Lecture 10A - Frequency Response

The amplifier block. Voltage and current amplifiers. Maximum power transfer. The decibel (dB). Frequency response of capacitively coupled circuits.

The Amplifier Block

The amplifier is a basic building block of analog electronic systems. Its response may be modelled without a knowledge of what is inside, which may change with technical innovation.

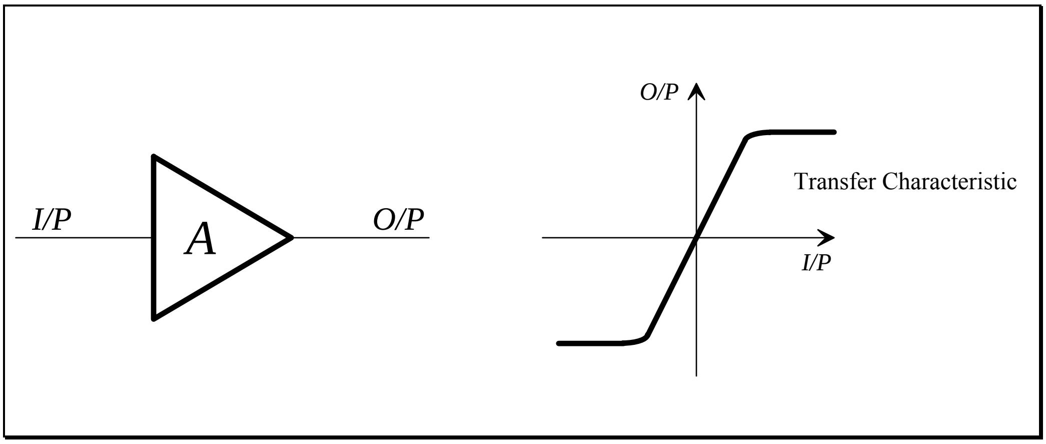

Figure 10A.1

If we treat an amplifier as a block, what do we have to know about it? The important parameters of an amplifier are:

- the open circuit gain A
- the input impedance Z_i
- the output impedance Z_o

To analyze the suitability of an amplifier in an application, we need to know the external circuit.

Voltage Amplifier

A voltage amplifier is used to amplify a voltage:

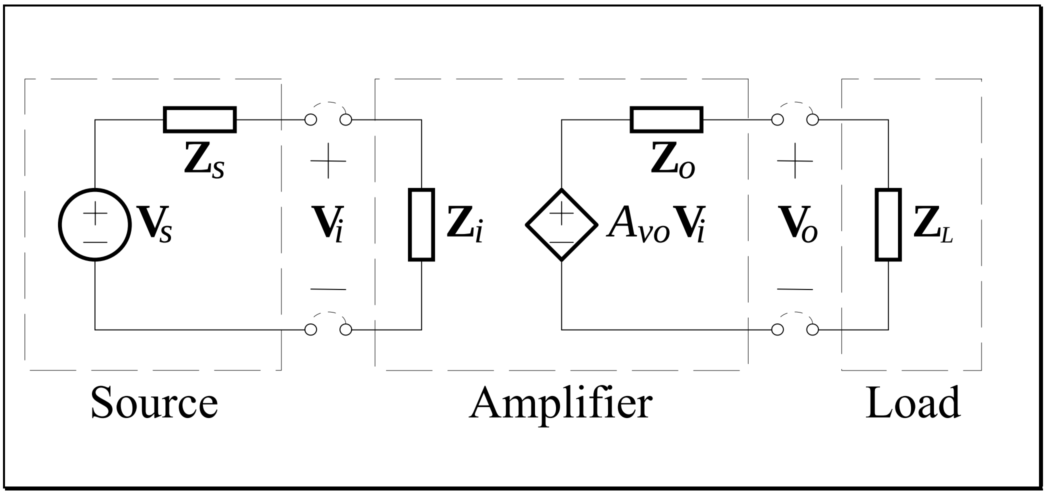

Figure 10A.2

where:

- A_{vo} = open circuit voltage gain
- $\mathbf{Z}_i$ = large
- $\mathbf{Z}_o$ = small

because:

$$\mathbf{V}_i = \frac{\mathbf{Z}_i}{\mathbf{Z}_i + \mathbf{Z}_s}\mathbf{V}_s \tag{10A.1a}$$

$$\mathbf{V}_L = \mathbf{V}_o = \frac{\mathbf{Z}_L}{\mathbf{Z}_L + \mathbf{Z}_o}A_{vo}\mathbf{V}_i \tag{10A.1b}$$

Current Amplifier

A current amplifier is used to amplify a current:

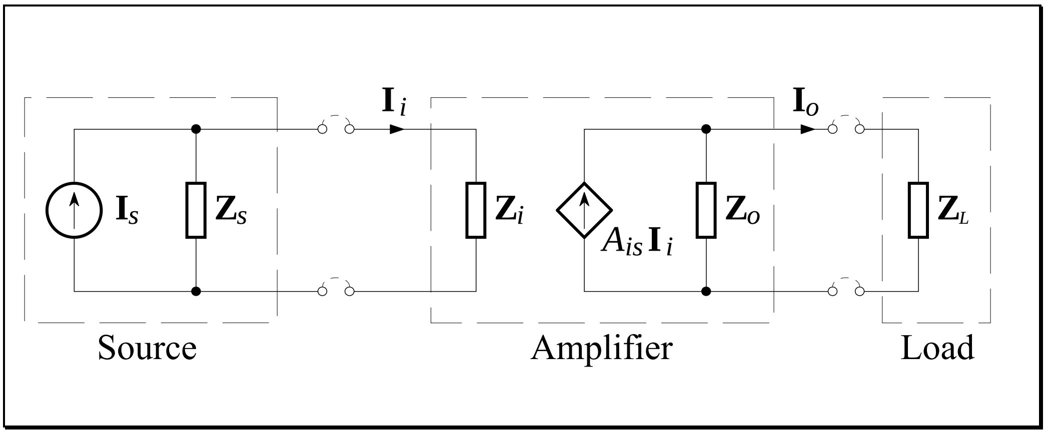

Figure 10A.3

where:

- A_{is} = short circuit current gain
- $\mathbf{Z}_i$ = small
- $\mathbf{Z}_o$ = large

because:

$$\mathbf{I}_i = \frac{\mathbf{Z}_s}{\mathbf{Z}_s + \mathbf{Z}_i} \mathbf{I}_s \tag{10A.2a}$$

$$\mathbf{I}_L = \mathbf{I}_o = \frac{\mathbf{Z}_o}{\mathbf{Z}_o + \mathbf{Z}_L} A_{is} \mathbf{I}_i \tag{10A.2b}$$

Maximum Power Transfer

Consider the following circuit:

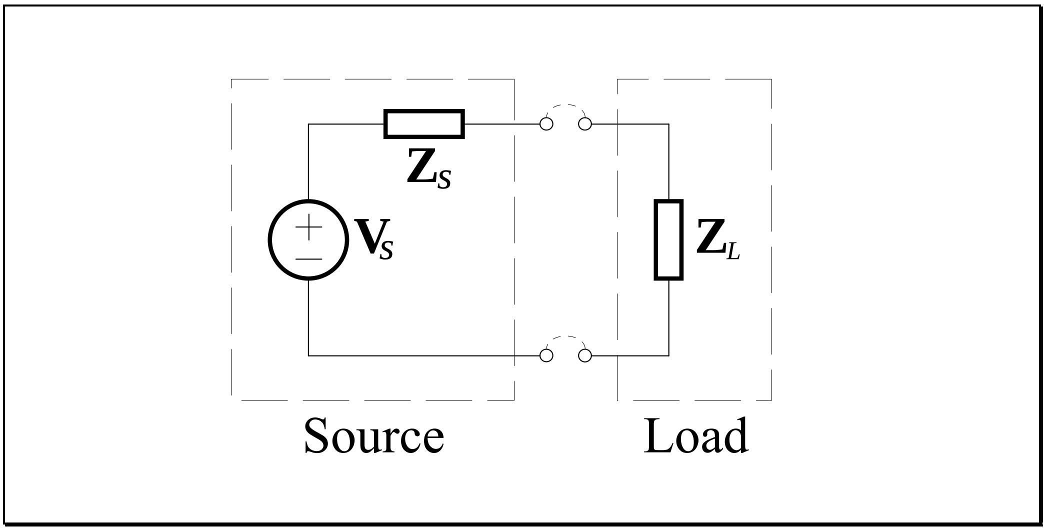

Figure 10A.4

To achieve maximum power in the load, what impedance do we choose? The load power is given by:

$$P_L = |\mathbf{I}_L|^2 R_L = \frac{|\mathbf{V}_s|^2}{|\mathbf{Z}_L + \mathbf{Z}_s|^2} R_L$$

$$= \frac{|\mathbf{V}_s|^2 R_L}{(R_L + R_s)^2 + (X_L + X_s)^2} \tag{10A.3}$$

To make the power as large as possible, make $(X_L + X_s)^2$ zero by choosing $X_L = -X_s$. We have seen before that to maximize power with just resistors, we choose $R_L = R_s$. In order to get maximum power to Z_L we select:

$$\mathbf{Z}_L = R_L + jX_L = R_s - jX_s = \mathbf{Z}_s^* \tag{10A.4}$$

The maximum power is therefore:

$$P_{L(\max)} = \frac{|\mathbf{V}_s|^2}{4R_L} \tag{10A.5}$$

Power matching is used in three situations:

- where the signal levels are very small, so any power lost gives a worse signal to noise ratio. e.g. in antenna to receiver connections in television, radio and radar.
- high frequency electronics
- where the signal levels are very large, where the maximum efficiency is desirable on economic grounds. e.g. a broadcast antenna.

The Decibel (dB)

Historically the Bel (named after Alexander Graham Bell – the inventor of the telephone) was used to define ratios of audio loudness i.e. ratios of power:

$$\text{Power gain (B)} = \log_{10} \frac{P_o}{P_i} \tag{10A.6}$$

Through the use of the metric system, a convenient unit to use was the decibel (dB):

$$\text{Power gain (dB)} = 10\log_{10} \frac{P_o}{P_i} \tag{10A.7}$$

The previous equation can also express the relationship between output power and input power for an amplifier. If we know what resistances the input and output power are dissipated in, and the voltages across them, then we can write:

$$\begin{aligned}\text{Power gain (dB)} &= 10\log_{10}\frac{\left|\mathbf{V}_o\right|^2 / R_0}{\left|\mathbf{V}_i\right|^2 / R_i} \\ &= 20\log_{10}\left|\frac{\mathbf{V}_o}{\mathbf{V}_i}\right| + 10\log_{10}\frac{R_i}{R_o}\end{aligned} \tag{10A.8}$$

In any circuits where $R_i = R_o$, then, and *only* then:

$$\begin{aligned}\text{Power gain (dB)} &= 20\log_{10}\left|\frac{\mathbf{V}_o}{\mathbf{V}_i}\right| \\ &= 20\log_{10}(\text{voltage gain})\end{aligned} \tag{10A.9}$$

In many systems (e.g. communication systems), this relationship was applicable, so eventually it became customary to express *voltage gain* in terms of the decibel.

The dB unit of power gain is useful when circuits are cascaded – you can add the voltage gains in dB instead of multiplying the standard voltage gains.

The frequency response curves of circuits are simple when the gain in decibels is plotted against frequency on a logarithmic scale, and again they are easily added when circuits are connected in cascade.

Frequency Response of Capacitively Coupled Circuits

The transistor amplifier looked at so far contains capacitors to couple the source and load to the amplifier. The amplifier performance is therefore dependent upon the frequency of the signal source. If we examine what happens to one of the amplifier's parameters when the frequency is varied, then we are looking at the frequency response of the parameter. The parameter we are most interested in at this stage is the gain of the amplifier.

Consider the output circuit of a voltage amplifier:

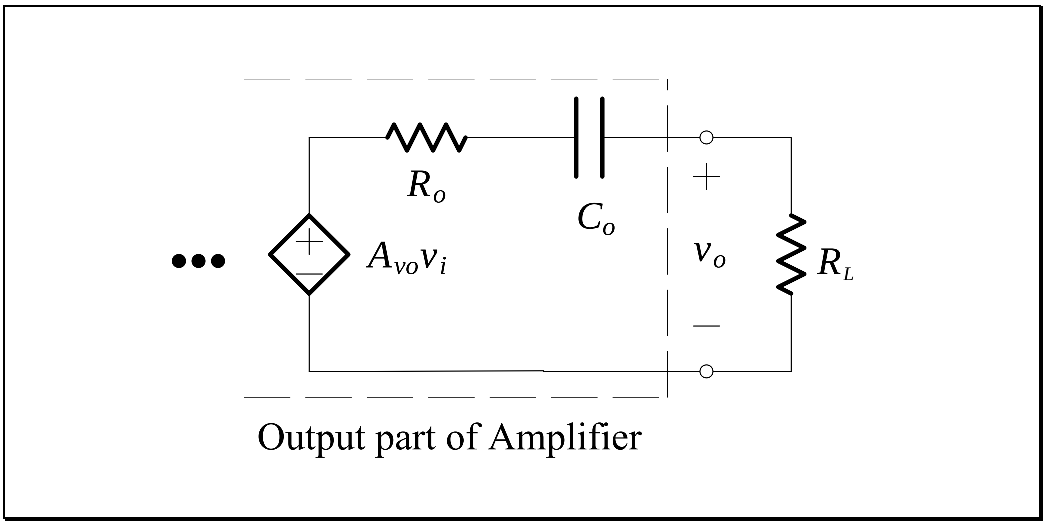

Figure 10A.5

The expression for the gain (as a function of frequency) can be derived:

$$\mathbf{V}_o = R_L\mathbf{I}_L = R_L \frac{A_{vo}\mathbf{V}_i}{(R_o + R_L) + 1/j\omega C_o}$$

$$\therefore \text{Gain } \mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = A_{vo} \frac{j\omega C_o R_L}{1 + j\omega C_o (R_o + R_L)} \qquad \text{(10A.10)}$$

Lets see how the expression for gain changes with the frequency. We will consider three cases.

Mid-Frequencies

The mid-frequency is chosen by the amplifier designer, and the values of the capacitors are chosen to suit. At mid-frequencies, we assume the capacitor has been chosen so that its reactance is very small compared to the resistors.

$$\left(R_o + R_L\right) >> \frac{1}{\omega C_o} \tag{10A.11a}$$

$$\text{Mid-frequency gain } A_{vM} = A_{vo}\frac{R_L}{R_o + R_L} \tag{10A.11b}$$

Corner Frequency

At a frequency ω_o such that:

$$\left(R_o + R_L\right) = \frac{1}{\omega_o C_o} \tag{10A.12}$$

the gain is:

$$\begin{aligned}\text{Gain } \mathbf{A}_v &= A_{vo}\frac{R_L}{\left(R_o + R_L\right) + \left(R_o + R_L\right)/j} \\ &= A_{vo}\frac{R_L}{\left(R_o + R_L\right)}\frac{1}{1-j} \\ &= \frac{A_{vM}\left(1+j\right)}{2} = \frac{A_{vM}}{\sqrt{2}}\angle 45^\circ\end{aligned} \tag{10A.13}$$

The gain is 70.7% of the mid-frequency gain, and the output leads the input by 45°.

Low Frequencies

The third region of interest is at very low frequencies, well below the corner frequency. At these frequencies:

$$\left(R_o + R_L\right) << \frac{1}{\omega C_o}$$

$$\begin{aligned} \text{Gain } \mathbf{A}_v &= A_{vo} \frac{R_L}{1/j\omega C_o} = jA_{vo}\omega R_L C_o \\ &= jA_{vM}\omega C_o\left(R_o + R_L\right) \\ &= jA_{vM}\frac{\omega}{\omega_o} = A_{vM}\frac{\omega}{\omega_o}\angle 90^\circ \end{aligned} \tag{10A.14}$$

At very low frequencies, the gain is proportional to the frequency. At DC the gain is zero, as expected.

At high frequencies, the frequency response of the whole amplifier must be considered. The input circuit, stray "feedback" capacitance, and capacitance internal to any transistors would have to be considered. The overall effect is a reduction in the gain, making it difficuolt to design amplifiers for high frequency operation.

Graphical Analysis

For graphical analysis, we plot the magnitude of the gain versus frequency and the phase of the gain versus frequency. These two plots together constitute a Bode* plot. We normally use logarithmic scales for each axis. For example:

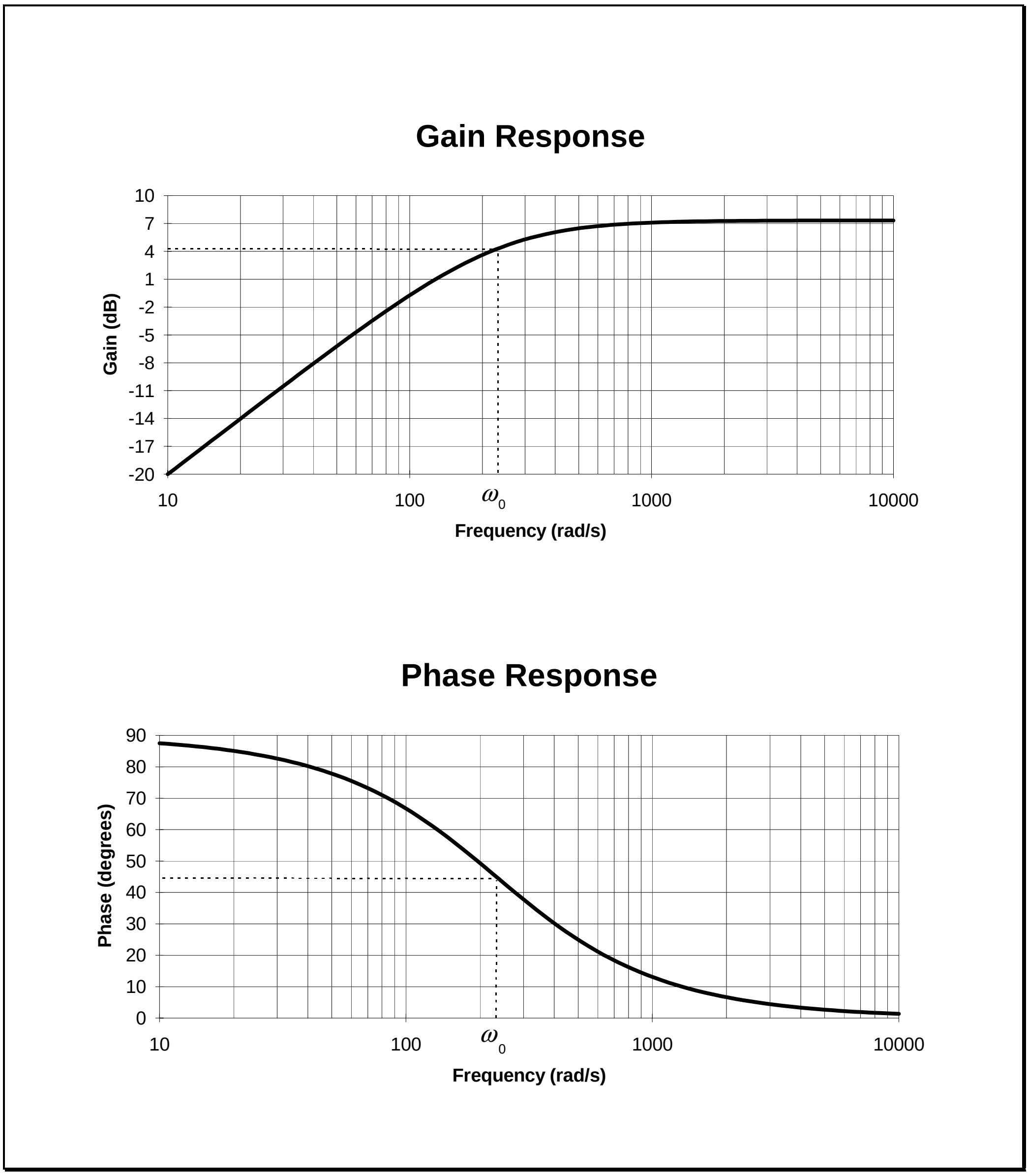

Figure 10A.6

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* Dr. Hendrik Bode grew up in Urbana, Illinois, USA, where his name is pronounced *boh-dee*. Purists insist on the original Dutch *boh-dah*. No one uses *bohd*.

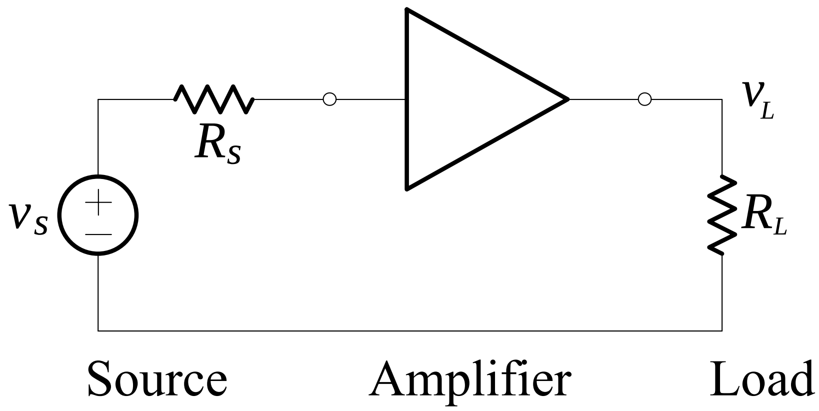
References

Sedra, A. and Smith, K.: *Microelectronic Circuits*, Saunders College Publishing, New York, 1991.

Problems

1.

The following circuit is used to perform tests on a voltage amplifier:



The following results were obtained:

v_S (volts RMS)	R_S (kΩ)	v_L (volts RMS)	R_L (kΩ)
0.1	0	8	∞
0.1	0	4	1
0.1	1000	2	1

Determine the amplifier's input and output resistance and its open-circuit voltage gain A_{vo}.

Draw the equivalent circuit.

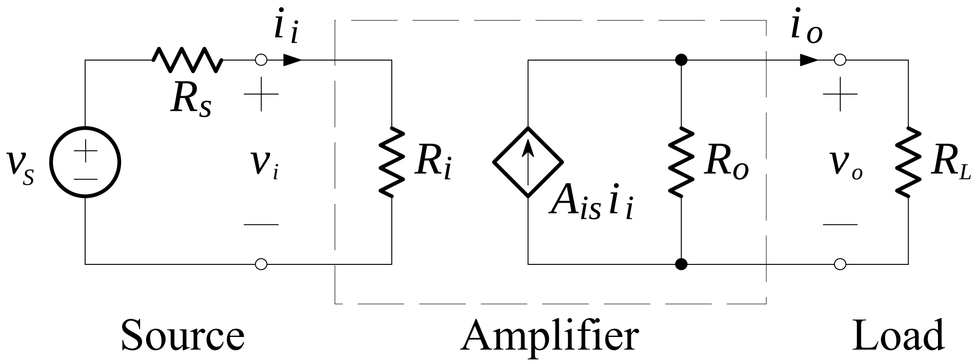
2.

A MOSFET is used in a single-stage common-source voltage amplifier that has a gain of 40 with a load resistor of $40\ \mathrm{k\Omega}$. If the load resistance is halved, the voltage gain drops to 30.

Determine the output resistance and the mutual conductance of the transistor.

3.

The diagram shows the Norton equivalent circuit of an amplifier:



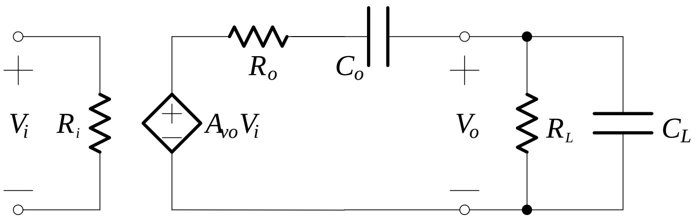
Derive an expression for the output current i_o in terms of $A_{is}i_i$, R_o and R_L.

If i_o is to be within $\pm 1\%$ of $A_{is}i_i$, what is the relation between R_o and R_L?

Determine $A_{is}i_i$ in terms of A_{vo}, v_i and R_o.

4.

For the following circuit:



$A_{vo} = 100$
$R_o = 1\text{ k}\Omega \quad C_o = 16\ \mu\text{F}$
$R_L = 9\text{ k}\Omega \quad C_L = 180\text{ pF}$

Determine the magnitude of the gain, $A_v = V_o/V_i$ at 1 Hz, 1 kHz and 1 MHz.

What is the amplifier's bandwidth between half-power points?

5.

A MOSFET is used in a single-stage common-source voltage amplifier with a load resistance of $45\text{ k}\Omega$. When the load resistor is halved, the voltage gain reduces to 91% of its original value.

Determine the voltage gain of the circuit, for both loads, if $g_m = 6\text{ mS}$.