Lecture 8B – The Force Equation

Force equation of singly excited electromechanical transducer. Electric field transducer. Electrostatic voltmeter.

Force Equation of Singly Excited Electromechanical Transducer

Demo

Discuss arrangement of contactor. Ask what will happen with DC and AC excitation. Demonstrate. Uses - contactors, meters, motors.

A simple electromechanical transducer could have the following arrangement:

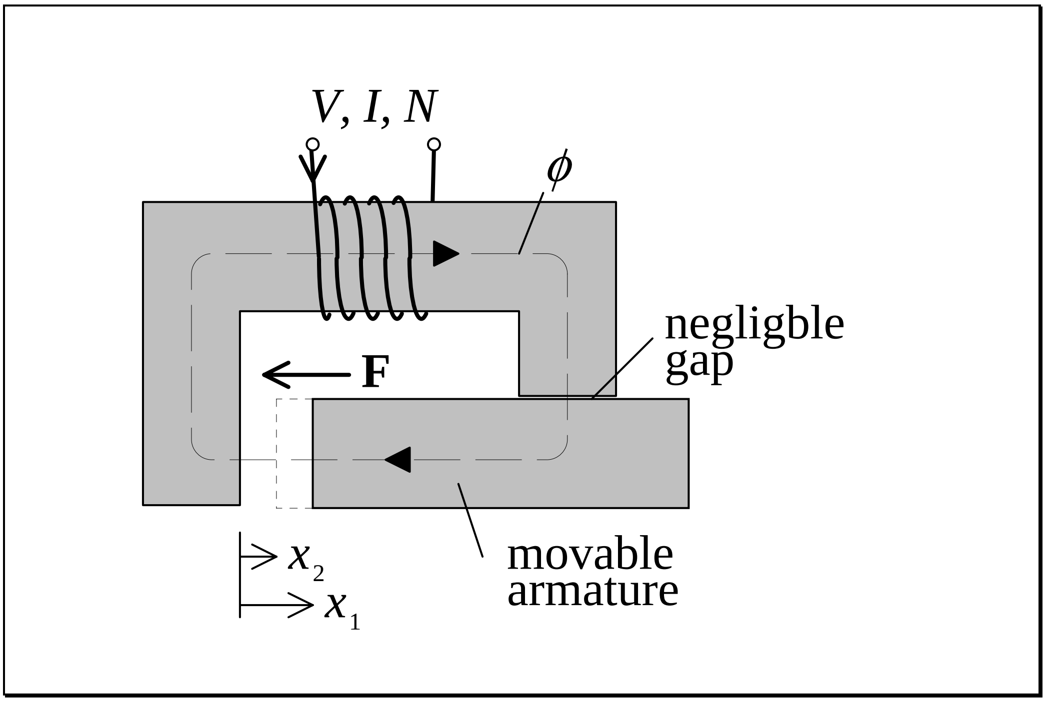


Figure 8B.1

It consists of a movable armature that is part of the core and one winding connected to a source. The source supplies current to the winding, which creates a flux in the core. The flux streams through the core and across the air gap.

8B.2

The magnetic and electric equivalent circuits for the above arrangement are:

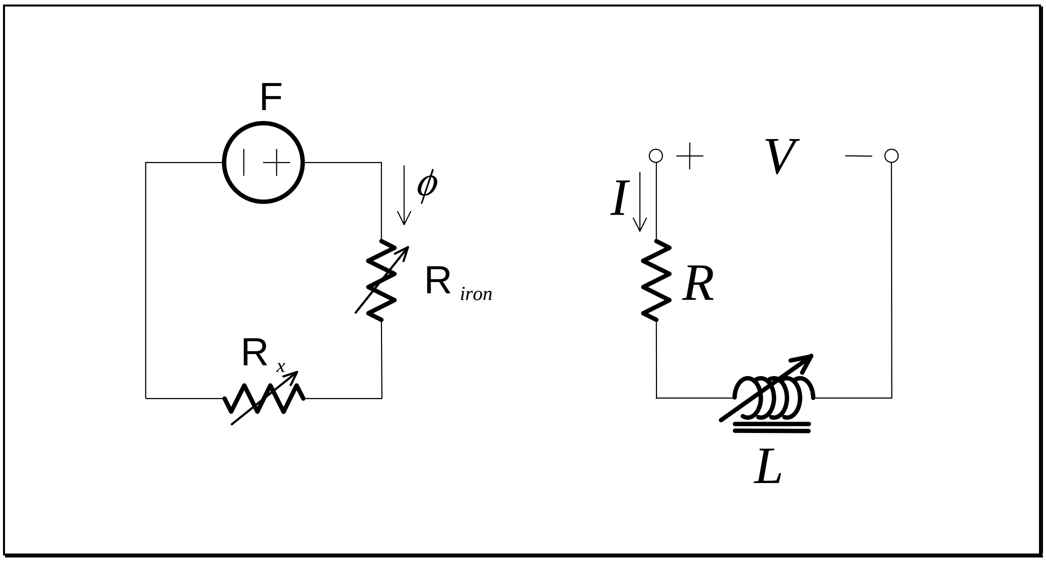


Figure 8B.2

The composite characteristic is almost linear due to the air gap. The characteristic varies as the distance *x* varies, since the gap width (reluctance, hence flux for a constant mmf) is changing.

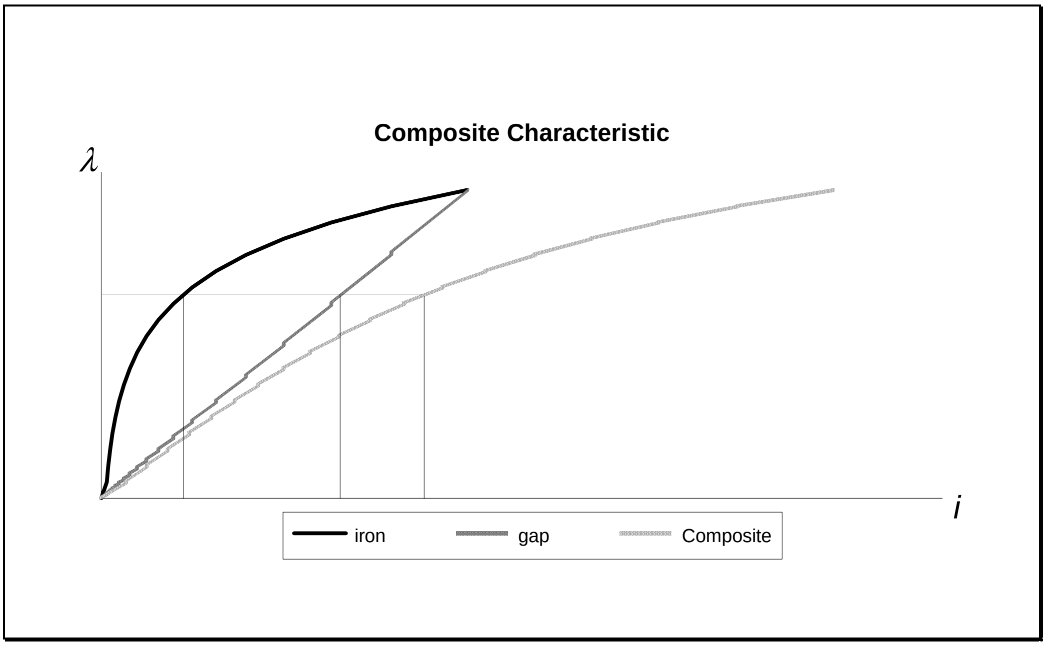


Figure 8B.3

Conservation of energy for the above system gives us:

Electrical energy		Mechanical energy		Field energy	(8B.1)
minus	=	plus	+	plus	
resistance losses		friction losses		core losses	

Expressed in symbols, this becomes:

$$W_e = W_m + W_f \tag{8B.2}$$

Suppose now that we are exciting the coil with a DC source. The steady state value of current is determined only by the resistance.

Electromechanical energy conversion occurs when the energy stored in the coupling field depends upon position. The armature can be imagined to have a north pole, while the fixed core has a south pole. The armature will be attracted to the fixed core.

If the armature moves slowly from position 1 to 2, the reluctance of the magnetic system is decreased. Since the mmf is constant, the flux must increase. To increase the flux, some energy is supplied to the magnetic field.

We assume the armature moves slowly in this case so that the emf produced by the changing flux is small and does not reduce the current. i.e. the current is constant.

In the time it takes the armature to move, an amount of electrical energy is supplied to the system:

$$\begin{aligned}\delta W_e &= VI\delta t - RI^2\delta t = \left(V - RI\right)I\delta t = eI\delta t \\ &= I\delta\lambda = I^2\delta L\end{aligned} \tag{8B.3}$$

We also remember that the amount of energy actually stored by the field in this time, for a linear lossless system is:

$$\delta W_f = \frac{1}{2} I \delta\lambda = \frac{1}{2} I^2 \delta L \tag{8B.4}$$

Assuming a lossless system (no core losses) is a good approximation because the losses are usually very small. According to our energy conservation relation, any energy delivered electrically and not stored in the field produces mechanical work:

$$\begin{aligned} \delta W_m &= \delta W_e - \delta W_f \\ &= I^2 \delta L - \frac{1}{2} I^2 \delta L \\ &= \frac{1}{2} I^2 \delta L \end{aligned} \tag{8B.5}$$

The work is done by the mechanical force that moves the armature through the distance δx:

$$\begin{aligned} \delta W_m &= F \delta x \\ F &= \frac{\delta W_m}{\delta x} \\ &= \frac{I^2}{2} \frac{\delta L}{\delta x} \end{aligned} \tag{8B.6}$$

Again, mechanical losses such as friction have been ignored, because they are small. In the limit, as δx becomes infinitesimally small:

$$F = \frac{I^2}{2}\frac{dL}{dx} = -\frac{\phi^2}{2}\frac{d\mathcal{R}}{dx} \tag{8B.7}$$

since

$$L = \frac{N^2}{\mathcal{R}} \quad \text{and} \quad \phi^2 = \frac{N^2 I^2}{\mathcal{R}^2}$$

This equation was derived for the DC case but it also applies to the AC case.

The force will act in a direction so as to increase the inductance of the system.

Electric Field Transducer

The electrical analogy to the previous magnetic system is the following system (the principle of the electrostatic speaker):

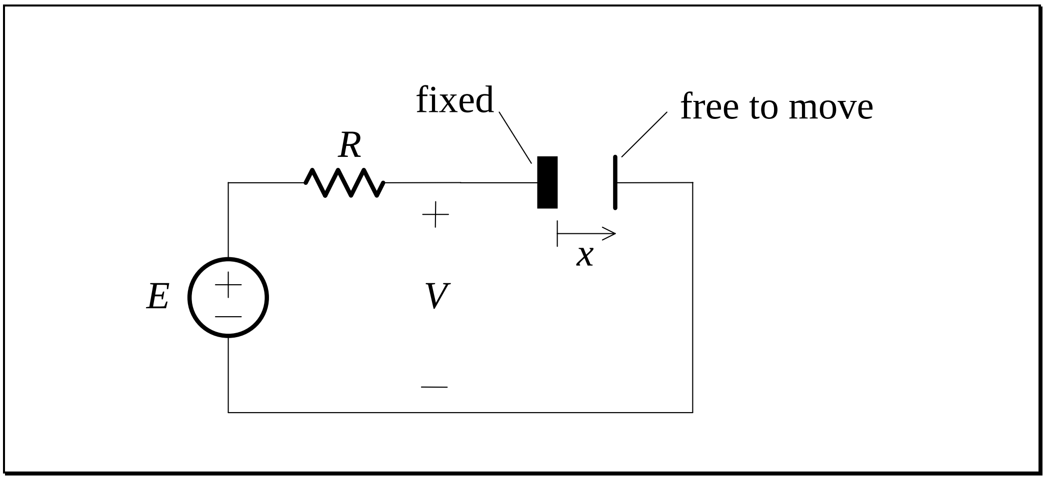


Figure 8B.4

The principle of operation is equivalent to the electromechanical device, except now the electric field is dominant – the magnetic field energy is ignored. This circuit is the dual of the electrical equivalent circuit for the electromechanical case.

We can replace *I* with *V*, and *L* with *C* to obtain:

$$F = \frac{V^2}{2}\frac{dC}{dx} \tag{8B.8}$$

The force will act in a direction so as to increase the capacitance of the system.

Electrostatic Voltmeter

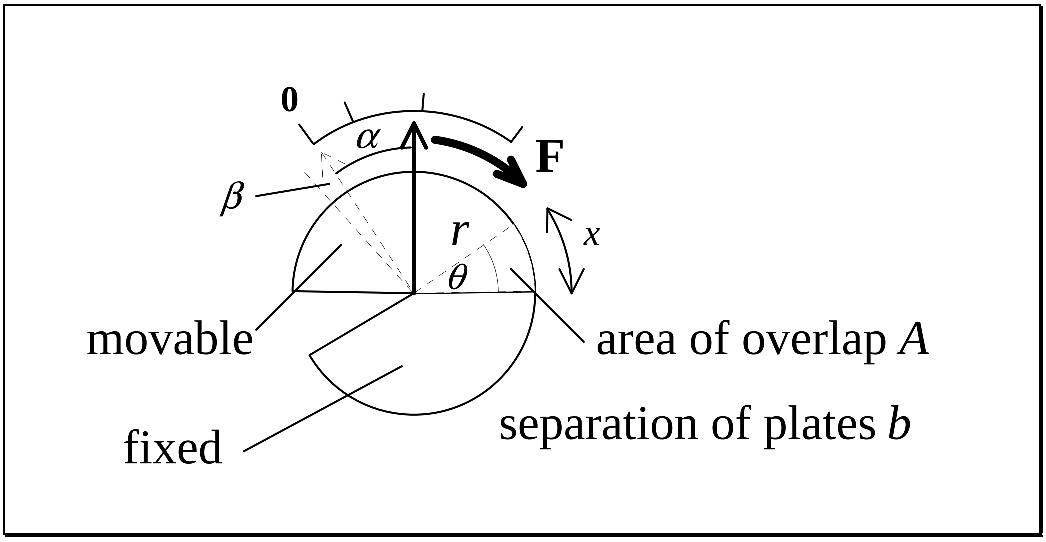


Figure 8B.5

The electric force equation applies. The force will tend to increase the capacitance, so θ increases.

The magnitude of the torque experienced by the movable plate is:

$$T = Fr = r\frac{V^2}{2}\frac{dC}{dx} = r\frac{V^2}{2}\frac{dC}{d\theta}\frac{d\theta}{dx} \tag{8B.9a}$$

$$x = r\theta, \quad \therefore \quad \frac{d\theta}{dx} = \frac{1}{r} \tag{8B.9b}$$

$$T = \frac{V^2}{2}\frac{dC}{d\theta} \tag{8B.9c}$$

i.e. in the force equation, replace *F*, *x* with *T*, θ.

If fringing is ignored:

$$C = \frac{\varepsilon_0 A}{b} = \frac{\varepsilon_0}{b}\frac{r^2\theta}{2} \tag{8B.10}$$

$$\frac{dC}{d\theta} = \frac{\varepsilon_0}{b}\frac{r^2}{2} = 2K_d$$

The electrical deflecting torque is:

$$T_d = K_d V^2 \tag{8B.11}$$

The meter responds to the square of the voltage.

To operate as a meter, the movement of the plate is restrained by a spring that has a constant torsional restraint:

$$T_r = K_r \alpha \tag{8B.12}$$

When the two opposing torques balance, the needle will be steady on the scale, and we know that:

$$\begin{aligned} T_r &= T_d \\ K_r \alpha &= K_d V^2 \\ \alpha &= K V^2 \end{aligned} \tag{8B.13}$$

The meter therefore has a square law (nonlinear) scale.

References

Slemon, G. and Straughen, A.: *Electric Machines*, Addison-Wesley Publishing Company, Inc., Sydney, 1982.

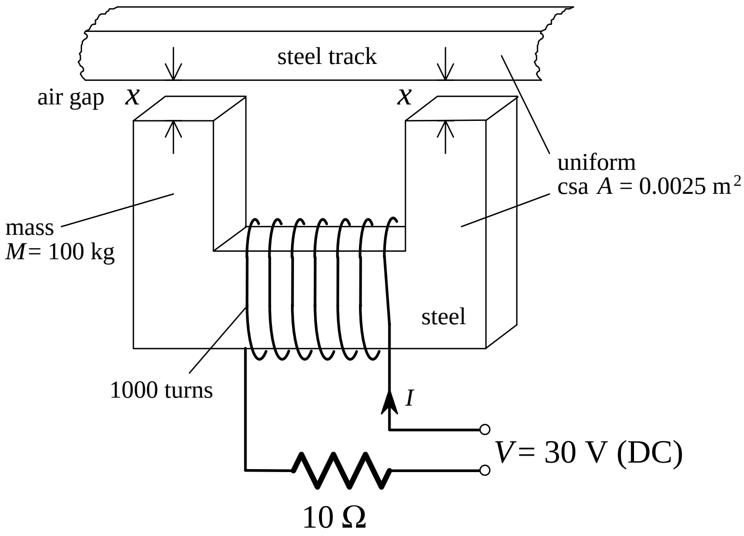
8B.8

Problems

1. [Magnetic suspension]

The device shown in the diagram consists of a steel track and a U-shaped bottom piece both of the same magnetic material, which can be considered ideal.

The bottom piece is prevented from falling by the magnetic field produced by current *I*.



(a) Draw the equivalent magnetic circuit and obtain an expression for the force *F* holding the bottom piece in terms of *x* (assume $\mu_{steel} = \infty$).

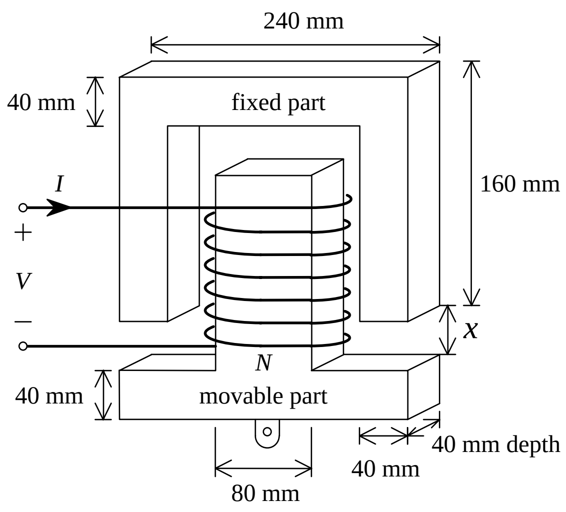
(b) At what value of *x* is *M* supported? Is the position stable? Sketch *F*~*x*. Give full explanations.

(c) How would the results of (a) and (b) be affected if fringing were considered?

2.

The steel magnetic actuator shown below is excited by DC, with $I = 2\ \text{A}$ and $N = 5000$ turns. The three gaps have the same length, x.

The mass density of the movable part is $7800\ \text{kgm}^{-3}$.



The movable part is at a position for which $B_{steel} = 1.5\ \text{T}$.

(a) Draw the magnetic equivalent circuit, assuming $\mu_{steel} = \infty$.

(b) Determine the air gap length, x, and the force exerted by the actuator.

(c) Determine the mass m (excluding actuator) which can be lifted against gravity.

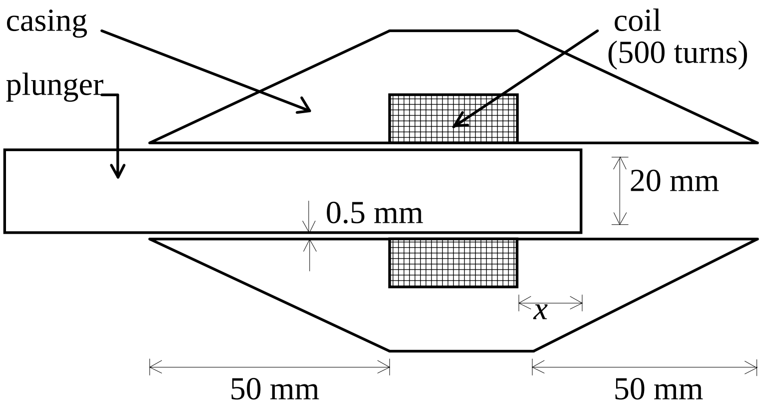
(d) Calculate the minimum current I needed to lift the unloaded actuator.

(e) Determine the initial acceleration of the unloaded actuator if it is released when $I = 0.3\ \text{A}$.

3.

The magnetic actuator shown in the diagram has a cylindrical casing and a cylindrical plunger, both of the same magnetic material, which saturates at a flux density $B = 1.6$ T. The plunger is free to move within the casing. The gap between the two is 0.5 mm. The diameter of the plunger is 20 mm.

The device is excited by a coil with $N = 500$ turns and carrying current I and is capable of producing a small force over a large distance.

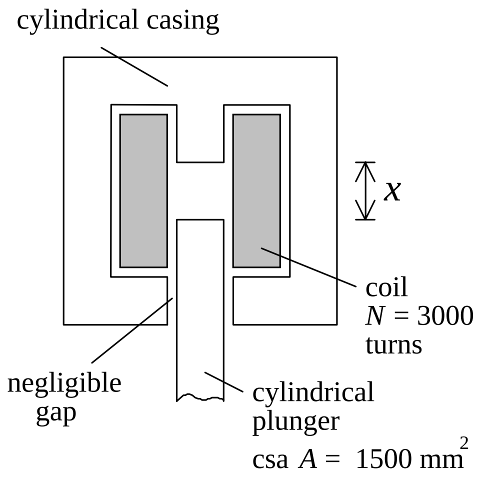


Determine the maximum coil current if B is not to exceed 1.6 T.

(Neglect leakage and fringing flux and assume the magnetic material to be perfect up to saturation).

4.

A magnetic actuator is constructed as shown:



The cylindrical plunger is free to move vertically. The coil resistance is $R = 8\,\Omega$. A 12 V DC supply is connected to the coil. The magnetic material is assumed perfect up to its saturation flux density of 1.6 T.

(a) Determine the x range over which the force on the plunger is essentially constant because the material is saturated.

(b) Determine the mechanical energy produced when x varies slowly from 10 mm to 0 mm.

(c) The plunger is allowed to close so quickly (from $x = 10\,\text{mm}$) that the change in flux linkage is negligible. Calculate the mechanical energy produced.

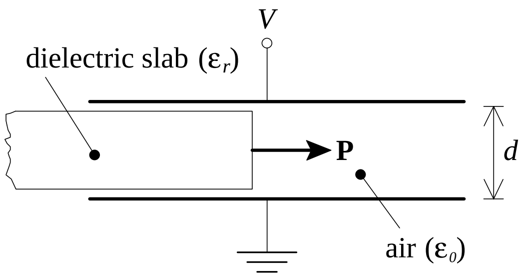
5.

A capacitor is formed by two parallel plates of csa $A = 0.02\ \text{m}^2$, x m apart. The dielectric is air. The plates are kept at a constant potential difference $V = 10\,\text{kV}$. Derive an expression for the force F between the plates as a function of x, and determine the energy converted to mechanical form as x is reduced from 10 mm to 5 mm.

8B.12

6.

A rectangular block of dielectric material (relative permittivity ε_r) is partially inserted between two much larger parallel plates.



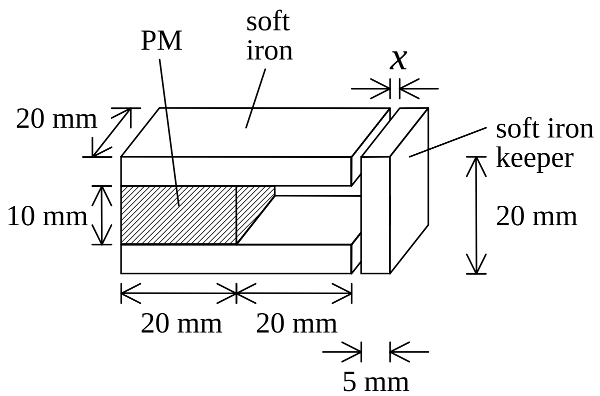
Show that the magnitude of the pressure on the RHS of the slab is:

$$P = \frac{\varepsilon_0(\varepsilon_r - 1)V^2}{2d^2}$$

Ignore fringing and leakage flux.

7.

A PM assembly to be used as a door holder was introduced in question 4B.5:



(a) Use the circuit derived in question 4B.5 (a) to obtain an expression for the force acting on the keeper as a function of x.

(b) Determine this force when x = 1 mm.