Lecture 10B – Bridges and Measurements

General bridge equations. Measurement of resistance, inductance and capacitance. Average and RMS values of periodic waveforms.

General Bridge Equations

The general bridge is constructed as follows:

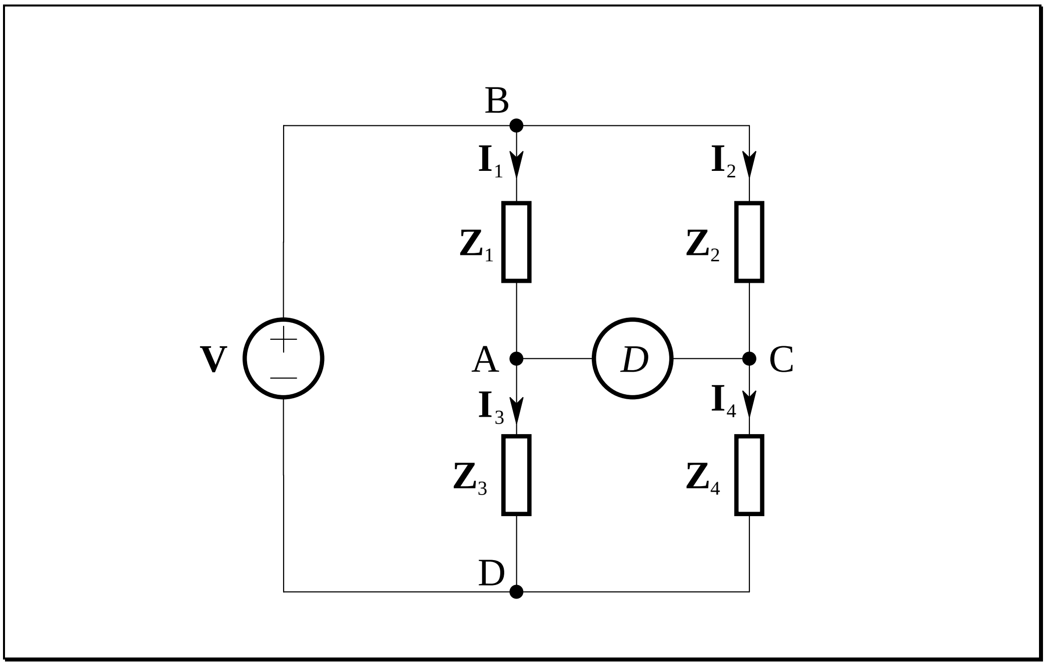


Figure 10B.1

The circuit is easy to analyze once there is no deflection in the detector *D*. When the circuit is in this state, it is said to be balanced, and:

$$\mathbf{V}_{AD} = \mathbf{V}_{CD}, \quad \mathbf{I}_1 = \mathbf{I}_3, \quad \mathbf{I}_2 = \mathbf{I}_4 \tag{10B.1}$$

Therefore:

$$\mathbf{Z}_1\mathbf{I}_1 = \mathbf{Z}_2\mathbf{I}_2 \quad \text{and} \quad \mathbf{Z}_3\mathbf{I}_1 = \mathbf{Z}_4\mathbf{I}_2 \tag{10B.2}$$

giving:

$$\frac{\mathbf{Z}_1}{\mathbf{Z}_2} = \frac{\mathbf{Z}_3}{\mathbf{Z}_4} \tag{10B.3}$$

Measurement of Resistance

Wheatstone bridge

The Wheatstone bridge (invented by Samuel Hunter Christie but popularized by Charles Wheatstone) is the simplest of all bridges, and is the most widely used method for the precision measurement of resistance. The Wheatstone bridge consists of four resistance arms, together with a source of current (a battery) and a detector (a galvanometer). The circuit is shown below:

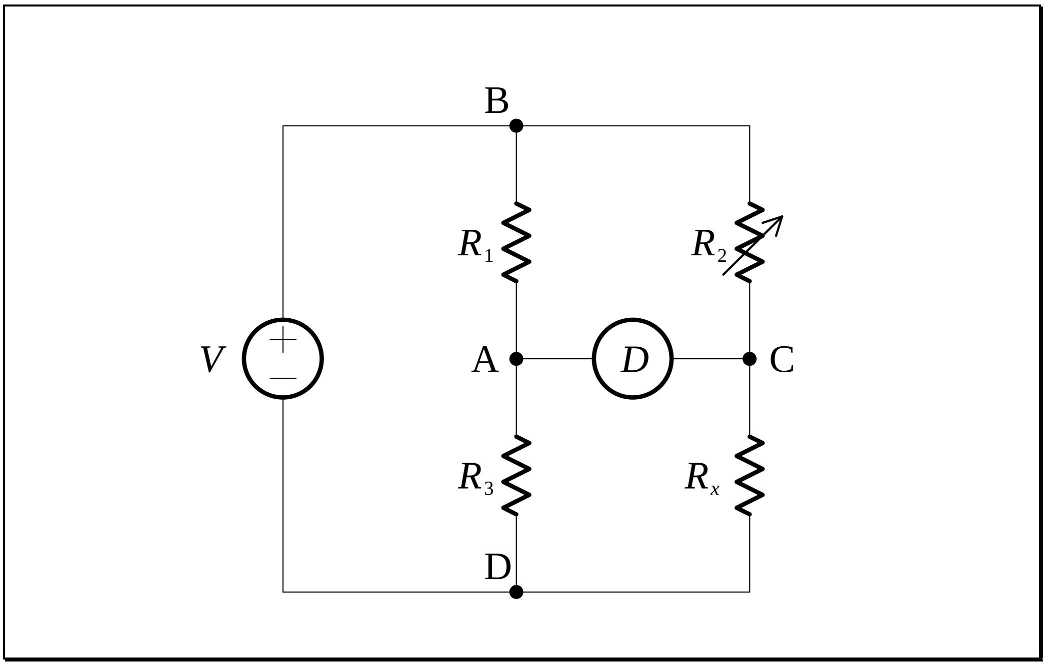


Figure 10B.2

It is used to measure resistance from 1 Ω to 10 MΩ. It has a low sensitivity for low values of resistance.

All the impedances of the general bridge are resistors, with R_2 being variable. R_1 and R_3 are at a fixed ratio. At balance:

$$R_x = \frac{R_3}{R_1} R_2 \tag{10B.4}$$

Kelvin bridge

The Kelvin bridge may be regarded as a modification of the Wheatstone bridge to secure increased accuracy in the measurement of low resistance. It is used to measure resistance from 100 nΩ to 1 Ω.

An understanding of the Kelvin arrangement may be obtained by a study of the difficulties that arise in a Wheatstone bridge in the measurement of resistances that are low enough for the resistance of leads and contacts to be appreciable in comparison. Consider the bridge shown below, where P represents the resistance of the lead that connects from R_2 to R_x:

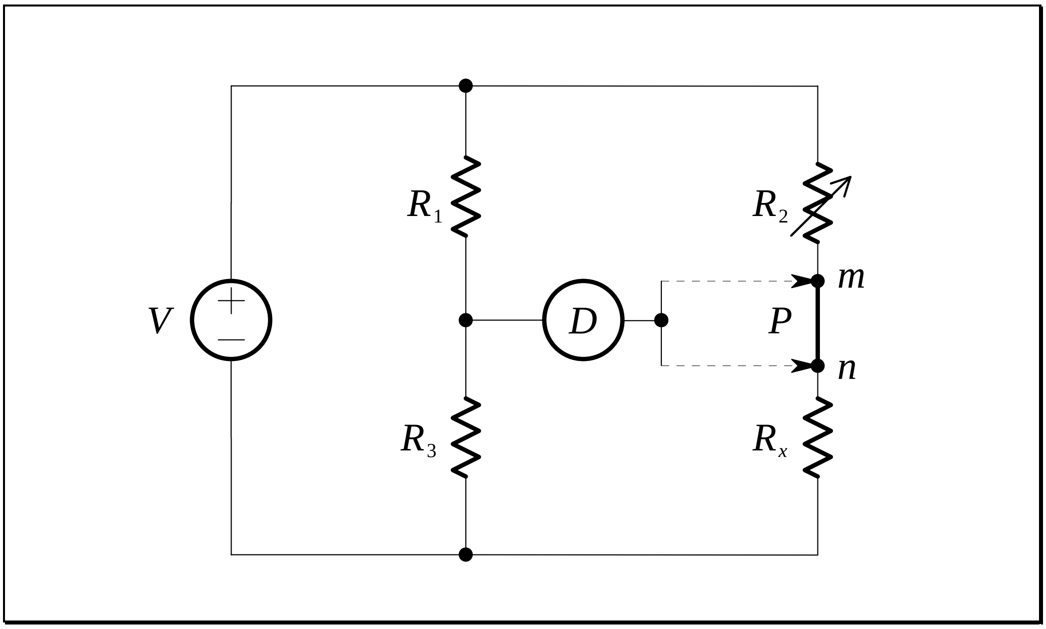


Figure 10B.3

Two possible connections for the galvanometer are indicated by the dotted lines. With connection to *m*, P is added to R_x, so that the computed value of the unknown is higher than R_x alone, if P is appreciable in comparison with R_x. On the other hand, if connection is made to *n*, R_x is in fact computed from the known value of R_2 only, and is accordingly lower than it should be.

10B.4

Suppose that instead of using point *m*, which gives a *high* result, or *n*, which makes the result *low*, we can slide the galvanometer connection along to any desired intermediate point, as shown below:

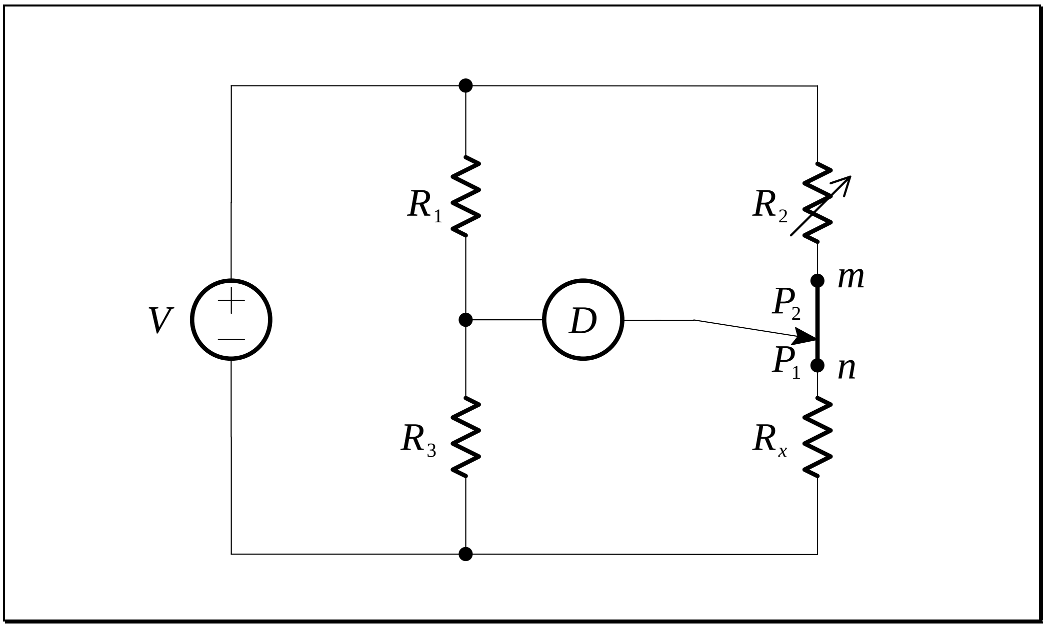


Figure 10B.4

From the usual balance relationship, we can write:

$$R_x + P_1 = \frac{R_3}{R_1}\left(R_2 + P_2\right) \tag{10B.5}$$

$$R_x = \frac{R_3}{R_1} R_2 + \frac{R_3}{R_1} P_2 - P_1$$

If the resistance of *P* is divided into two parts such that

$$\frac{P_1}{P_2} = \frac{R_3}{R_1} \tag{10B.6}$$

then the presence of P *causes no error* in the result, since substituting Eq. (10B.6) into Eq. (10B.5) gives:

$$R_x = \frac{R_3}{R_1} R_2 \tag{10B.7}$$

The final balanced condition has the same formula as the Wheatstone bridge.

The process described here is obviously not a practical way of achieving the desired result, as we would have trouble in determining the correct point for the galvanometer connection. It does, however, suggest the simple modification that we connect two actual resistance units of the correct ratio between points *m* and *n*, and connect the galvanometer to their junction. This is the Kelvin bridge arrangement, shown below:

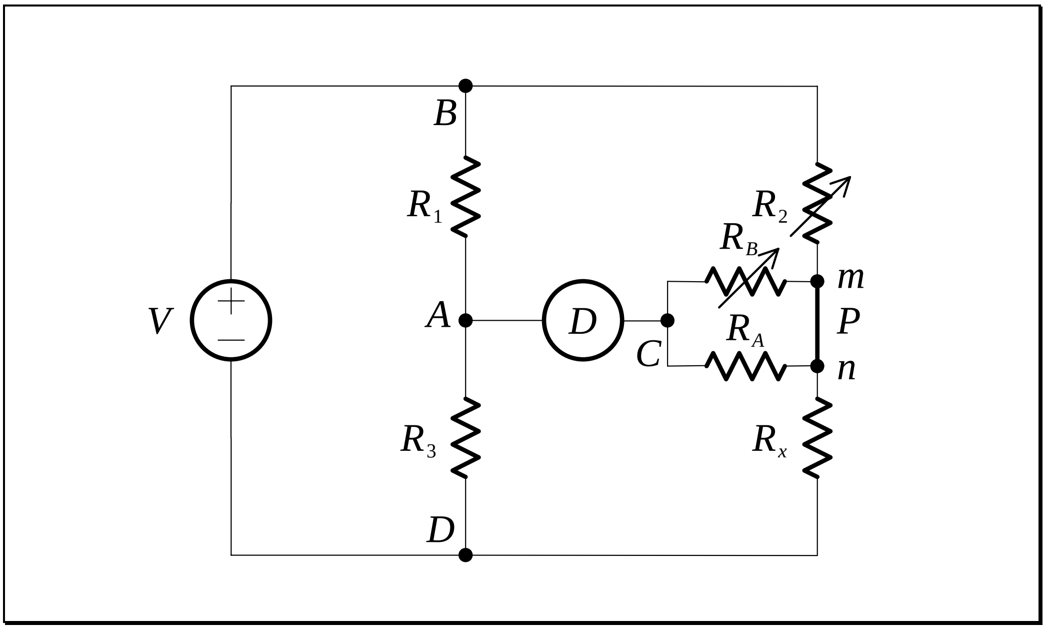


Figure 10B.5

To operate the bridge, a balance is performed in the normal way. Then the link P is removed to see whether the detector still indicates balance. If it is out of balance then resistor R_B is varied to balance the resulting Wheatstone bridge (R_2 and R_x are negligible in comparison to R_A and R_B in this case).

10B.6

With the link P in place, the balance condition gives $V_{BA} = V_{BmC}$. The voltage V_{BA} is given by:

$$\begin{aligned} V_{BA} &= \frac{R_1}{R_1 + R_3} V \\ &= \frac{R_1}{R_1 + R_3} I_2 \left(R_2 + R_x + \frac{(R_A + R_B)P}{R_A + R_B + P} \right) \end{aligned} \tag{10B.8}$$

since no current exists in the galvanometer branch. Similarly, using the current divider rule, we get:

$$V_{BmC} = I_2 \left(R_2 + R_B \frac{P}{R_A + R_B + P} \right) \tag{10B.9}$$

If these two values are equated and R_x made the subject, the result is:

$$R_x = \frac{R_3}{R_1} R_2 + \frac{R_B P}{R_A + R_B + P} \left(\frac{R_3}{R_1} - \frac{R_A}{R_B} \right) \tag{10B.10}$$

Now, if $R_3/R_1 = R_A/R_B$, the equation becomes:

$$R_x = \frac{R_3}{R_1} R_2 \tag{10B.11}$$

This indicates that the resistance of the connection P (which carries most of the current) has no effect, provided that the two sets of ratio arms have equal ratios.

Four-Terminal Resistor

One complication in the construction of the Kelvin bridge is the fact that in actual practice the resistance P includes not only the ohmic resistance of the connecting wire, but also the contact resistance between wire and binding post. Contact resistance is a variable and uncertain element, as it depends on such things as the cleanness of the surfaces and the amount of pressure between them. This uncertainty can be removed by using a *four-terminal* resistor:

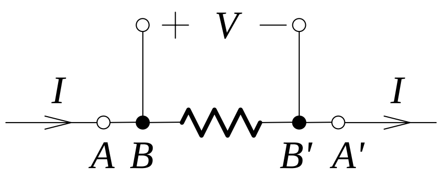


Figure 10B.6

One pair of terminals, AA', is used to lead the current to and from the resistor. The voltage drop is measured between the other pair of terminals, BB'. The voltage V is thus I times the resistance from B to B', and does not include any contact drop that may be present at terminals A and A'.

Resistors of low values are measured in terms of the resistance between the potential terminals, which thus becomes perfectly definite in value and independent of contact drop at the current terminals. (Contact drop at the potential terminals need not be a source of error, as the current crossing these contacts is usually extremely small – or even zero for null methods.)

Measurement of Inductance

There are a few different bridges to measure inductance. The bridges not only measure the inductance of a real inductor, they also measure the resistance associated with a real inductor.

Maxwell bridge

The Maxwell bridge consists of the following arrangement:

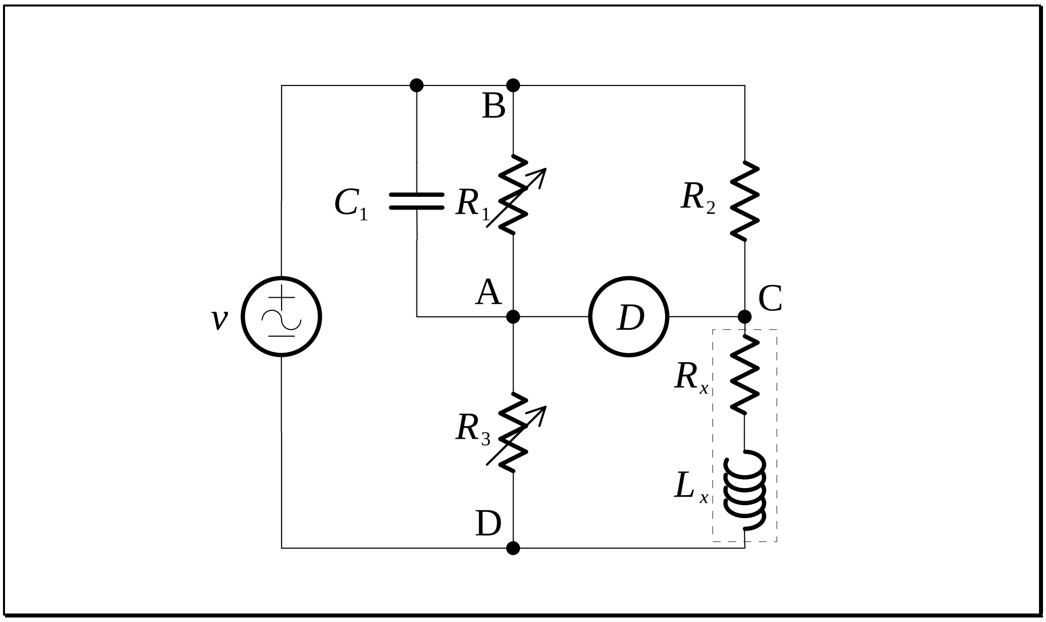


Figure 10B.7

At balance, R_x and L_x do not depend upon the frequency of the AC supply, thus eliminating a possible source of error. Another advantage is that it permits measurement of inductance in terms of capacitance. A capacitor can be made to have a more precise value than an inductor, since they effectively have no external field, are more compact, and are easier to shield.

A disadvantage is that it requires inconvenient large resistors to measure high Q coils, and balancing R_x and L_x is iterative.

Hay bridge

The Hay bridge is similar to the Maxwell bridge, except the capacitor has a series resistance, instead of a parallel resistance:

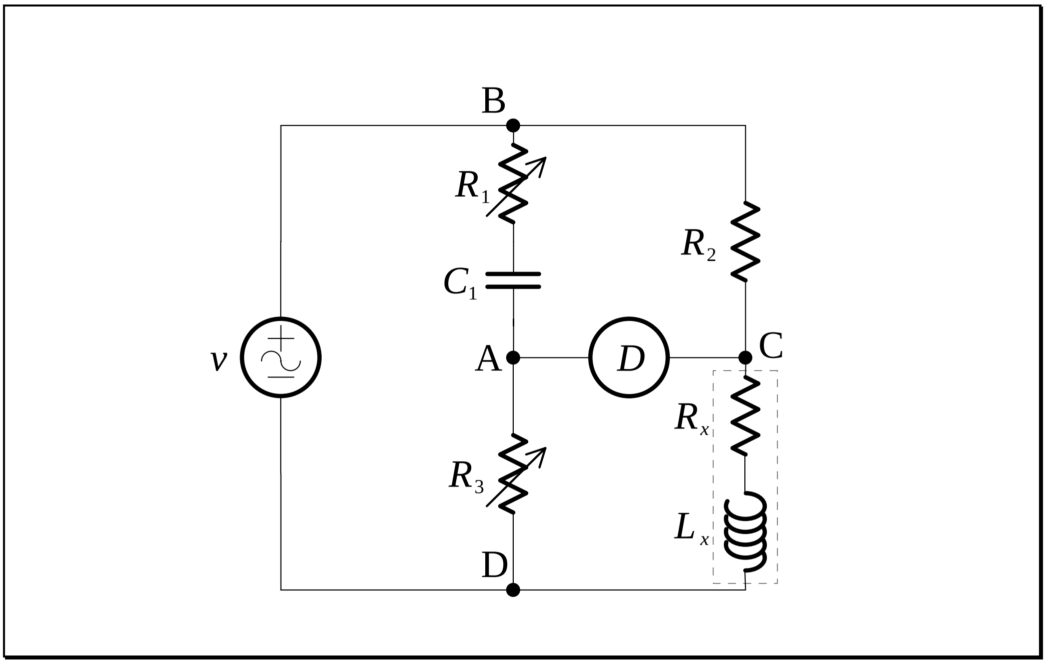


Figure 10B.8

This gives more convenient values of resistance and better balancing for high Q coils. For high Q coils, the frequency dependence is not a serious concern, because the terms involving frequency are small. For low Q coils, the frequency is important, and it is better to use the Maxwell bridge.

Measurement of Capacitance

The measurement of capacitance is carried out by a comparison bridge. This means we compare the value of the unknown capacitance with a known capacitance:

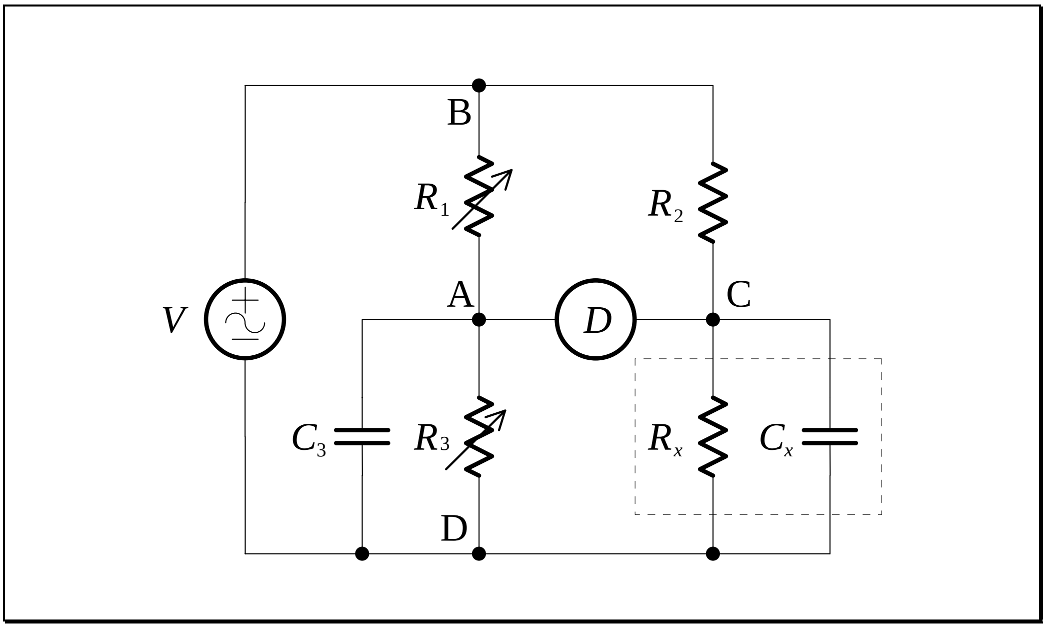


Figure 10B.9

Average and RMS Values of Periodic Waveforms

Consider a current waveform $i(t)$ of period T. We define:

- Average or mean:

$$I_{AV} = \frac{1}{T}\int_0^T i(t)dt \tag{10B.12}$$

- RMS (root of mean of squares) or effective value (DC value producing same energy in time T):

$$W = \int_0^T p dt = \int_0^T Ri^2 dt = RI_{RMS}^2 T$$

$$I_{RMS} = \sqrt{\frac{1}{T}\int_0^T i^2 dt} \tag{10B.13}$$

- Form factor and crest factor:

$$ff = \frac{RMS}{AV} \tag{10B.14a}$$

$$cf = \frac{\text{peak}}{RMS} \tag{10B.6b}$$

Any periodic wave can be decomposed into a sum of sine waves, with different amplitude and phase. This is a Fourier series:

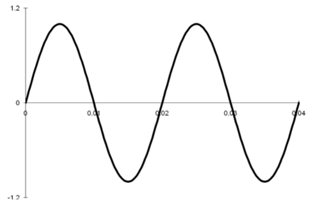
$$i(t) = I_{AV} + \sum_{n=1}^{\infty} \hat{I}_n \cos(n\omega_0 t + \theta_n) \tag{10B.15}$$

$$I_{RMS} = \sqrt{I_{AV}^2 + \sum_{n=1}^{\infty} I_{nRMS}^2} \tag{10B.7b}$$

10B.12

Examples

(i)



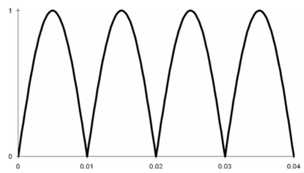
$$I_{AV} = 0$$

$$I_{RMS} = \sqrt{\frac{1}{T}\int_0^T \left(\hat{I}\sin\omega t\right)^2 dt}$$

$$= \hat{I}\sqrt{\frac{1}{2T}\int_0^T (1-\cos 2\omega t)dt}$$

$$= \frac{\hat{I}}{\sqrt{2}}$$

(ii)



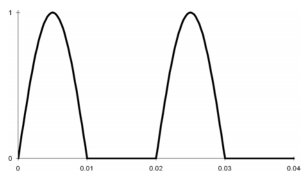
$$I_{AV} = \frac{\hat{I}}{T/2}\int_0^{T/2}\sin\omega t dt$$

$$= \frac{\hat{I}\omega}{\pi}\left[-\frac{\cos\omega t}{\omega}\right]_0^{\pi/\omega}$$

$$= \frac{2\hat{I}}{\pi}$$

$I_{RMS} = I_{RMS}$ sine wave, $\quad ff = \frac{\pi}{2\sqrt{2}} = 1.11$

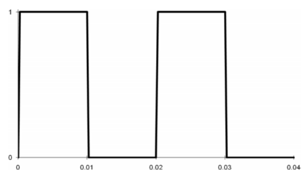
(iii)



$$I_{AV} = \frac{\hat{I}}{\pi}$$

$$I_{RMS} = \frac{\hat{I}}{2}$$

(iv)

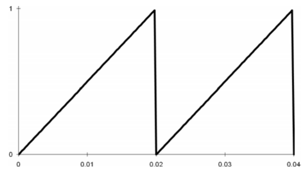


$$I_{AV} = \frac{\hat{I}}{2}$$

$$I_{RMS} = \frac{\hat{I}}{\sqrt{2}}$$

(v)

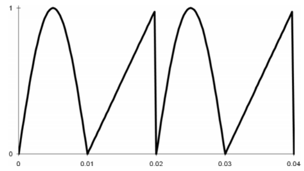
$$I_{AV} = \frac{\hat{I}}{2}$$



$$I_{RMS} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{\hat{I}}{T}t\right)^2 dt} = \frac{\hat{I}}{\sqrt{3}}$$

(vi)

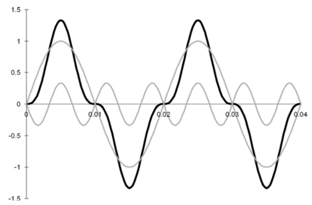
$$I_{AV} = \hat{I}\left(\frac{1}{\pi} + \frac{1}{4}\right)$$



$$I_{RMS} = \hat{I}\sqrt{\frac{1}{4} + \frac{1}{6}}$$

(vii)

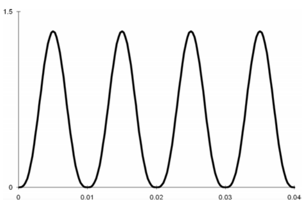
$$I_{AV} = 0$$



$$I_{RMS} = \sqrt{I_{1RMS}^2 + I_{3RMS}^2}$$
$$= \sqrt{\frac{\hat{I}_1^2}{2} + \frac{\hat{I}_3^2}{2}}$$

(vii)

$$I_{AV} = \frac{2\hat{I}_1}{\pi} + \frac{2\hat{I}_3}{3\pi}$$



$$I_{RMS} = \sqrt{I_{1RMS}^2 + I_{3RMS}^2}$$
$$= \sqrt{\frac{\hat{I}_1^2}{2} + \frac{\hat{I}_3^2}{2}}$$

References

Jones, D. and Chin, A.: *Electronic Instruments and Measurements*, John Wiley & Sons, Inc., New York, 1983.

Stout, M.: *Basic Electrical Measurements 2nd Ed.*, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1960.

Problems

1.

Derive the equation for the Kelvin bridge given by Eq. (10B.10).

2.

Derive equations for R_x and L_x in the Maxwell bridge.

3.

Derive equations for R_x and L_x in the Hay bridge.