Lecture 8A – The MOSFET Voltage Amplifier

Small signal equivalent circuit. The common-source amplifier. The common drain (or source follower) amplifier.

Small Signal Equivalent Circuit

When we looked at the MOSFET previously, we saw that the drain current i_D was controlled by the voltage between the gate and source. The voltage v_{GS} could increase or decrease the depth of the channel. There was a point where the current through the MOSFET could increase no further – this was termed saturation.

In real devices, saturation is not ideal. If we increase the applied voltage, v_{DS}, we also increase the drain current, i_D, by a small amount. The drain current, in the saturation region, is therefore dependent upon v_{GS} and v_{DS}. For small variations in v_{GS} and v_{DS} we have:

$$\Delta i_D = \frac{\partial i_D}{\partial v_{GS}} \Delta v_{GS} + \frac{\partial i_D}{\partial v_{DS}} \Delta v_{DS} \qquad (8A.1)$$

or:

$$\boxed{i_d = g_m v_{gs} + g_o v_{ds}} \qquad (8A.2)$$

where:

$$\begin{aligned} g_m &= \text{slope of } i_D \sim v_{GS} \text{ characteristic at } Q\text{-point} \\ &= \text{transconductance} \end{aligned} \qquad (8A.3a)$$

$$\begin{aligned} g_o &= \text{slope of } i_D \sim v_{DS} \text{ characteristic at } Q\text{-point} \\ &= \text{output conductance} \end{aligned} \qquad (8A.3b)$$

Then for g_m in Eq. (8A.3a), a graphical interpretation is:

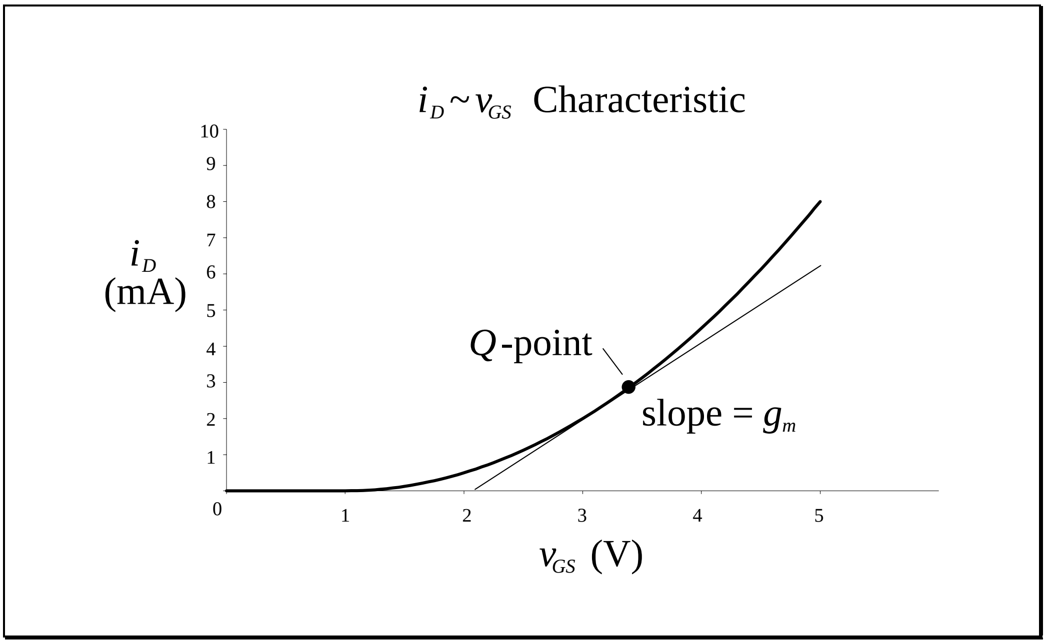

Figure 8A.1

and for g_o in Eq. (8A.3b):

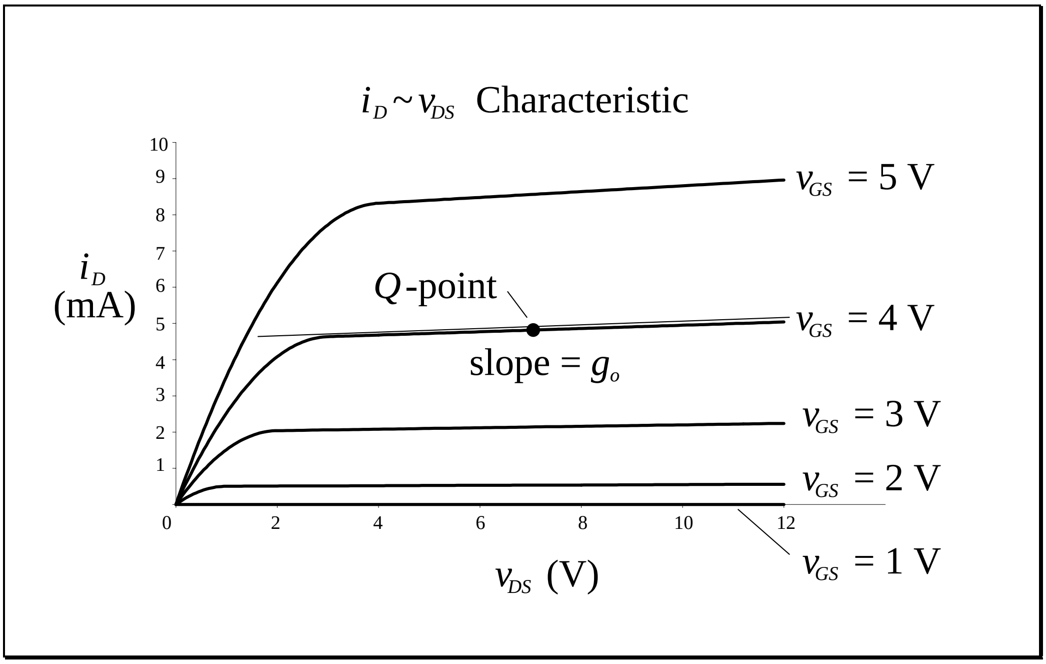

Figure 8A.2

If the signal is large, then the linear approximation given in Eq. (8A.2) is not very good, and we have to use a "large-signal" model of the MOSFET.

The formula of Eq. (8A.2) that relates small variations in v_{GS} and v_{DS} to i_D can be put into circuit form. With small signals applied, the MOSFET characteristics look linear, and we can effectively model the MOSFET by an *AC small-signal equivalent circuit*:

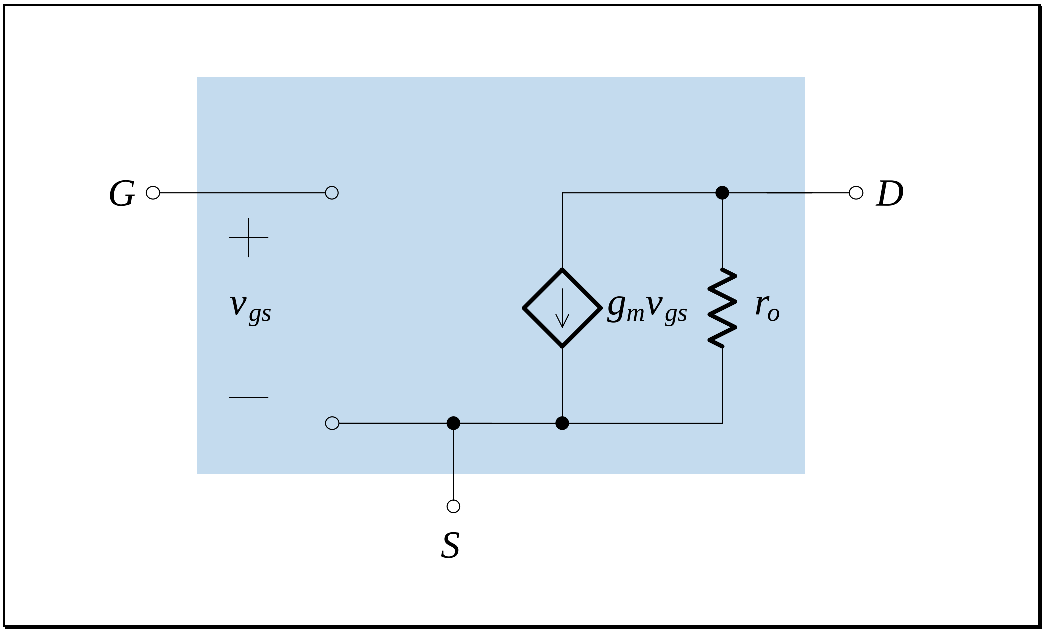

Figure 8A.3

Confrim that an analysis of the above circuit will lead to Eq. (8A.2). Since r_o is large, it is common to ignore it in a *first-order hand analysis*. When this is the case, the equivalent circuit can be converted to a T equivalent-circuit:

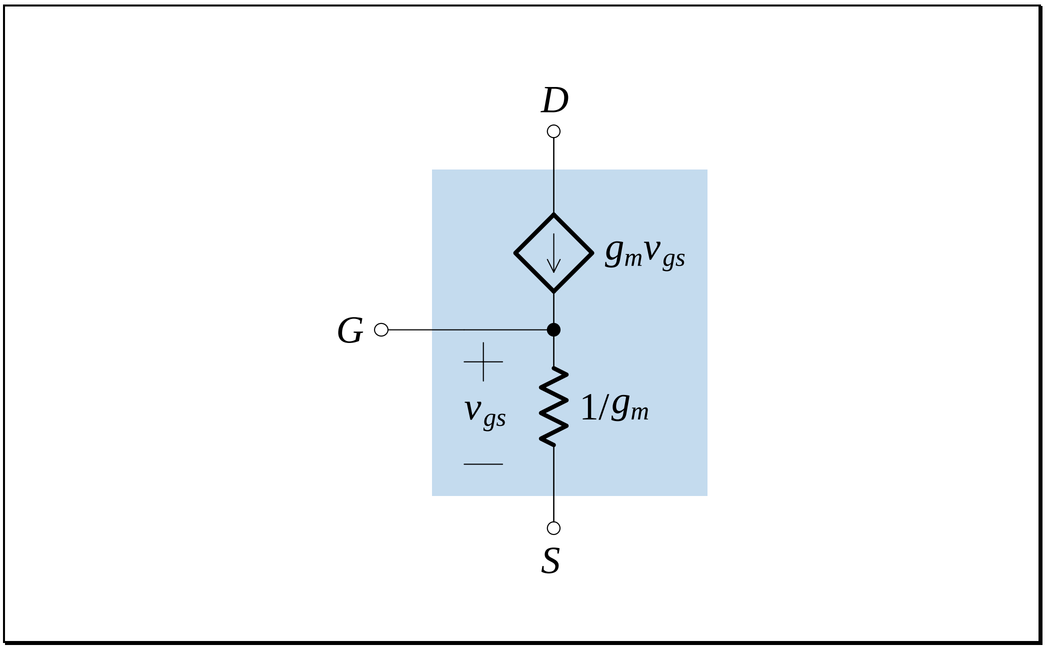

Figure 8A.4

Since there are depletion regions inside the MOSFET, there is charge separation. This results in a capacitance. These capacitances are due to the reverse-biased junctions and are of the order of 1 to 3 pF.

With small signals applied, the characteristic looks linear, and we can effectively model the MOSFET by an AC small signal equivalent circuit:

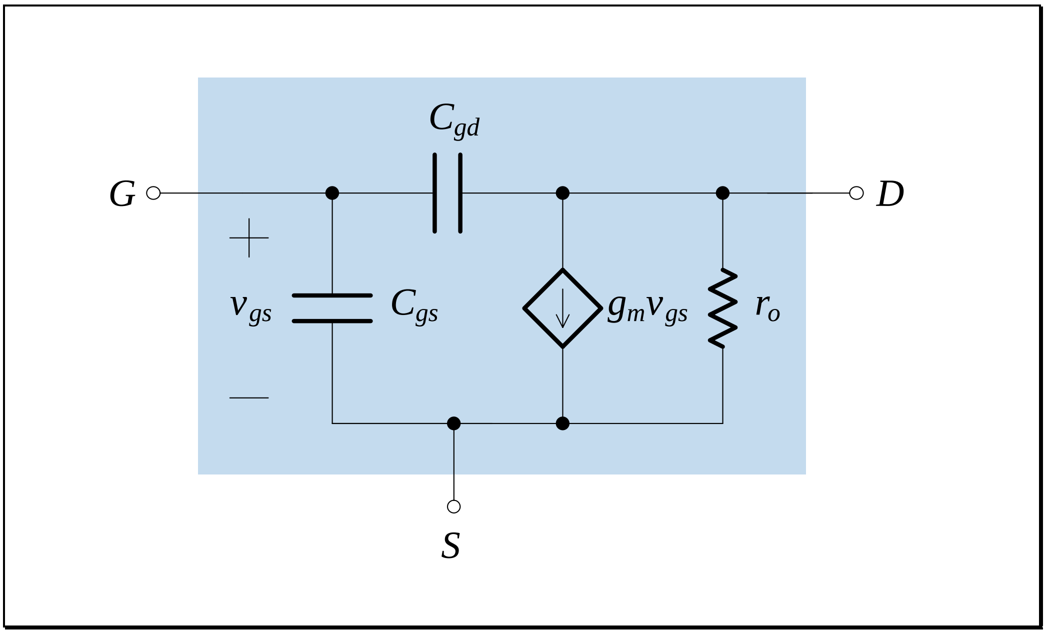

Figure 8A.5

The capacitors are ignored except when working at very high frequencies, where their reactance has an effect (> 100 kHz).

We can replace a MOSFET by its appropriate small-signal equivalent circuit when we perform an AC analysis of a properly DC-biased MOSFET.

We can now look at how to build various amplifiers using the MOSFET.

The Common-Source Amplifier

The common-source amplifier is:

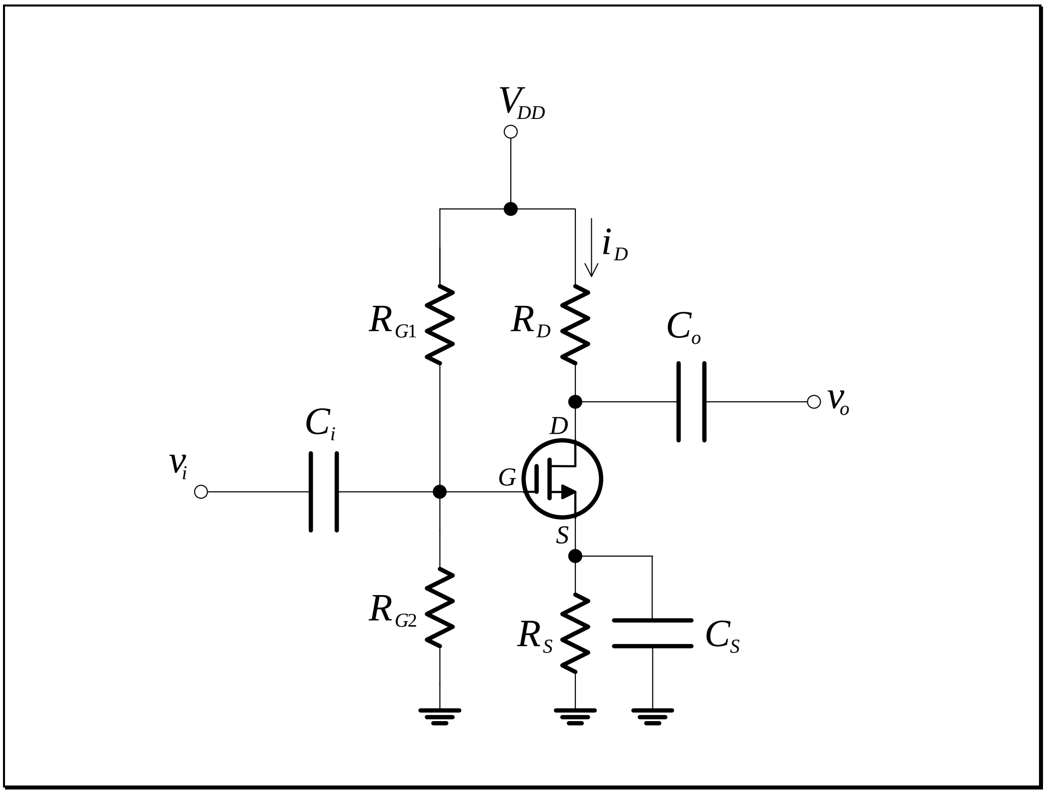

Figure 8A.6

The first part of analysing such a circuit is to determine the DC, or bias conditions on the MOSFET. The second part is to perform an AC analyis.

DC Analysis

Note that for DC all capacitors are open circuits. This simplifies the DC analysis considerably.

Since the gate current is zero, the voltage at the gate is simply determined by the voltage divider formed by R_{G1} and R_{G2}:

$$V_G = \frac{R_{G2}}{R_{G1} + R_{G2}} V_{DD} \tag{8A.4}$$

The voltage at the source is simply:

$$V_S = R_S I_D \tag{8A.5}$$

Thus, the gate-to-source voltage is:

$$V_{GS} = V_G - V_S = V_G - R_S I_D \tag{8A.6}$$

If this positive gate-to-source voltage exceeds the threshold voltage, the NMOS transistor will be turned on. We do not know, however, whether the transistor is operating in the saturation region or the triode region. If we assume it is operating in the saturation region, then to find the MOSFET's drain current, we can rearrange the Eq. (8A.6) to get the equation of a *load line*:

$$I_D = -\frac{1}{R_S}\left(V_{GS} - V_G\right) \tag{8A.7}$$

and graph it on the $i_D \sim v_{GS}$ characteristic:

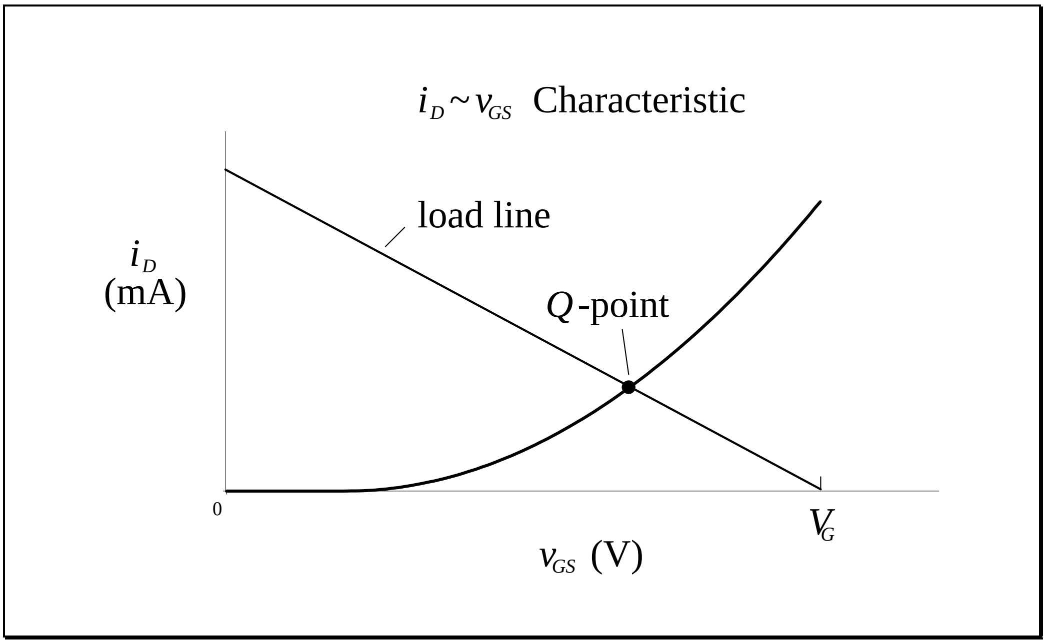

Figure 8A.7

Alternatively, we can find an algebraic solution:

$$\begin{aligned} I_D &= K\left(V_{GS} - V_t\right)^2 \\ &= K\left(V_G - R_S I_D - V_t\right)^2 \end{aligned} \quad (8A.8)$$

which results in the following quadratic equation:

$$R_S^2 I_D^2 + -\left[2K\left(V_G - V_t\right)R_S + 1\right]I_D + K\left(V_G - V_t\right)^2 = 0 \quad (8A.9)$$

Out of the two solutions to this quadratic, only one will intersect the $i_D \sim v_{GS}$ characteristic in the saturation region.

After finding I_D we can then find the drain voltage:

$$V_D = V_{DD} - R_D I_D \quad (8A.10)$$

Lastly, we need to check that $V_D > V_G - V_t$, i.e. that the transistor is indeed operating in the saturation region.

DC Design

KVL, starting from the common and going up through the channel gives:

$$R_S I_D + V_{DS} + R_D I_D = V_{DD}$$

$$I_D = -\frac{1}{R_D + R_S}\left(V_{DS} - V_{DD}\right) \tag{8A.11}$$

This is the equation of a load line. The intersection of this line on the $i_D \sim v_{DS}$ characteristic gives the *Q*-point.

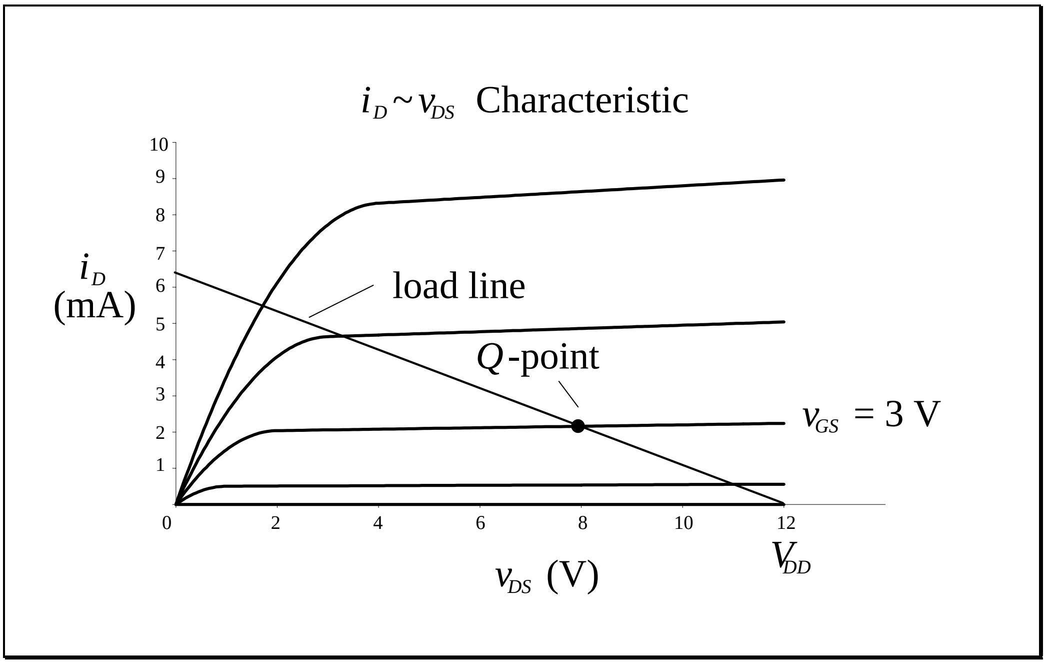

Figure 8A.8

When designing an amplifier, we use the rule of thumb that one-third of the power supply voltage, V_{DD}, appears across each of R_D, the transistor (i.e., V_{DS}) and R_S. This choice minimises variations in the *Q*-point as the threshold voltage of the MOSFET varies (from device to device), as well as providing roughly equal excursions of the output voltage when a small signal is applied.

It is possible to design the DC bias conditions algebraically by assuming that the transistor is in saturation, and using the applicable MOSFET voltage and current relations.

AC Analysis

AC analysis of electronic circuits relies on the fact that the circuit is linear and we can thus use superposition. Then the DC supply appears as a short circuit (i.e. an independent supply of 0 V). We also assume that the capacitors behave as perfect short circuits for the frequency at which our input signal is applied.

For small AC signals, the MOSFET small-signal equivalent circuit can be used in the AC analysis. We note also that the capacitor C_S by-passes (effectively shorts) the resistor R_S. The DC supply is equivalent to a short circuit so that resistors R_D and R_{G1} are connected to common. The input and output capacitors C_i and C_o are used to couple the input and output voltages without disturbing the DC bias conditions, and are assumed to have zero reactance at "mid-band frequencies".

The AC small-signal equivalent circuit of the amplifier is then:

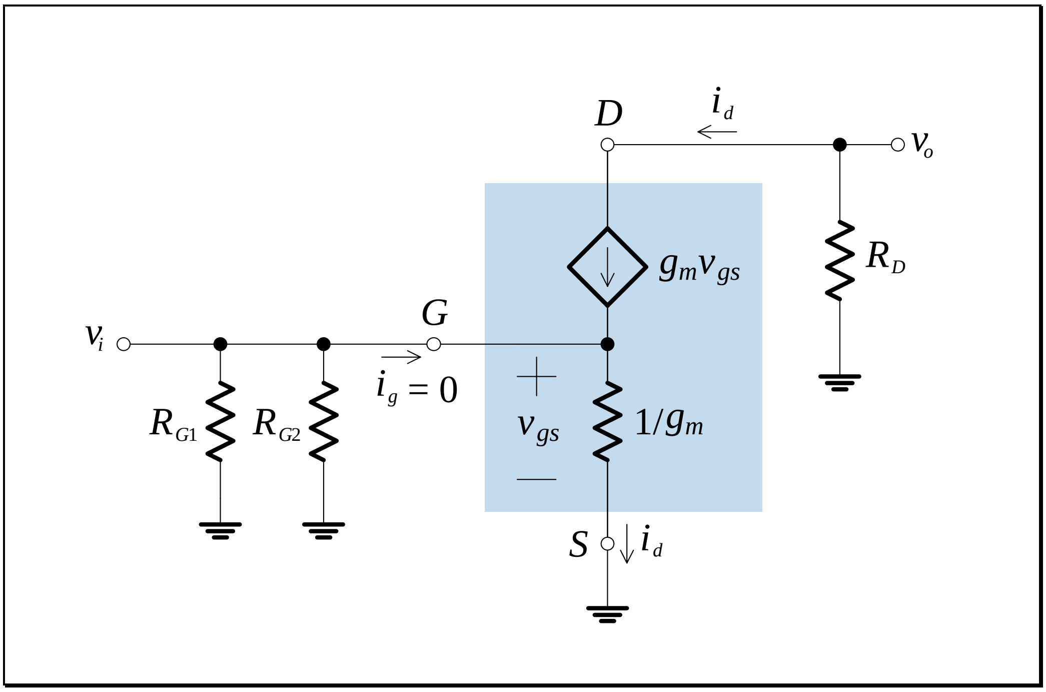

Figure 8A.9

Note that we have simply "dropped in" the small-signal equivalent circuit of the MOSFET – in this case the T equivalent.

We can now see that the input signal appears directly across the gate-to-source junction of the transistor:

$$v_{gs} = v_i \tag{8A.12}$$

and that the drain current is:

$$i_d = g_m v_{gs} \tag{8A.13}$$

The output voltage can now be found from:

$$\begin{aligned} v_o &= -i_d R_d \\ &= -g_m R_d v_i \end{aligned} \tag{8A.14}$$

What features of this circuit are we interested in? We are trying to create a voltage amplifier, so the quantities of interest are: the input impedance, the open circuit voltage gain, and the output impedance.

Thus the open-circuit (no load) voltage gain is:

$$\boxed{A_{vo} = \frac{v_o}{v_i} = -g_m R_d} \tag{8A.15}$$

The input resistance, as seen by the signal source, is, by inspection:

$$\boxed{R_{\text{in}} = R_{G1} \parallel R_{G2}} \tag{8A.16}$$

The output resistance is obtained by setting the input signal source to zero and observing the resistance "seen looking back into the output terminal":

$$\boxed{R_{\text{out}} = R_D} \tag{8A.17}$$

This value of output resistance is quite large, and is unsuitable in many applications.

We can now draw an equivalent circuit of the voltage amplifier:

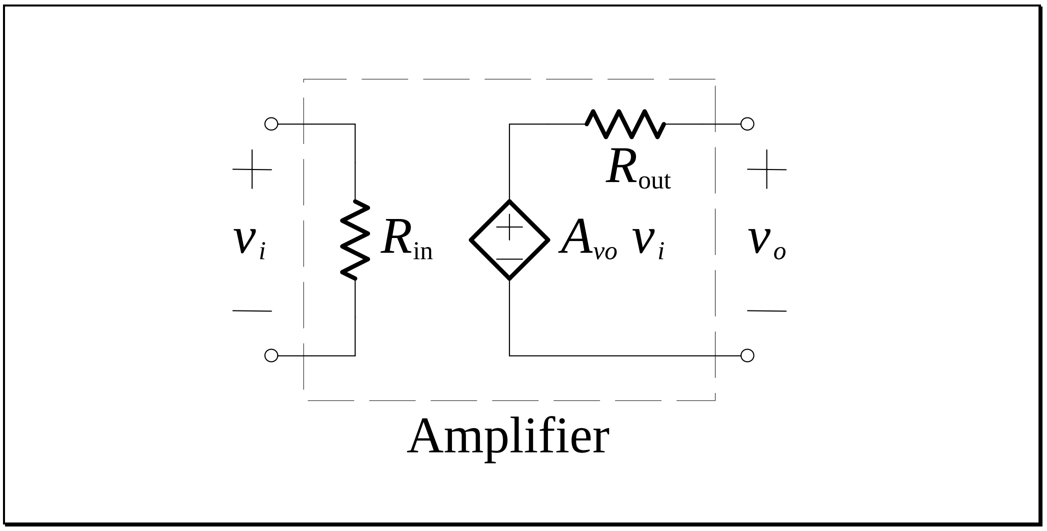

Figure 8A.10

This equivalent circuit can be used to study the performance of the amplifier when a real source (with input resistance) and a real load are connected:

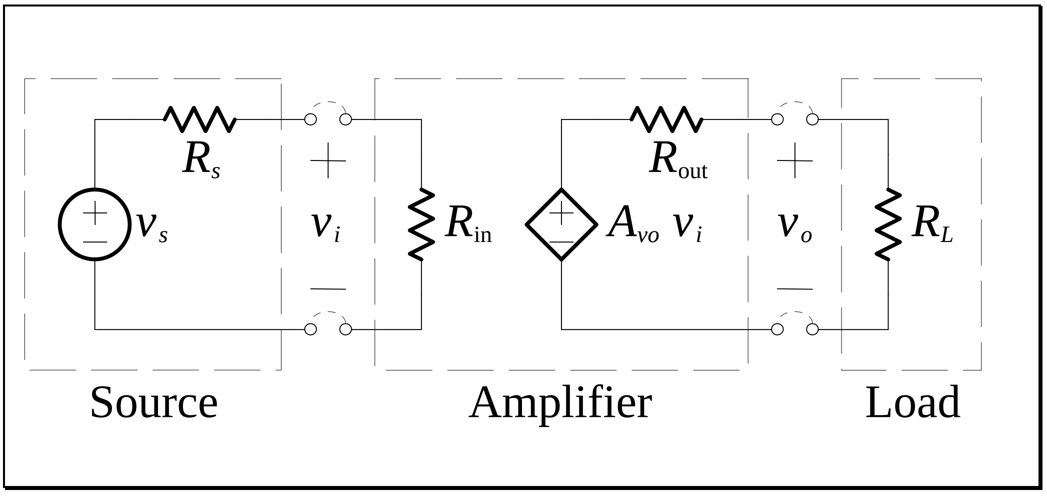

Figure 8A.11

A quick analysis shows that:

$$A_v = \frac{v_o}{v_s} = A_{vo} \frac{R_L}{R_L + R_{\text{out}}} \frac{R_{\text{in}}}{R_{\text{in}} + R_s} \qquad (8A.18)$$

8A.12

AC Design

As can be seen from Eqs. (8A.15) to (8A.17), the design of the amplifier from the small signal perspective is concerned with choosing a suitable drain resistance, R_D, and bias point g_m.

The Common Drain (or Source Follower) Amplifier

The common-drain amplifier is:

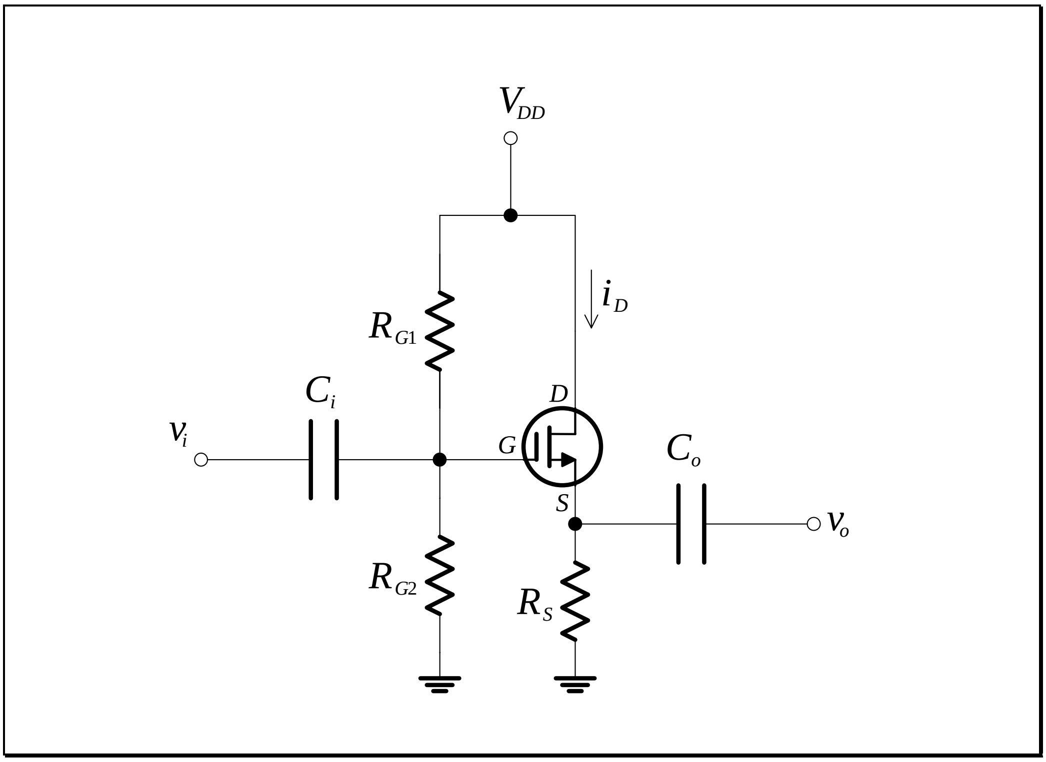

Figure 8A.12

Here, the output is taken from the source terminal, and the drain, for AC is connected to the common.

The small signal equivalent circuit for the common-drain amplifier is:

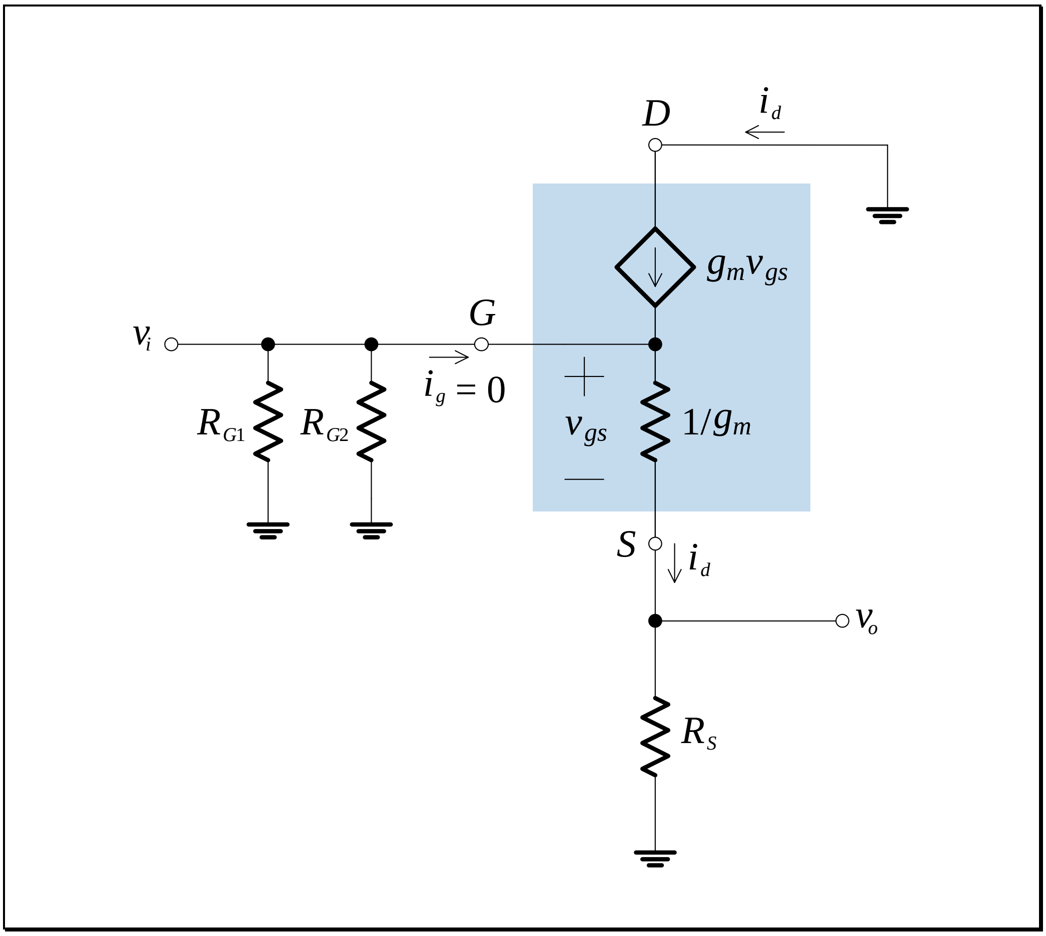

Figure 8A.13

Analysis of the circuit is straightforward:

$$v_o = \frac{R_S}{R_S + \dfrac{1}{g_m}} v_i \tag{8A.19}$$

which gives:

$$A_{vo} = \frac{v_o}{v_i} = \frac{R_S}{R_S + \dfrac{1}{g_m}} \tag{8A.20}$$

Normally, $R_S >> 1/g_m$, causing the open-circuit voltage gain to be approximately unity. Thus the voltage at the source follows that at the gate, giving the circuit its popular name of *source follower*.

The input resistance is:

$$R_{\text{in}} = R_{G1} \parallel R_{G2} \tag{8A.21}$$

and the output resistance is:

$$R_{\text{out}} = R_S \parallel \frac{1}{g_m} \tag{8A.22}$$

Normally, $R_S >> 1/g_m$, which means $R_{out} \approx 1/g_m$ is moderately low. Thus the common-drain amplifier presents a low output resistance on its output and it can be used a unity-gain voltage buffer amplifier.

References

Sedra, A. and Smith, K.: *Microelectronic Circuits, 5th Ed.*, Oxford University Press, New York, 2004.