1 Basic Circuits

Contents

1.2

Introduction

Electric circuit theory and electromagnetic theory are the two fundamental theories upon which all branches of electrical engineering are built. Many branches of electrical engineering, such as power, electric machines, control, electronics, communications, and instrumentation, are based on electric circuit theory. Circuit theory is also valuable to students specializing in other branches of the physical sciences because circuits are a good model for the study of energy systems in general, and because of the applied mathematics, physics, and topology involved.

Electronic circuits are used extensively in the modern world – society in its present form could not exist without them! They are used in *communication systems* (such as televisions, telephones, and the Internet), *digital systems* (such as personal computers, embedded microcontrollers, smart phones), and *industrial systems* (such as robotic and process control systems). The study of electronics is therefore critical to electrical engineering and related professions.

One goal in this subject is to learn various analytical techniques and computer software applications for describing the behaviour of electric circuits. Another goal is to study various uses and applications of electronic circuits.

We will start by revising some basic concepts, such as KVL, KCL and Ohm's Law. We will then introduce the concept of the electronic amplifier, and then study a device called an operational amplifier (op-amp for short), which has been used as the building block for modern analog electronic circuitry since its invention in the 1960's.

1.1 Current

Charge in motion represents a *current*. The current present in a discrete path, such as a metallic wire, has both a magnitude and a direction associated with it – it is a measure of the rate at which charge is moving past a given reference point in a specified direction. Current is symbolised by *i* and thus:

Current defined as the rate of change of charge moving past a reference

$$i = \frac{dq}{dt} \tag{1.1}$$

The unit of current is the ampere (A) and is equivalent to Cs^{-1}. In a circuit current is represented by an arrow:

Representation of current in a circuit

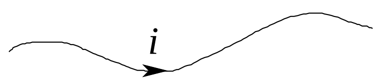


Figure 1.1

The arrow does not indicate the "actual" direction of charge flow, but is simply part of a convention that allows us to talk about the current in an unambiguous manner.

Correct usage of the term "current"

The use of terms such as "a current flows through the resistor" is a tautology and should not be used, since this is saying a "a charge flow flows through the resistor". The correct way to describe such a situation is "there is a current in the resistor".

DC and AC defined

A current which is constant is termed a direct current, or simply DC. Examples of direct currents are those that exist in circuits with a chemical battery as the source. A sinusoidal current is often referred to as alternating current, or AC[1]. Alternating current is found in the normal household electricity supply.

[1] Later we shall also see that a periodic current (e.g. a square wave), with no DC term, can also be referred to as an alternating current.

1.2 Voltage

A *voltage* exists between two points in a circuit when energy is required to move a charge between the two points. The unit of voltage is the volt (V) and is equivalent to JC^{-1}. In a circuit, voltage is represented by a pair of +/- signs:

Representation of voltage in a circuit

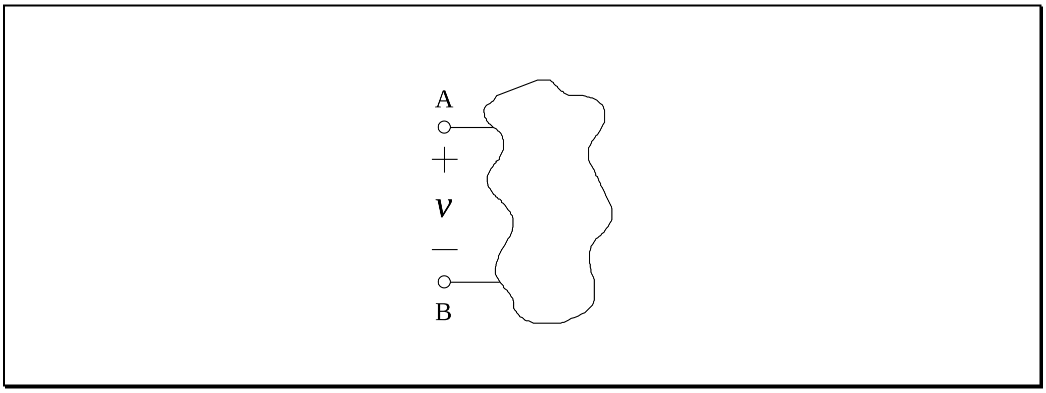
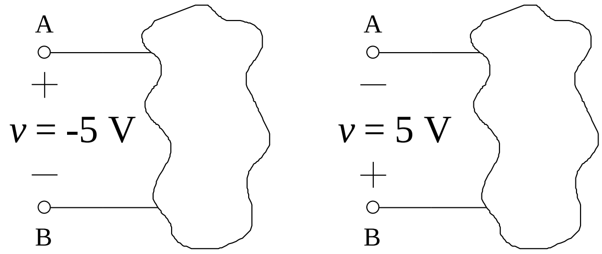


Figure 1.2

Once again, the plus-minus pair does not indicate the "actual" voltage polarity.

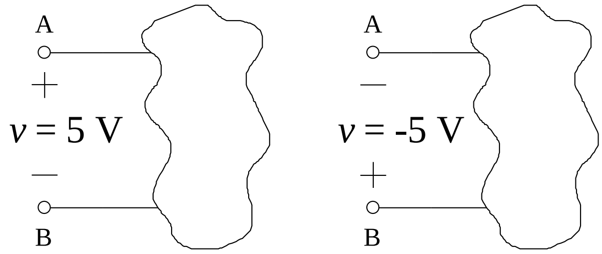
EXAMPLE 1.1 Voltage Polarity

Note the voltages across the circuit elements below:



(a) (b)

In both (a) and (b), terminal B is 5 V positive with respect to terminal A.



(c) (d)

In both (c) and (d), terminal A is 5 V positive with respect to terminal B.

1.3 Circuit Elements and Types of Circuits

A circuit element is an *idealised* mathematical model of a two-terminal electrical device that is completely characterised by its voltage-current relationship. Although ideal circuit elements are not "off-the-shelf" circuit components, their importance lies in the fact that they can be interconnected (on paper or on a computer) to approximate actual circuits that are composed of nonideal elements and assorted electrical components – thus allowing for the analysis of such circuits.

Ideal circuit elements are used to model real circuit elements

Circuit elements can be categorised as either *active* or *passive*.

1.3.1 Active Circuit Elements

Active circuit elements *can* deliver a non-zero average power indefinitely. There are four types of active circuit element, and all of them are termed an *ideal source*. They are:

Active circuit element defined

- the independent voltage source
- the independent current source
- the dependent voltage source
- the dependent current source

1.3.2 Passive Circuit Elements

Passive circuit elements *cannot* deliver a non-zero average power indefinitely. Some passive elements are capable of storing energy, and therefore delivering power back into a circuit at some later time, but they cannot do so indefinitely.

Passive circuit element defined

There are three types of passive circuit element. They are:

- the resistor
- the inductor
- the capacitor

1.3.3 Types of Circuits

The interconnection of two or more circuit elements forms an electrical *network*. If the network contains at least one closed path, it is also an electrical *circuit*. A network that contains at least one active element, i.e. an independent or dependent source, is an *active* network. A network that does not contain any active elements is a *passive* network.

Network and circuit defined

Active and passive circuits defined

1.4 Independent Sources

Independent sources are *ideal* circuit elements that possess a voltage or current value that is independent of the behaviour of the circuits to which they belong.

1.4.1 The Independent Voltage Source

Independent voltage source defined

An independent voltage source is characterised by a terminal voltage which is completely independent of the current through it. The representation of an independent voltage source is shown below:

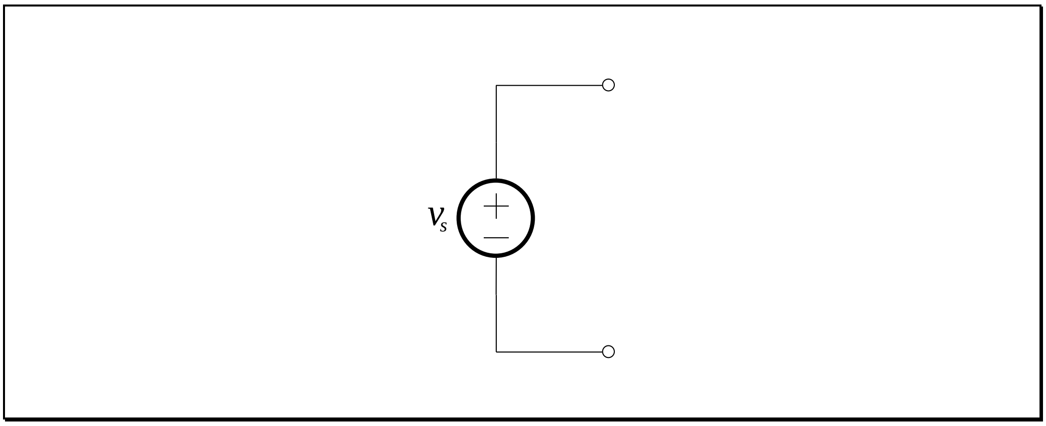


Figure 1.3

An ideal battery is equivalent to an independent voltage source that has a constant value

If the value of the voltage source is constant, that is, does not change with time, then we can also represent it as an *ideal battery*:

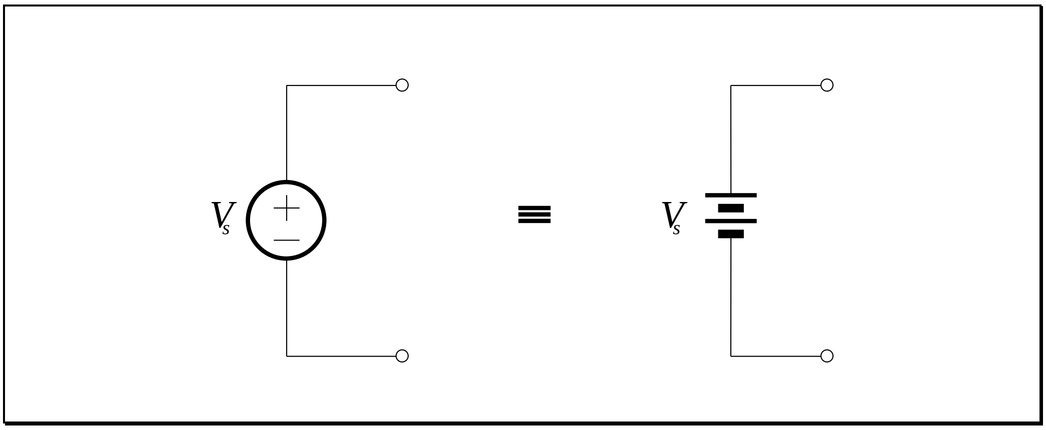


Figure 1.4

Although a "real" battery is not ideal, there are many circumstances under which an ideal battery is a very good approximation.

In general, however, the voltage produced by an ideal voltage source will be a function of time. In this case we represent the voltage symbolically as $v(t)$.

A few typical voltage waveforms are shown below. The waveforms in (a) and (b) are typical-looking amplitude modulation (AM) and frequency modulation (FM) signals, respectively. Both types of signals are used in consumer radio communications. The sinusoid shown in (c) has a wide variety of uses; for example, this is the shape of ordinary household voltage. A "pulse train", such as that in (d), can be used to drive DC motors at a variable speed.

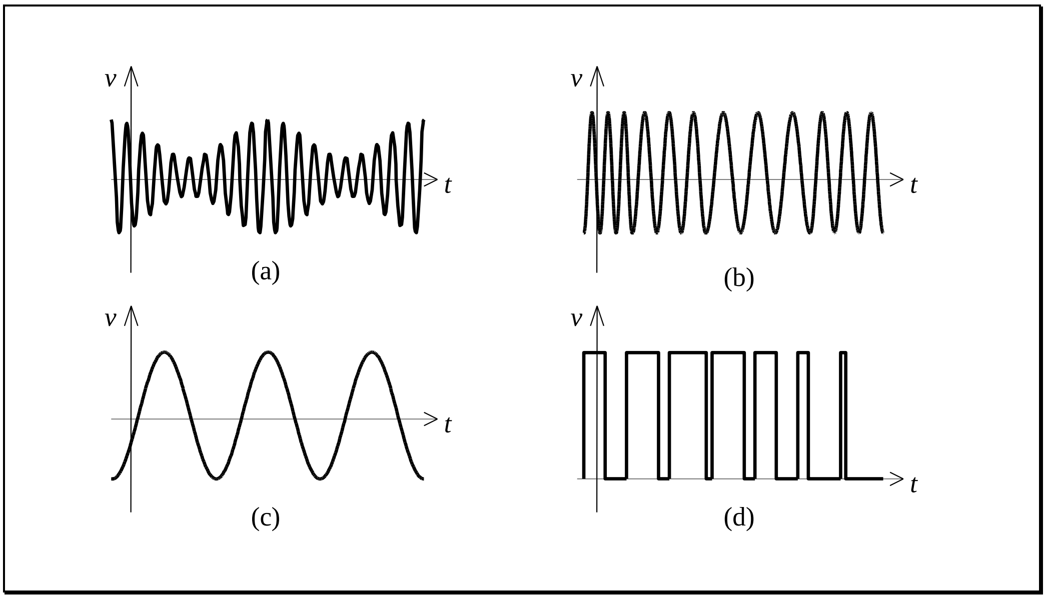


Figure 1.5

Since the voltage produced by a source is in general a function of time, then the most general representation of an ideal voltage source is as shown below:

The most general representation of an ideal independent voltage source

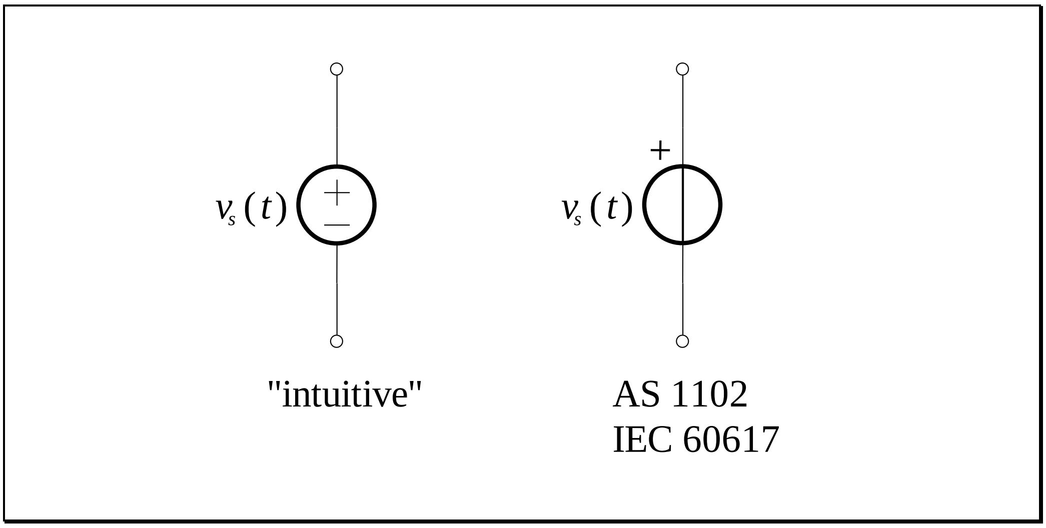


Figure 1.6

1.4.2 The Independent Current Source

Independent current source defined

An independent current source establishes a current which is independent of the voltage across it. The representation of an independent current source is shown below:

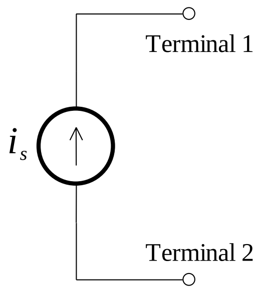


Figure 1.7

In other words, an *ideal current source* is a device that, *when connected to anything*, will always push i_s out of terminal 1 and pull i_s into terminal 2.

Since the current produced by a source is in general a function of time, then the most general representation of an ideal current source is as shown below:

The most general representation of an ideal independent current source

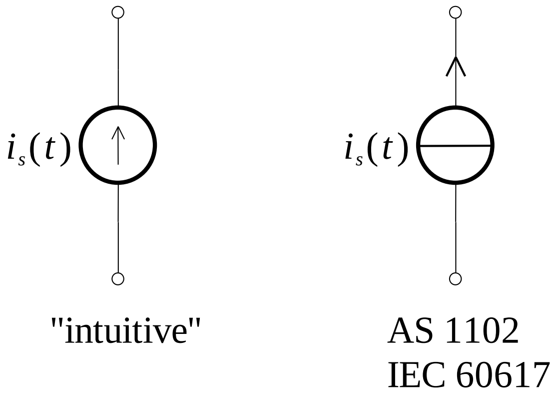


Figure 1.8

1.5 The Resistor and Ohm's Law

In 1827 the German physicist George Ohm published a pamphlet entitled "The Galvanic Circuit Investigated Mathematically". It contained one of the first efforts to measure currents and voltages and to describe and relate them mathematically. One result was a statement of the fundamental relationship we now call Ohm's Law.

Consider a uniform cylinder of conducting material, to which a voltage has been connected. The voltage will cause charge to flow, i.e. a current:

A simple resistor

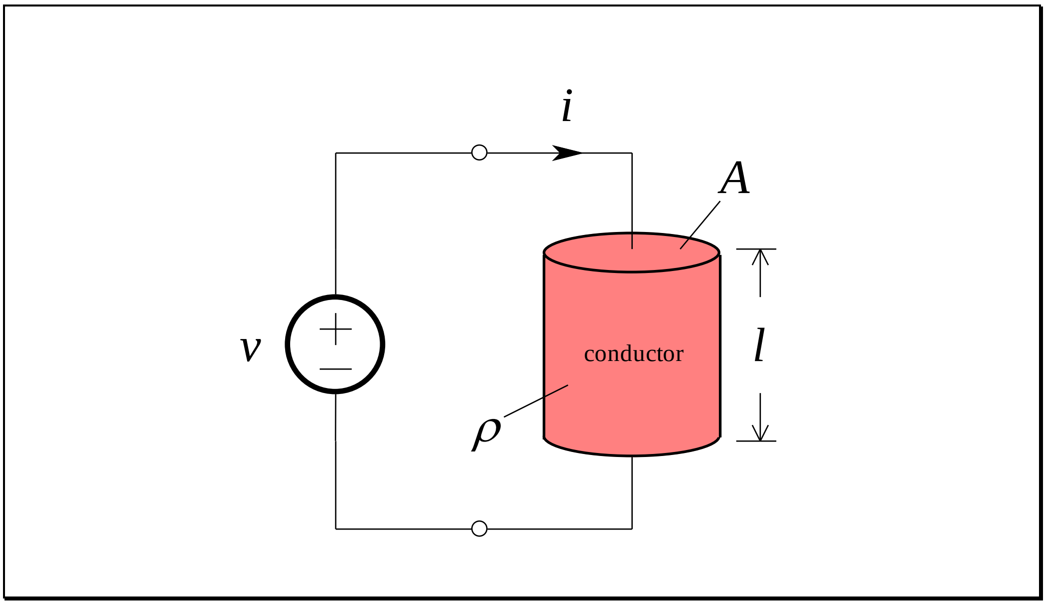


Figure 1.9

Ohm found that in many conducting materials, such as metal, the current is always proportional to the voltage. Since voltage and current are directly proportional, there exists a proportionality constant R, called *resistance*, such that:

Ohm's Law

$$\boxed{v = Ri} \tag{1.2}$$

This is Ohm's Law. The unit of resistance (volts per ampere) is referred to as the *ohm*, and is denoted by the capital Greek letter omega, **Ω**.

We refer to a construction in which Ohm's Law is obeyed as a *resistor*.

The resistor is a linear circuit element

The ideal resistor relationship is a *straight line through the origin*:

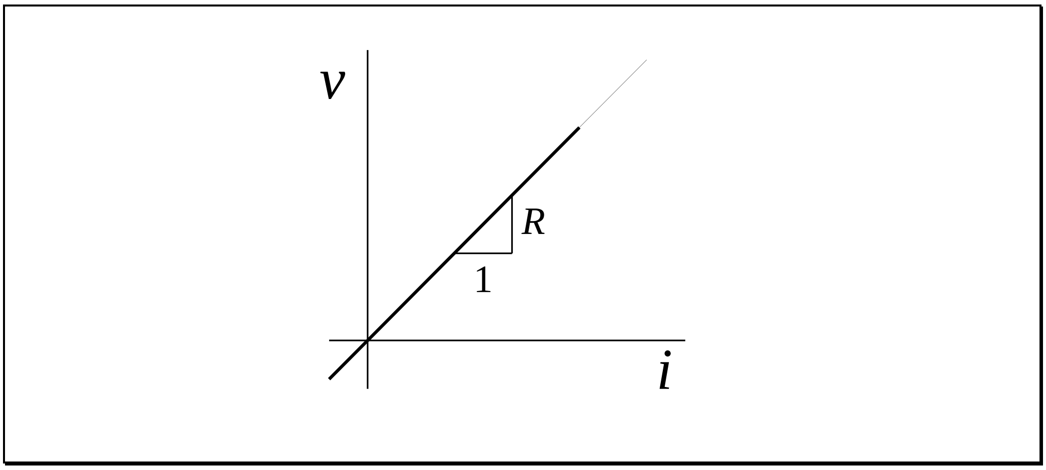


Figure 1.10

Even though resistance is defined as $R = v/i$, it should be noted that R is a purely geometric property, and depends only on the conductor shape and the material used in the construction. For example, it can be shown for a uniform resistor that the resistance is given by:

The resistance of a uniform resistor

$$R = \frac{\rho l}{A} \tag{1.3}$$

where l is the length of the resistor, and A is the cross-sectional area. The *resistivity*, ρ, is a constant of the conducting material used to make the resistor.

The circuit symbol for the resistor

The circuit symbol for the resistor is shown below, together with the direction of current and polarity of voltage that make Ohm's Law algebraically correct:

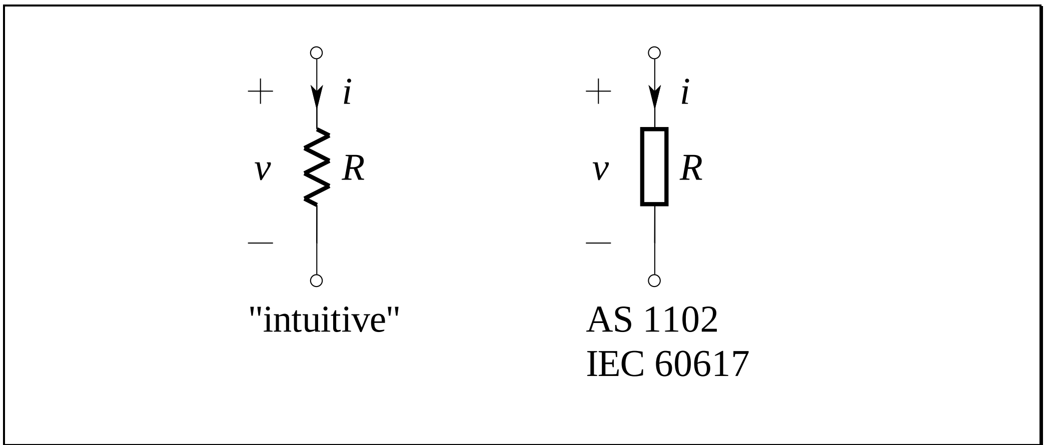
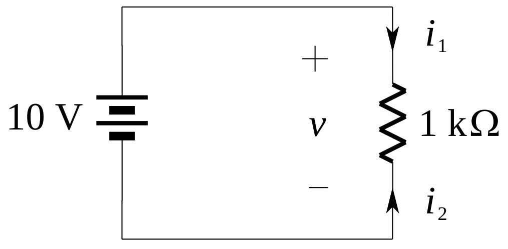


Figure 1.11

EXAMPLE 1.2 Ohm's Law with a Voltage Source

Consider the circuit shown below.



The voltage across the 1 kΩ resistor is, by definition of an ideal voltage source, $v(t)=10\text{ V}$. Thus, by Ohm's Law, we get:

$$i_1=\frac{v}{R}=\frac{10}{1000}=0.01\text{ A}=10\text{ mA}$$

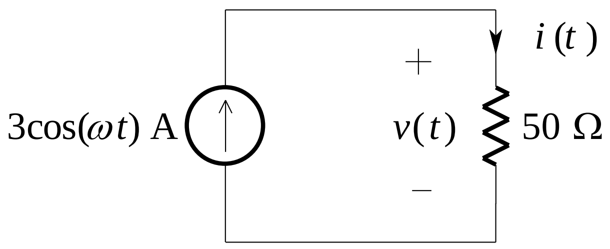
and:

$$i_2=\frac{-v}{R}=\frac{-10}{1000}=-0.01\text{ A}=-10\text{ mA}$$

Note that $i_2=-i_1$, as expected.

EXAMPLE 1.3 Ohm's Law with a Current Source

Consider the circuit shown below.



Ohm's Law yields:

$$\begin{aligned} v(t) &= Ri(t) \\ &= 50\times 3\cos(\omega t) \\ &= 150\cos(\omega t)\text{ V} \end{aligned}$$

1.5.1 The Short-Circuit

Consider a resistor whose value is zero ohms. An equivalent representation of such a resistance, called a *short-circuit*, is shown below:

The short-circuit

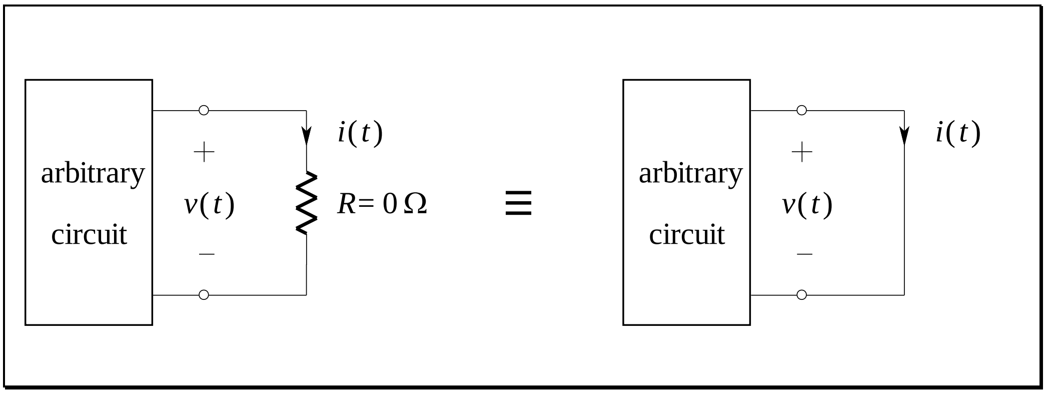


Figure 1.12

By Ohm's Law:

$$\begin{aligned} v &= Ri \\ &= 0i \\ &= 0\text{ V} \end{aligned} \tag{1.4}$$

Thus, no matter what finite value $i(t)$ has, $v(t)$ will be zero. Hence, we see that a *zero-ohm resistor* is equivalent to an *ideal voltage source whose value is zero volts*, provided that the current through it is finite.

1.5.2 The Open-Circuit

Consider a resistor having infinite resistance. An equivalent representation of such a resistance, called an *open-circuit*, is shown below:

The open-circuit

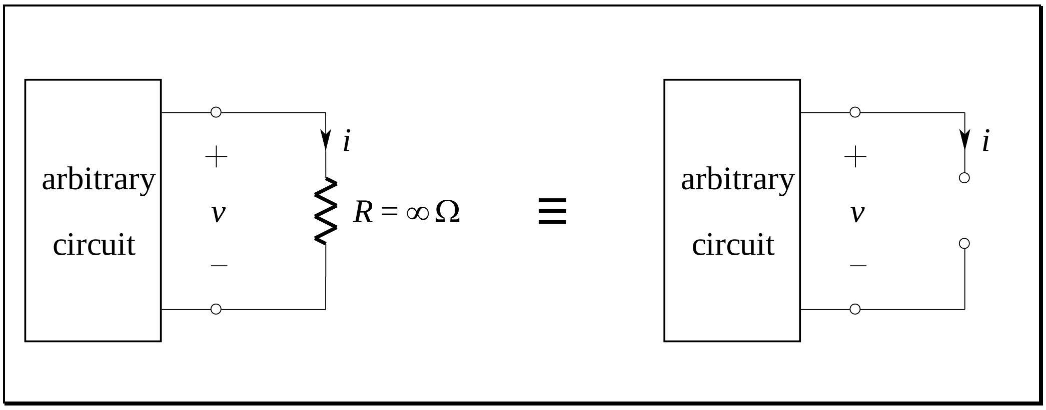


Figure 1.13

By Ohm's Law:

$$\begin{aligned} i &= \frac{v}{R} \\ &= \frac{v}{\infty} \\ &= 0\text{ A} \end{aligned} \tag{1.5}$$

Thus, no matter what finite value $v(t)$ has, $i(t)$ will be zero. Thus, we may conclude that an *infinite resistance* is equivalent to an *ideal current source whose value is zero amperes*, provided that the voltage across it is finite.

1.5.3 Conductance

The reciprocal of resistance is called the *conductance*, *G*:

Conductance defined

$$\boxed{G = \frac{1}{R}} \tag{1.6}$$

The unit of conductance is the *siemen*, and is abbreviated S. The same circuit symbol is used to represent both resistance and conductance.

1.6 Practical Resistors

There are many different types of resistor construction. Some are shown below:

Some types of resistors

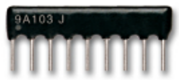
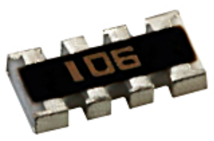
 carbon composition	 carbon film	 metal film
 wire wound	 wire wound with heat sink	 array
 chip - thick film	 chip - thin film	 chip array

Figure 1.14 – Some types of resistors

The "through-hole" resistors are used by hobbyists and for prototyping real designs. Their material and construction dictate several of their properties, such as accuracy, stability and pulse handling capability.

The wire wound resistors are made for accuracy, stability and high power applications. The array is used where space is a premium and is normally used in digital logic designs where the use of "pull-up" resistors is required.

Modern electronics utilises "surface-mount" components. There are two varieties of surface-mount chip resistor – thick film and thin film.

1.6.1 Preferred Values and the Decade Progression

Fundamental standardization practices require the selection of *preferred* values within the ranges available. Standard values may at first sight seem to be strangely numbered. There is, however, a beautiful logic behind them, dictated by the tolerance ranges available.

The *decade progression* of preferred values is based on preferred numbers generated by a geometric progression, repeated in succeeding decades. In 1963, the International Electrotechnical Commission (IEC) standardized the preferred number series for resistors and capacitors (standard IEC 60063). It is based on the fact that we can linearly space values along a logarithmic scale so a percentage change of a value results in a linear change on the logarithmic scale.

Component values have been standardized by the IEC

For example, if 6 values per decade are desired, the common ratio is $\sqrt[6]{10} \approx 1.468$. The six rounded-off values become 100, 150, 220, 330, 470, 680.

Component values are spaced equidistantly on a logarithmic scale

1.6.2 The 'E' Series Values

The IEC set the number of values for resistors (and capacitors) per decade based on their tolerance. These tolerances are 0.5%, 1%, 2%, 5%, 10%, 20% and 40% and are respectively known as the E192, E96, E48, E24, E12, E6 and E3 series, the number indicating the quantity of values per decade in that series. For example, if resistors have a tolerance of 5%, a series of 24 values can be assigned to a single decade multiple (e.g. 100 to 999) knowing that the possible extreme values of each resistor overlap the extreme values of adjacent resistors in the same series.

The 'E' series values explained

Any of the numbers in a series can be applied to any decade multiple set. Thus, for instance, multiplying 220 by each decade multiple (0.1, 1, 10 100, 1000 etc.) produces values of 22, 220, 2 200, 22 000, 220 000 etc.

The 'E' series of preferred resistor and capacitor values according to IEC 60063 are reproduced in Table 1.1.

0.5% E192	1% E96	2% E48	0.5% E192	1% E96	2% E48	0.5% E192	1% E96	2% E48	0.5% E192	1% E96	2% E48	0.5% E192	1% E96	2% E48
100	100	100	169	169	169	287	287	287	487	487	487	825	825	825
101			172			291			493			835		
102	102		174	174		294	294		499	499		845	845	
104			176			298			505			856		
105	105	105	178	178	178	301	301	301	511	511	511	866	866	866
106			180			305			517			876		
107	107		182	182		309	309		523	523		887	887	
109			184			312			530			898		
110	110	110	187	187	187	316	316	316	536	536	536	909	909	909
111			189			320			542			920		
113	113		191	191		324	324		549	549		931	931	
114			196			328			556			942		
115	115	115	196	196	196	332	332	332	562	562	562	953	953	953
117			198			336			569			965		
118	118		200	200		340	340		576	576		976	976	
120			203			344			583			988		
121	121	121	205	205	205	348	348	348	590	590	590			
123			208			352			597					
124	124		210	210		357	357		604	604				
126			213			361			612					
127	127	127	215	215	215	365	365	365	619	619	619			
129			218			370			626					
130	130		221	221		374	374		634	634				
132			223			379			642					
133	133	133	226	226	226	383	383	383	649	649	649			
135			229			388			657					
137	137		232	232		392	392		665	665				
138			234			397			673					
140	140	140	237	237	237	402	402	402	681	681	681			
142			240			407			690					
143	143		243	243		412	412		698	698				
145			246			417			706					
147	147	147	249	249	249	422	422	422	715	715	715			
149			252			427			723					
150	150		255	255		432	432		732	732				
152			259			437			741					
154	154	154	261	261	261	442	442	442	750	750	750			
156			264			448			759					
158	158		267	267		453	453		768	768				
160			271			459			777					
162	162	162	274	274	274	464	464	464	787	787	787			
164			277			470			796					
165	165		280	280		475	475		806	806				
167			284			481			816					

5% E24	10% E12	20% E6	40% E3
100	100	100	100
110			
120	120		
130			
150	150	150	
160			
180	180		
200			
220	220	220	220
240			
270	270		
300			
330	330	330	
360			
390	390		
430			
470	470	470	470
510			
560	560		
620			
680	680	680	
750			
820	820		
910			

Table 1.1 – IEC standard 'E' series of values in a decade

1.6.3 Marking Codes

The IEC also defines how manufacturers should mark the values of resistors and capacitors in the standard called IEC 60062. The colours used on fixed leaded resistors are shown below:

IEC labelling for leaded resistors

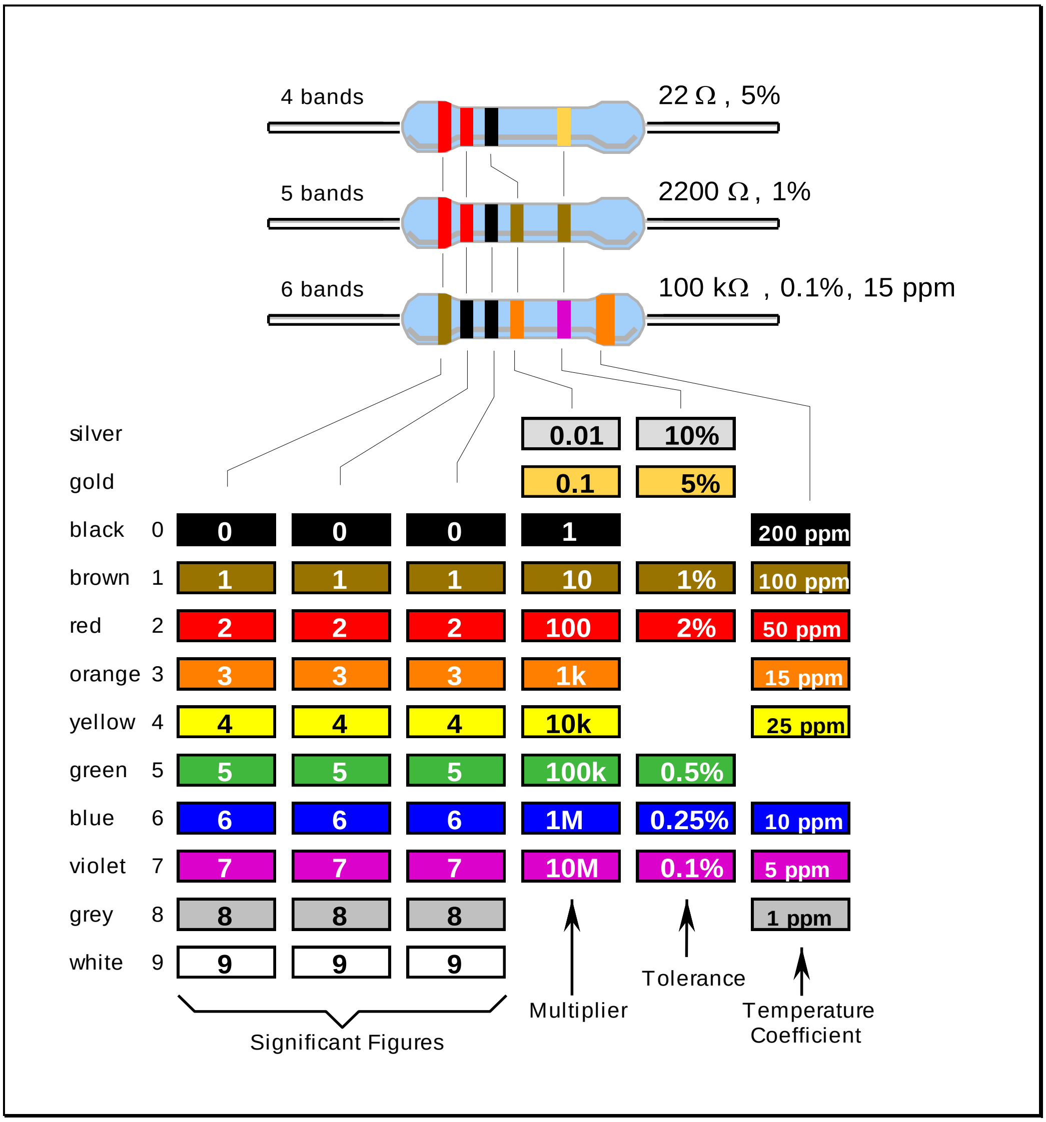


Figure 1.15 – Colour code marking of leaded resistors

The resistance *colour code* consists of three or four colour bands and is followed by a band representing the tolerance. The temperature coefficient band, if provided, is to the right of the tolerance band and is usually a wide band positioned on the end cap.

The resistance colour code includes the first two or three significant figures of the resistance value (in ohms), followed by a multiplier. This is a factor by which the significant-figure value must be multiplied to find the actual resistance value. (i.e. the number of zeros to be added after the significant figures).

Whether two or three significant figures are represented depends on the tolerance: ±5% and wider require two band; ±2% and tighter requires three bands. The significant figures refer to the first two or three digits of the resistance value of the standard series of values in a decade, in accordance with IEC 60063 as indicated in the relevant data sheets and shown in Table 1.1.

The colours used and their basic numerical meanings are recognized internationally for any colour coding used in electronics, not just resistors, but some capacitors, diodes, cabling and other items.

The resistor colour code explained

The colours are easy to remember: Black is the absence of any colour, and therefore represents the absence of any quantity, 0. White (light) is made up of all colours, and so represents the largest number, 9. In between, we have the colours of the rainbow: red, orange, yellow, green, blue and violet. These take up the numbers from 2 to 7. A colour in between black and red would be brown, which has the number 1. A colour intermediate to violet and white is grey, which represents the number 8.

When resistors are labelled in diagrams, such as schematics, IEC 60062 calls for the significant figures to be printed as such, but the decimal point is replaced with the SI prefix of the multiplier. Examples of such labelling are shown below:

IEC labelling for diagrams

Resistor Value	IEC Labelling
0.1 Ω	0R1
1 Ω	1R0
22 Ω	22R
3.3 kΩ	3K3
100 kΩ	100K
4.5 MΩ	4M5

Note how the decimal point is expressed, that the ohm symbol is shown as an R, and that 1000 is shown as a capital K. The use of a letter instead of a decimal point solves a printing problem – the decimal point in a number may not always be printed clearly, and the alternative display method is intended to help misinterpretation of component values in circuit diagrams and parts lists.

We use a letter in place of a decimal point for labelling component values

In circuit diagrams and constructional charts, a resistor's numerical identity, or *designator*, is usually prefixed by 'R'. For example, R15 simply means resistor number 15.

A portion of a schematic diagram showing designators and IEC labelling is shown below:

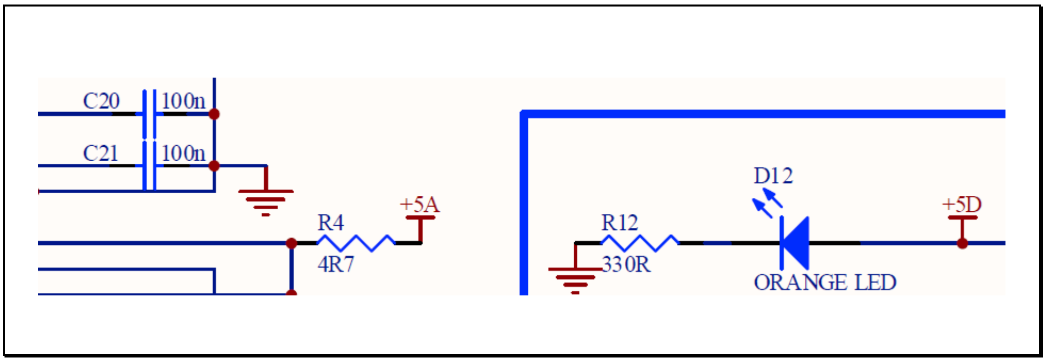


Figure 1.16 – Schematic portion showing IEC labelling

Note that resistor R4 has the value 4.7 Ω and resistor R12 has the value 330 Ω.

1.7 Summary

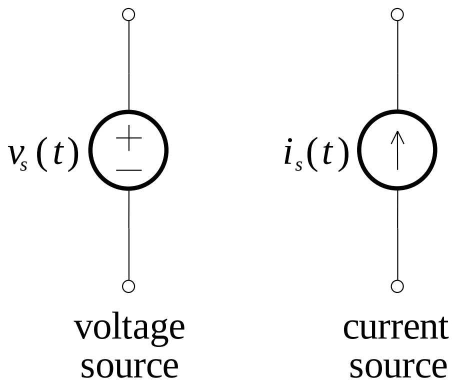
- Current is defined as the rate of flow of charge past a certain cross-sectional area:

$$i = \frac{dq}{dt}$$

- Voltage is defined as the work done per unit charge in moving it from one point to another in a circuit.

- A circuit element is an *idealised* mathematical model of a two-terminal electrical device that is completely characterised by its voltage-current relationship. Active circuit elements *can* deliver a non-zero average power indefinitely, whilst passive circuit elements *cannot*. A connection of circuit elements is called a *network*. If the network contains at least one closed path, it is also an electrical *circuit*.

- Independent sources are *ideal* circuit elements that possess a voltage or current value that is independent of the behaviour of the circuits to which they belong. There are two types, voltage and current:



- The resistor is a linear passive circuit element that obeys Ohm's Law:

$$v = Ri$$

A resistance of $0\,\Omega$ is known as a *short-circuit*.

A resistance of $\infty\,\Omega$ is known as an *open-circuit*.

The reciprocal of resistance is called the *conductance*:

$$G = \frac{1}{R}$$

- Practical resistors come in a large variety of shapes, materials and construction which dictate several of their properties, such as accuracy, stability, pulse handling capability, resistor value, size and cost.

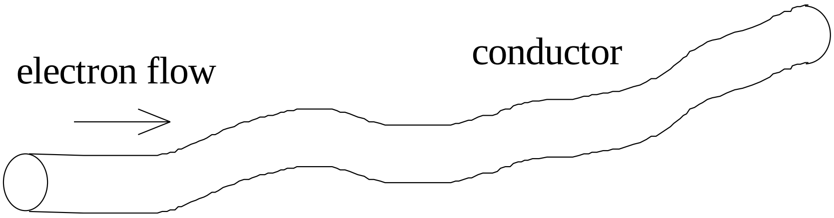
1.8 References

Hayt, W. & Kemmerly, J.: *Engineering Circuit Analysis*, 3rd *Ed*., McGraw-Hill, 1984.

Exercises

1.

A large number of electrons are moving through a conductor:



The number varies with time *t* seconds.

(a) What is the direction of current?

(b) If the total charge to pass a certain point on the conductor varies according to the equation:

$$q(t) = 3\left(1 - e^{-100t}\right) \text{ mC}$$

then find the current in amperes as a function of time.

(c) When will the current be 200 mA?

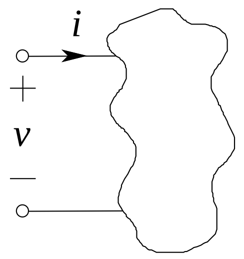
(d) If the conductor has a uniform diameter of 1 mm throughout its length, find the current density as a function of time (express as A/mm^2).

(e) Sketch charge and current as functions of time.

(f) How many electrons are moving through the conductor at time t = 50 ms?

2.

The total charge that has entered the upper terminal of the element below is given by $5\sin 1000\pi t\ \mu\text{C}$.



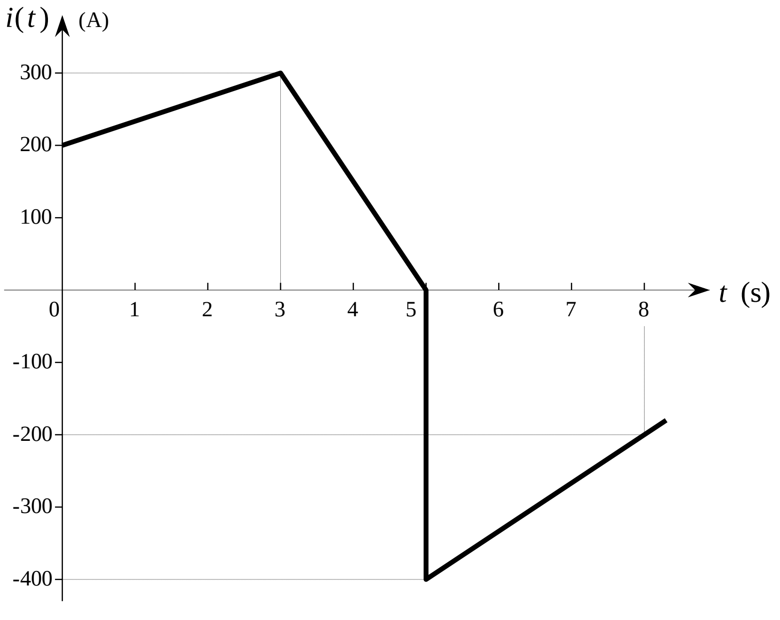
(a) How much charge enters that terminal between $t=-0.5\text{ ms}$ and $t=0.5\text{ ms}$?

(b) How much charge leaves the lower terminal in the same time interval?

(c) Find i at $t=0.2\text{ ms}$.

3.

For the current waveform shown below:



determine the total charge transferred between $t=0$ and $t=$:

(a) 4 s (b) 7 s

4.

The charging current supplied to a 12 V automotive battery enters its positive terminal. It is given as a function of time by:

$$i = \begin{cases} 0 & t < 0 \\ 4e^{-t/10000}\,\text{A} & 0 \le t \le 15000\ \text{s} \\ 0 & t > 15000\ \text{s} \end{cases}$$

(a) What is the total charge delivered to the battery in the 15000 s charging interval?

(b) What is the maximum power absorbed by the battery?

(c) What is the total energy supplied?

(d) What is the average power delivered in the 15000 s interval?

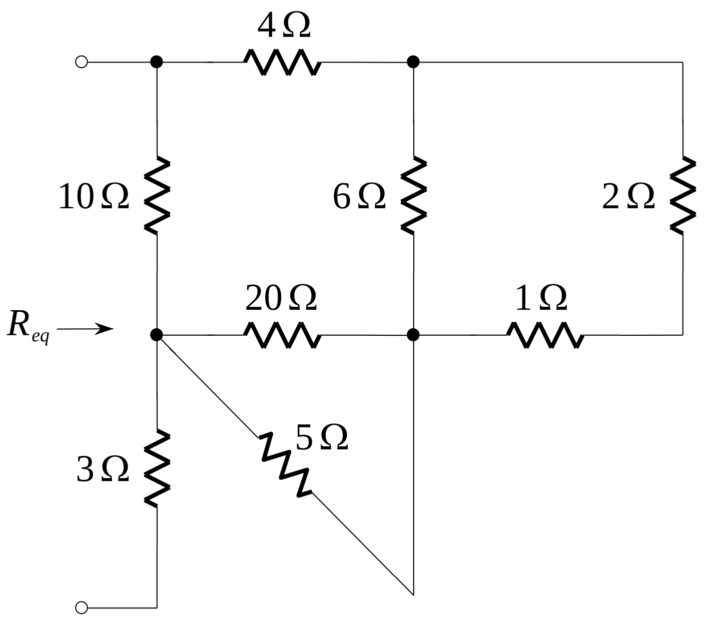
5.

The voltage v has its positive reference at terminal A of a certain circuit element. The power absorbed by the circuit element is $4(t-1)^2$ W for $t > 0$. If $v = (2t-2)\,\text{V}$ for $t > 0$, how much charge enters terminal A between $t = 0$ and $t = 2\,\text{s}$?

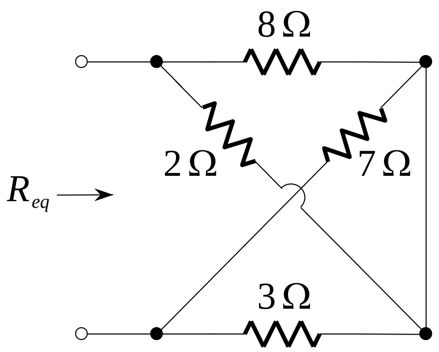
6.

Find R_{eq} for each of the networks shown below:

(a)



(b)



7.

Determine the necessary values of v and i in the circuit shown below: