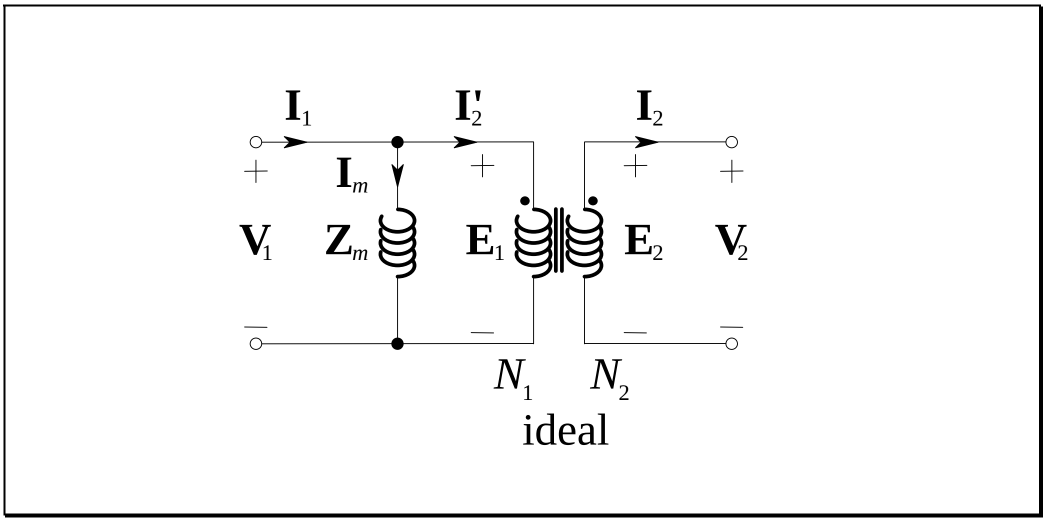


Figure 7B.6

The source is assumed to be sinusoidal. The flux in the core is given by Faraday's Law:

$$\begin{aligned}\phi &= \frac{1}{N}\int_0^t e_1 dt \\ &= \frac{1}{N}\int_0^t v_1 dt \\ &= \frac{1}{N}\int_0^t \hat{V}_1 \cos\omega t dt \\ &= \frac{\hat{V}_1}{N\omega}\sin\omega t\end{aligned} \tag{7B.1}$$

The flux therefore lags the voltage by 90º.

KCL at the input also gives:

$$\begin{aligned}i_1 = i_m + i_2' &= i_m + \frac{N_2}{N_1} i_2 \\ &= \hat{I}_m \sin\omega t + \frac{N_2}{N_1}\hat{I}_2 \cos(\omega t - \theta) \\ &= \hat{I}_1 \cos(\omega t - \beta)\end{aligned} \tag{7B.2}$$

The corresponding waveforms for the voltage, flux and current are:

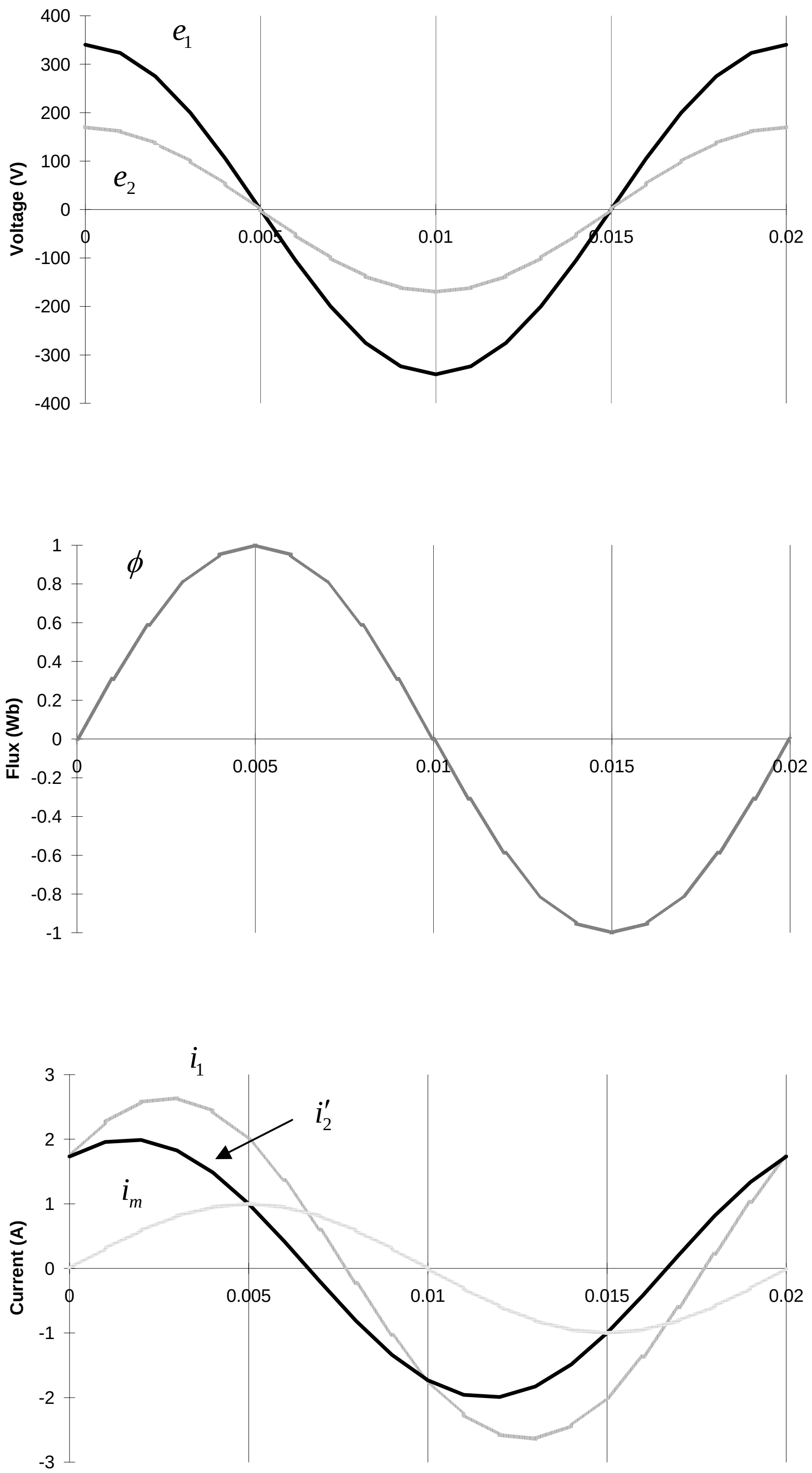


Figure 7B.7

Phasor Diagram

Under sinusoidal excitation in the linear region of operation, the equivalent circuit of the transformer with all quantities referred to the primary side can be represented with phasor voltages and currents and impedances:

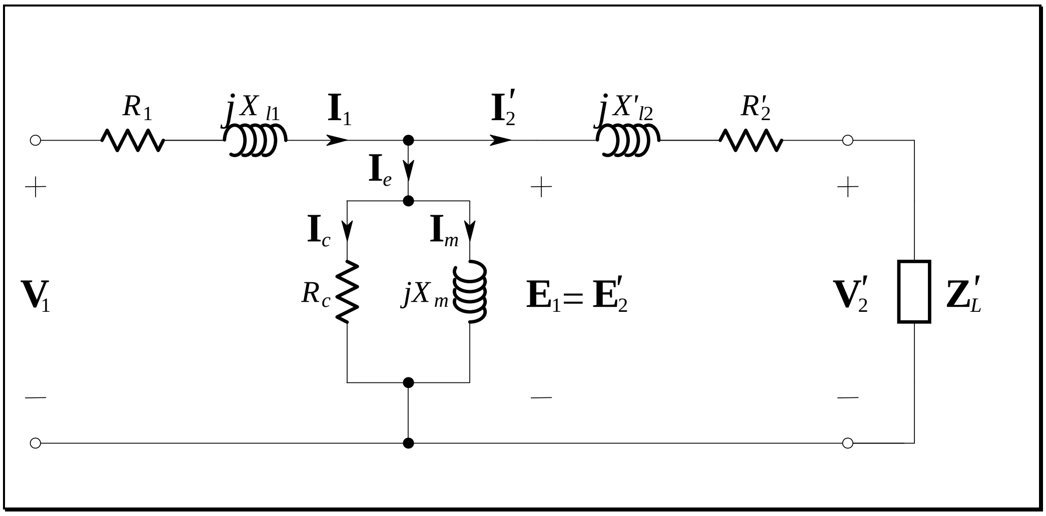


Figure 7B.8

Note that the variables and parameters on the secondary side of the transformer have been "referred" to the primary side of the transformer by using the relations derived earlier for the ideal transformer:

$$\mathbf{E}'_2 = \frac{N_1}{N_2}\mathbf{E}_2 = \mathbf{E}_1$$

$$\mathbf{I}'_2 = \frac{N_2}{N_1}\mathbf{I}_2$$

$$\mathbf{Z}'_2 = \left(\frac{N_1}{N_2}\right)^2 \mathbf{Z}_2 \qquad (7B.3)$$

The phasor diagram for the transformer is developed as follows. Draw $\mathbf{V}'_2$ as the reference. The current $\mathbf{I}'_2$ will be at some angle to this, depending upon the type of load. Assume a load circuit which possesses both resistance and inductance, and therefore operates at a lagging power factor.

Then KVL applied around the right-hand mesh of the transformer equivalent circuit gives:

$$\mathbf{E}_1 = \mathbf{E}_2' = \mathbf{V}_2' + \left(R_2' + jX_{l2}'\right)\mathbf{I}_2' \tag{7B.4}$$

Now we know $\mathbf{E}_1$, the exciting current can be determined by:

$$\mathbf{I}_e = \mathbf{I}_c + \mathbf{I}_m = \frac{\mathbf{E}_1}{R_c} + \frac{\mathbf{E}_1}{jX_m} \tag{7B.5}$$

Note that $\mathbf{I}_c$ is in phase with the voltage $\mathbf{E}_1$, and $\mathbf{I}_m$ lags $\mathbf{E}_1$ by 90°. The total primary current $\mathbf{I}_1$ can now be found from KCL:

$$\mathbf{I}_1 = \mathbf{I}_2' + \mathbf{I}_e \tag{7B.6}$$

Finally, KVL around the left-hand mesh gives:

$$\mathbf{V}_1 = \mathbf{E}_1 + \left(R_1 + jX_{l1}\right)\mathbf{I}_1 \tag{7B.7}$$

The complete phasor diagram showing all voltages and currents is:

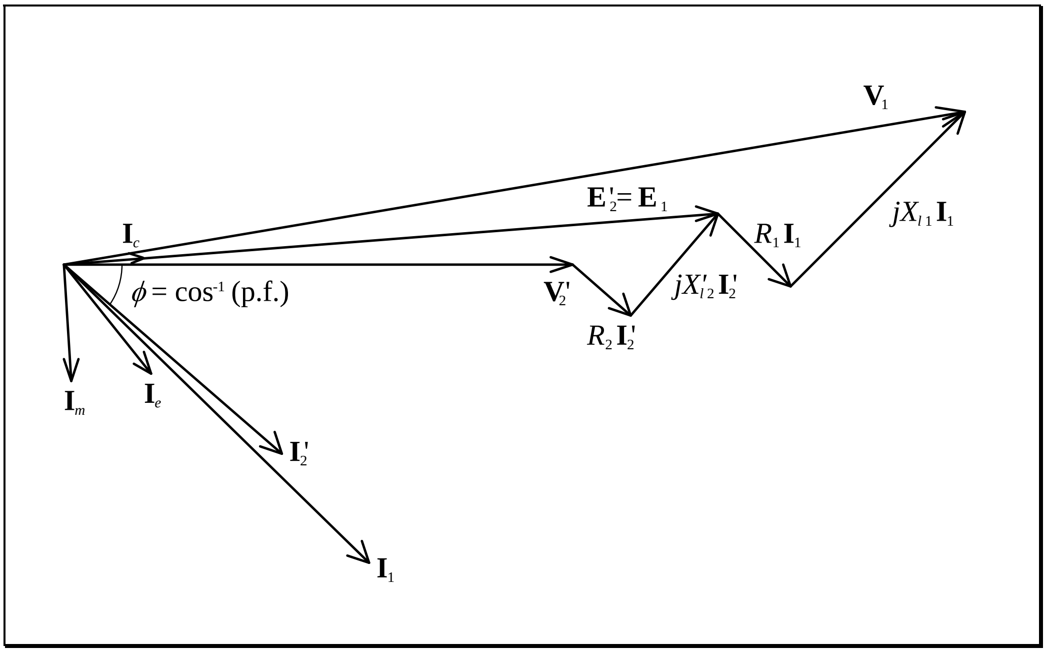


Figure 7B.9

7B.8

Example

A 20-kVA, 2200:220-V, 50 Hz, single-phase transformer has the following equivalent-circuit parameters referred to the high-voltage side of the transformer:

$$R_1 = 2.51\,\Omega \qquad R_2' = 3.11\,\Omega$$
$$X_{l1} = 10.9\,\Omega \qquad X_{l2}' = 10.9\,\Omega$$
$$X_m = 25.1\,\text{k}\Omega$$

The transformer is supplying 15 kVA at 220 volts and a lagging power factor of 0.85. We would like to determine the required voltage at the primary of the transformer.

The equivalent circuit of Figure 7B.8 and phasor diagram of Figure 7B.9 are applicable, if we assume $R_c = 0$. Note that the transformer is not supplying its rated power. Also note that the rating gives the nominal ratio of terminal voltages – that is, it gives the turns ratio of the ideal transformer.

We proceed with the analysis as follows:

$$\mathbf{V}_2 = 220\angle 0°\ \text{V}$$
$$\mathbf{V}_2' = \frac{2200}{220}\mathbf{V}_2 = 2200\angle 0°\ \text{V}$$
$$\left|\mathbf{I}_2\right| = \frac{15\times 10^3}{220} = 68.2\ \text{A}$$
$$\cos^{-1} 0.85 = 31.7°$$
$$\mathbf{I}_2 = 68.2\angle -31.7°\ \text{A}$$
$$\mathbf{I}_2' = \frac{220}{2200}\times 68.2\angle -31.7° = 6.82\angle -31.7°\ \text{A}$$

We now have all the secondary variables and parameters referred to the primary side. From Figure 7B.8, we have:

$$\mathbf{E}_1 = \mathbf{V}_2' + \left(R_2' + jX_{l2}'\right)\mathbf{I}_2' = 2200\angle 0° + \left(3.11 + j10.9\right)6.82\angle -31.7° = 2260\angle 1.3°\ \text{V}$$
$$\mathbf{I}_m = \frac{\mathbf{E}_1}{jX_m} = \frac{2260\angle 1.3°}{25100\angle 90°} = 0.090\angle -88.7°\ \text{A}$$

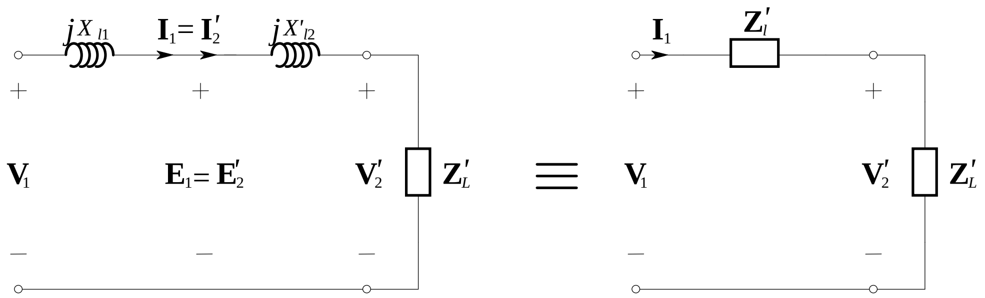
Note that $\mathbf{I}_m$ is very small compared with $\mathbf{I}_2'$. We now find the total primary current and voltage:

$$\mathbf{I}_1 = \mathbf{I}_m + \mathbf{I}_2' = 0.090\angle -88.7° + 6.82\angle -31.7° = 6.87\angle -32.3° \text{ A}$$
$$\mathbf{V}_1 = \mathbf{E}_1 + \left(R_1 + jX_{l1}\right)\mathbf{I}_1 = 2260\angle 1.3° + \left(2.51 + j10.9\right)6.87\angle -32.3° = 2311\angle 2.6° \text{ V}$$

Thus $\left|\mathbf{V}_1\right| = 2311\text{V}$, as compared with the rated or nameplate value of 2200 V. The additional voltage of 111 V is needed to "overcome" the impedance of the transformer.

Note also that the phasor diagram of Figure 7B.9 has greatly exaggerated the typical losses in a transformer for the sake of clarity of the drawing.

Also note that if we ignore the losses (resistances), and ignore the magnetizing current, we have a transformer model which looks like:



Analyzing, we get:

$$\mathbf{V}_1 = \mathbf{V}_2' + j\left(X_{l1} + X_{l2}'\right)\mathbf{I}_2' = 2200\angle 0° + j2\times 10.9\times 6.82\angle -31.7° = 2282\angle 3.2° \text{ V}$$

This is not significantly different to the real voltage (a 1.3% error in terms of magnitude), which justifies modelling power transformers as just a leakage reactance under normal conditions of operation.

Losses and Efficiency

Iron Loss

This is the term used for any core loss – it includes hysteresis and eddy current losses. For most materials, the power loss is:

$$P_i \propto f\hat{B}^{1.6} \tag{7B.8}$$

It is independent of the load current.

Copper Loss

This is the term used for heating loss due to the resistance of the windings.

$$P_c = R_1 I_1^{\,2} + R_2 I_2^{\,2} \tag{7B.9}$$

It is dependent upon the load current.

Efficiency

Is a measure of how well a device converts its input to desired output. For a transformer, efficiency is defined as:

$$\begin{aligned}\eta &= \frac{\text{output power}}{\text{input power}} \times 100\% \\ &= \frac{\text{output power}}{\text{output power} + \text{losses}} \times 100\% \end{aligned} \tag{7B.10}$$

Measurement of Transformer Parameters

Open-Circuit Test

We leave the secondary of the transformer as on open circuit and apply the rated voltage on the primary side. With an open circuit on the secondary, we have already seen that the transformer is an iron-cored inductor:

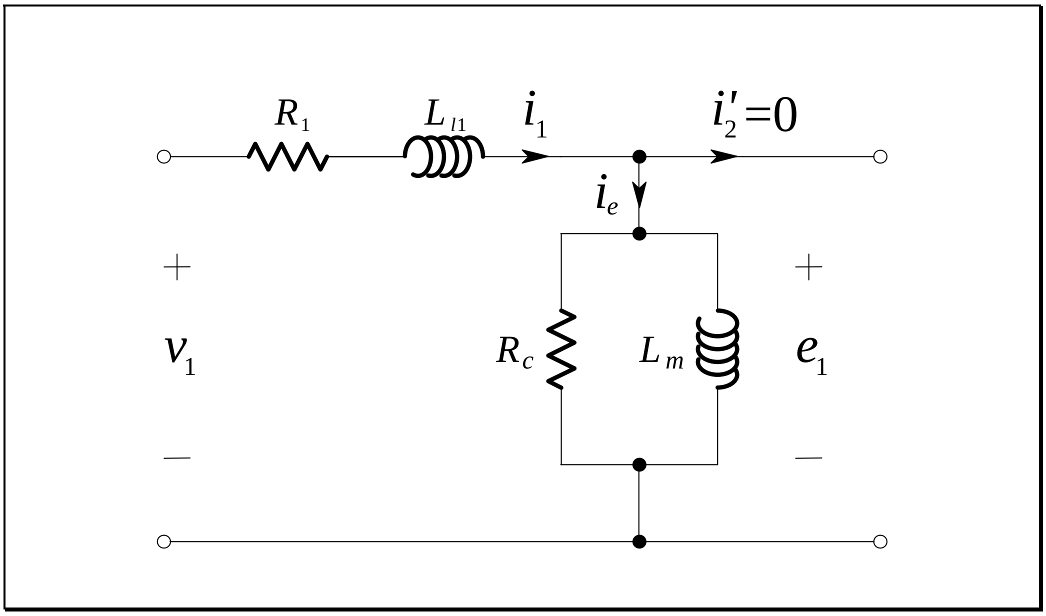


Figure 7B.10

The magnetizing branch has a higher impedance than the primary winding resistance and leakage inductance. They appear in series, so we can ignore the winding resistance and leakage inductance with a small error:

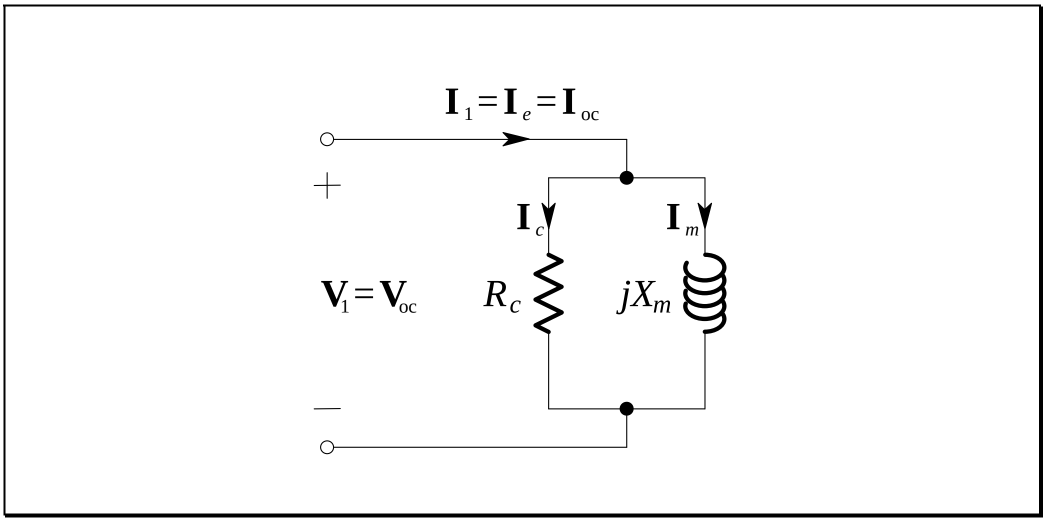


Figure 7B.11

The phasor diagram for this test is:

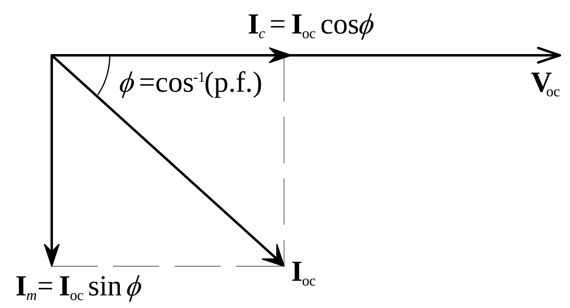


Figure 7B.12

If we measure the average power P, RMS voltage magnitude $|\mathbf{V}_{oc}|$ and RMS current magnitude $|\mathbf{I}_{oc}|$ then:

$$\cos\phi = \frac{P_{oc}}{|\mathbf{V}_{oc}||\mathbf{I}_{oc}|} \tag{7B.11a}$$

$$R_c = \frac{|\mathbf{V}_{oc}|}{|\mathbf{I}_{oc}|\cos\phi} = \frac{|\mathbf{V}_{oc}|^2}{P_{oc}} \tag{7B.11b}$$

$$X_m = \frac{|\mathbf{V}_{oc}|}{|\mathbf{I}_{oc}|\sin\phi} \tag{7B.11c}$$

This gives us the magnetizing branch at rated voltage.

Short-Circuit Test

We apply a short circuit to the secondary of the transformer and increase the primary voltage until we achieve rated current in each winding. With a short on the secondary, we can reflect the secondary resistance and leakage inductance to the primary side:

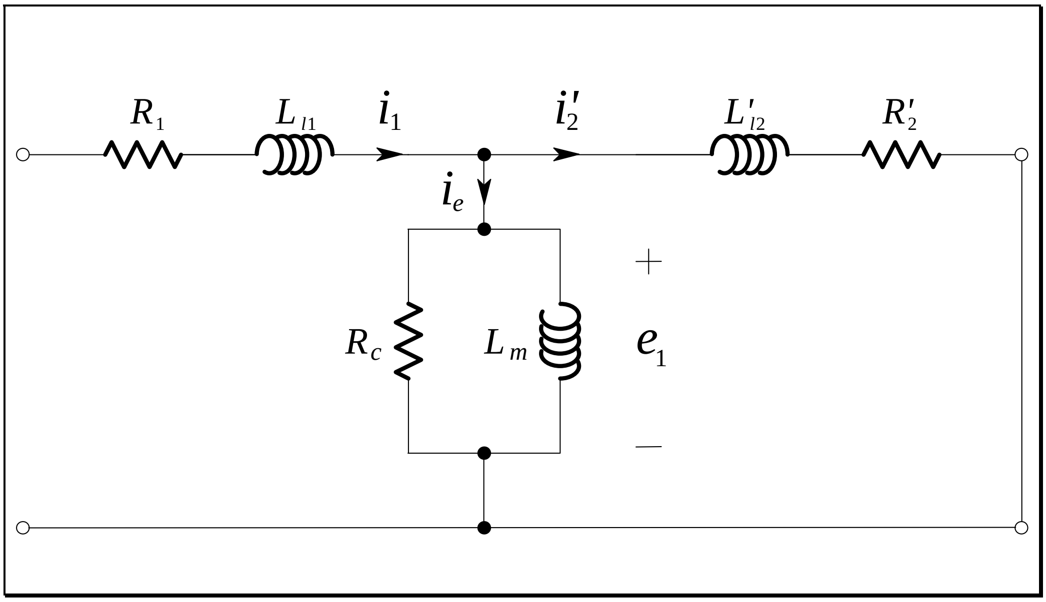


Figure 7B.13

The magnetizing branch has a higher impedance than the secondary impedance referred to the primary. They appear in parallel, so we can ignore the magnetizing branch with a small error:

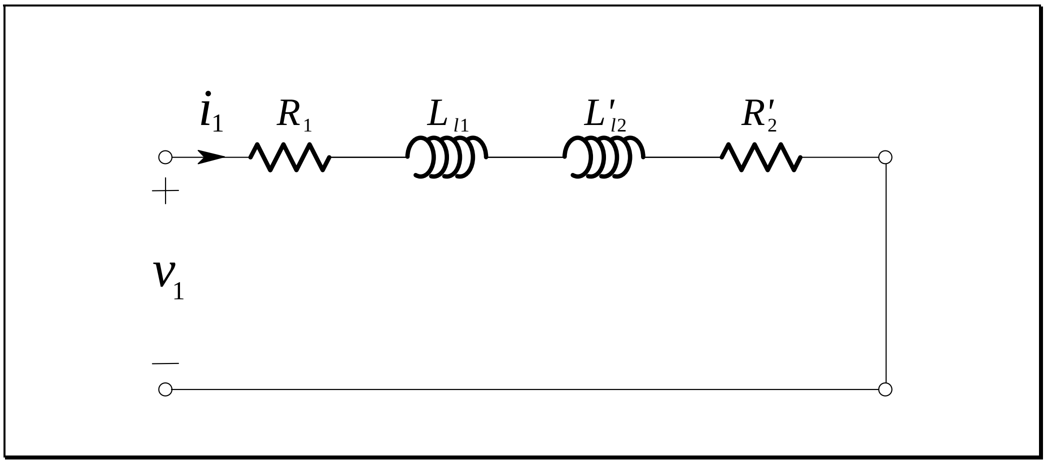


Figure 7B.14

7B.14

The equivalent circuit for this test is effectively that shown below:

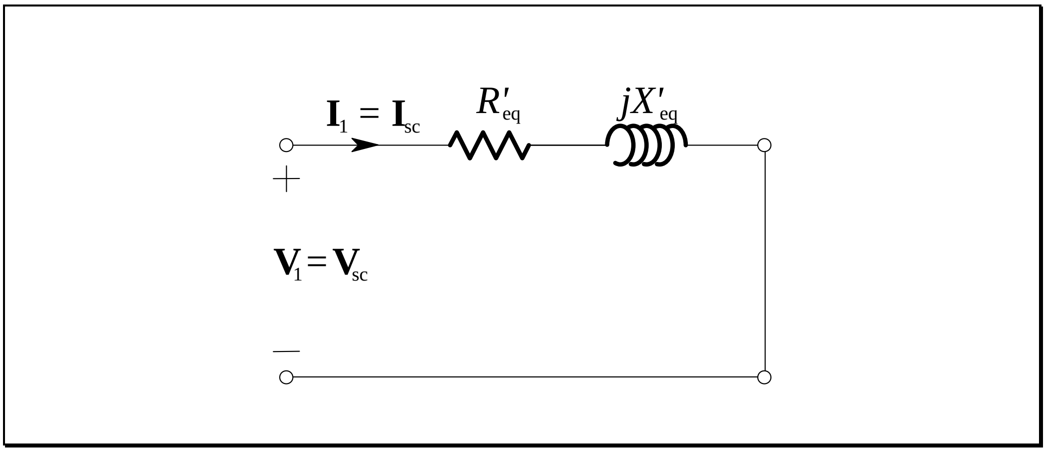


Figure 7B.15

where R'_{eq} and X'_{eq} are called the *equivalent resistance* and *equivalent leakage reactance* and are defined by:

$$R'_{eq} = R_1 + R'_2 \tag{7B.12a}$$

$$X'_{eq} = X_{l1} + X'_{l2} \tag{7B.12b}$$

It is then usually assumed (because the paths for the leakage flux of both windings are approximately the same) that:

$$X_{l1} = X'_{l2} = \frac{X'_{eq}}{2} \tag{7B.13}$$

The phasor diagram for this test is:

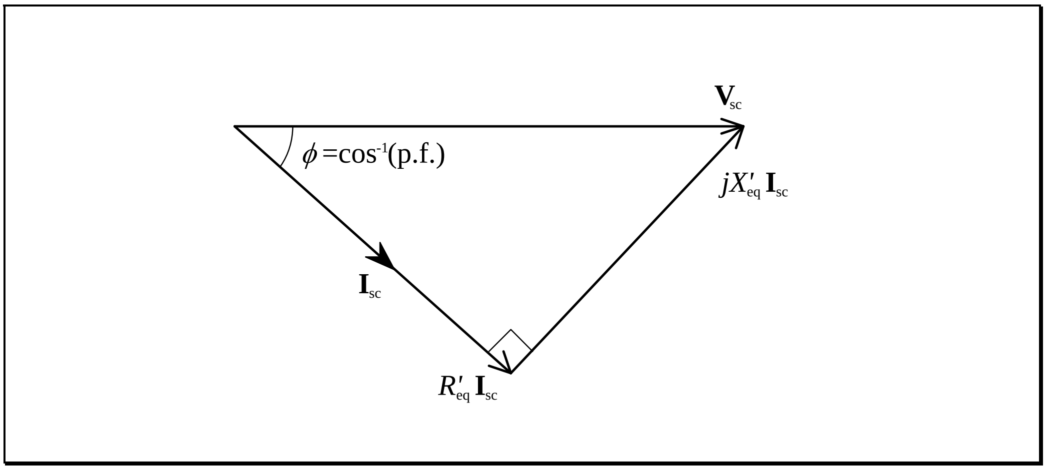


Figure 7B.16

If we measure the average power P, RMS voltage magnitude $|\mathbf{V}_{sc}|$ and RMS current magnitude $|\mathbf{I}_{sc}|$ then:

$$\cos\phi = \frac{P_{sc}}{|\mathbf{V}_{sc}||\mathbf{I}_{sc}|} \tag{7B.14a}$$

$$R'_{eq} = \frac{|\mathbf{V}_{sc}|\cos\phi}{|\mathbf{I}_{sc}|} = \frac{P_{sc}}{|\mathbf{I}_{sc}|^2} \tag{7B.14b}$$

$$X'_{eq} = \frac{|\mathbf{V}_{sc}|\sin\phi}{|\mathbf{I}_{sc}|} \tag{7B.14c}$$

Winding-Resistance Measurements

R_1 and R_2 may be measured directly using a multimeter. Such measurements give the resistance of the windings to direct current, and it may be that these differ appreciably from the resistance to alternating current owing to non-uniform distribution of alternating currents in the conductors. This may be checked by determining R'_{eq} from the short-circuit test and comparing it with the equivalent DC winding resistance referred to the primary side of the transformer. If R_1 and R_2 are the measured DC values, then the equivalent DC resistance is:

$$R'_{DC} = R_1 + \left(\frac{N_1}{N_2}\right)^2 R_2 \tag{7B.15}$$

Should R'_{DC} differ appreciably from R'_{eq}, then the AC winding resistances referred to the primary side may be determined by dividing R'_{eq} in the ratio of the two terms on the right-hand side of Eq. (7B.15).

Current and Voltage Excitation

For any magnetic system with one electrical circuit (applies to transformer with open circuited secondary), KVL around the loop gives:

$$v = Ri + e$$
$$e = \frac{d\lambda}{dt} \tag{7B.16}$$

There are two extreme cases we consider.

Case 1 - Current Excitation

R is large so that $e \approx 0$ (exact for DC). Then:

$$v \approx Ri \tag{7B.17}$$

and the system is said to be current excited. That is, we vary the voltage source which directly varies the current according to the above relationship. The flux in the system will then be determined from the $\lambda \sim i$ characteristic.

Case 2 - Voltage Excitation

R is small (applies to AC only) so that:

$$v \approx \frac{d\lambda}{dt} \tag{7B.18}$$

and the system is said to be voltage excited. That is, we vary the voltage source which directly varies the flux according to the above relationship. The current in the system will then be determined from the $\lambda \sim i$ characteristic.

3rd Harmonics

The non-linear $\lambda \sim i$ characteristic gives rise to unusual waveforms. Since the waveforms are periodic, we can make these strange waveforms by summating sine waves of different frequency, amplitude and phase. This is known as Fourier synthesis. To a close approximation, the magnetizing currents in most iron cores can be considered to be made of a fundamental (50 Hz) and a 3rd harmonic (150 Hz).

References

Slemon, G. and Straughen, A.: *Electric Machines*, Addison-Wesley Publishing Company, Inc., Sydney, 1982.