2 Circuit Laws

Contents

2.2

Introduction

Electric circuit theory is one of the fundamental theories upon which all branches of electrical engineering are built. Many branches of electrical engineering, such as power, electric machines, control, electronics, communications, and instrumentation, are based on electric circuit theory. Circuit theory is also valuable to students specializing in other branches of the physical sciences because circuits are a good model for the study of energy systems in general, and because of the applied mathematics, physics, and topology involved.

2.1 Kirchhoff's Current Law

A connection of two or more elements is called a *node*. An example of a node is depicted in the partial circuit shown below:

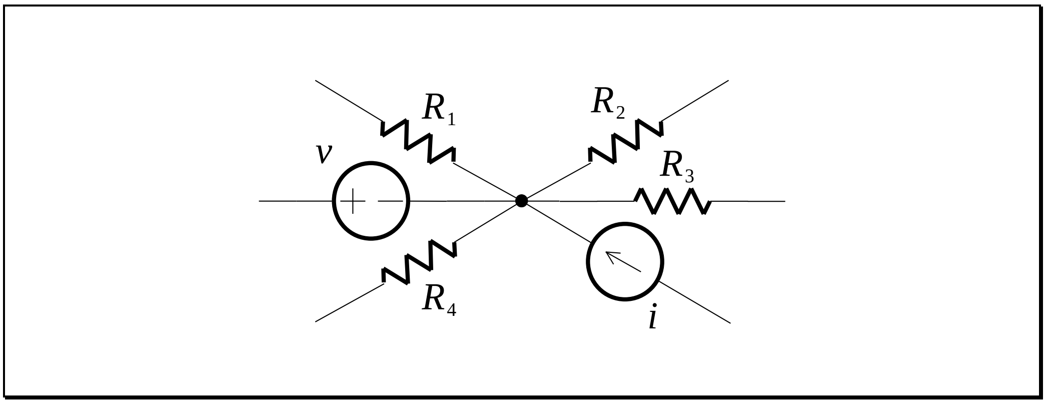


Figure 2.1

Even if the figure is redrawn to make it appear that there may be more than one node, as in the figure below, the connection of the six elements actually constitutes only one node.

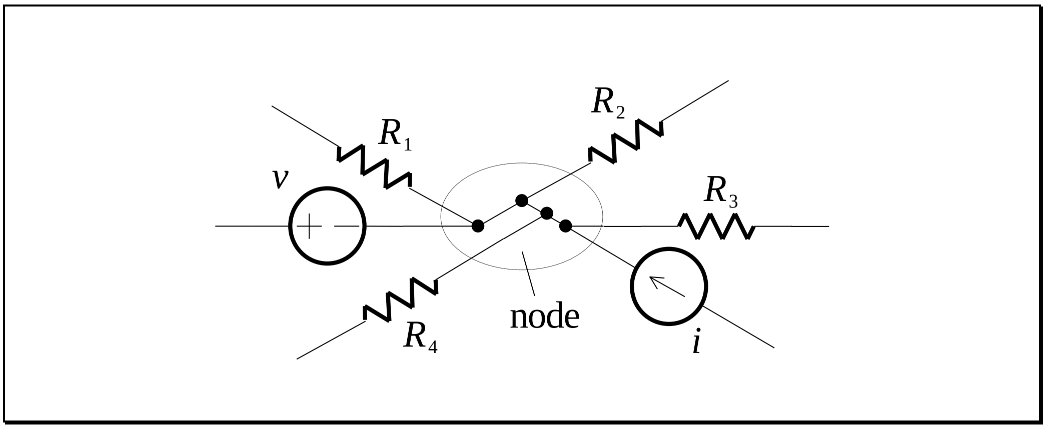


Figure 2.2

2.4

Kirchhoff's Current Law (KCL) is essentially the law of conservation of electric charge. If currents directed out of a node are positive in sense, and currents directed into a node are negative in sense (or vice versa), then KCL can be stated as follows:

KCL defined

> **KCL: At any node of a circuit, the currents algebraically sum to zero.** (2.1)

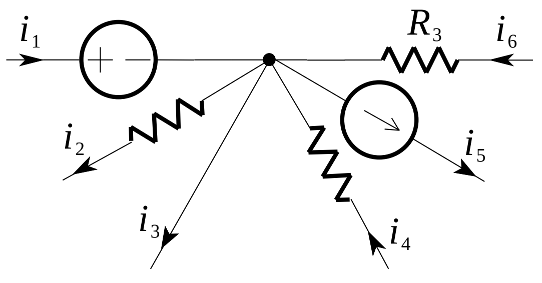
If there are *n* elements attached to a node then, in symbols, KCL is:

$$\sum_{k=1}^{n} i_k = 0 \tag{2.2}$$

KCL can also be stated as: The sum of the currents entering a node is equal to the sum of the currents leaving a node.

EXAMPLE 2.1 Kirchhoff's Current Law for a Node

As an example of KCL, consider a portion of some circuit, shown below:



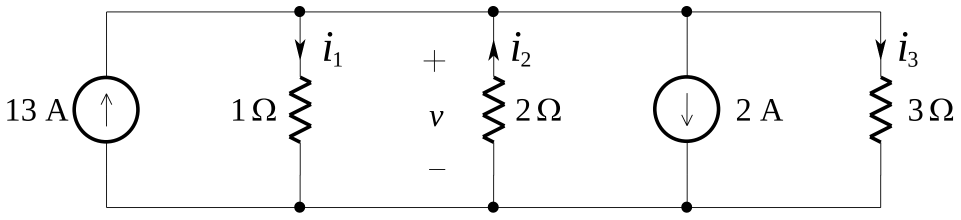
Choosing the positive sense to be leaving, we apply KCL at the node and obtain the equation:

$$-i_1 + i_2 + i_3 - i_4 + i_5 - i_6 = 0$$

Note that even if one of the elements – the one which carries i_3 – is a short-circuit, KCL holds. In other words, KCL applies regardless of the nature of the elements in the circuit.

EXAMPLE 2.2 Kirchhoff's Current Law for a Two-Node Circuit

We want to find the voltage *v*, in the two-node circuit shown below:



The directions of i_1, i_2, i_3 and the polarity of v were chosen arbitrarily (the directions of the 13 A and 2 A sources are given). By KCL (at either of the two nodes), we have:

$$-13 + i_1 - i_2 + 2 + i_3 = 0$$

From this we can write:

$$i_1 - i_2 + i_3 = 11$$

By Ohm's Law:

$$i_1 = \frac{v}{1} \qquad i_2 = \frac{-v}{2} \qquad i_3 = \frac{v}{3}$$

Substituting these into the previous equation yields:

$$\begin{aligned} \left(\frac{v}{1}\right) - \left(\frac{-v}{2}\right) + \left(\frac{v}{3}\right) &= 11 \\ v + \frac{v}{2} + \frac{v}{3} &= 11 \\ \frac{6v + 3v + 2v}{6} &= 11 \\ \frac{11v}{6} &= 11 \\ v &= 6\text{ V} \end{aligned}$$

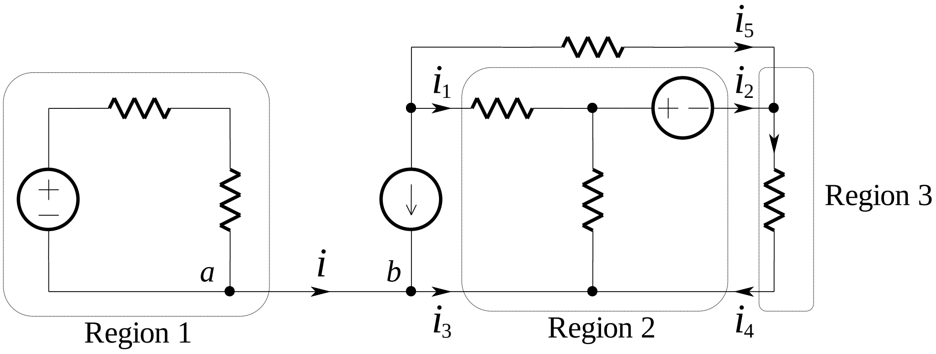
Having solved for *v*, we can now find that:

$$i_1 = \frac{v}{1} = \frac{6}{1} = 6\text{ A} \qquad \mathrm{i}_2 = -\frac{v}{2} = -\frac{6}{2} = -3\text{ A} \qquad \mathrm{i}_3 = \frac{v}{3} = \frac{6}{3} = 2\text{ A}$$

Just as KCL applies to any node of a circuit, so must KCL hold for any closed region, i.e. to satisfy the physical law of conservation of charge, the total current leaving (or entering) a region must be zero.

EXAMPLE 2.3 Kirchhoff's Current Law for a Closed Region

In the circuit shown below, three regions have been identified:



Applying KCL to Region 1, we get:

$$i = 0$$

For Region 2:

$$i_1 + i_3 + i_4 = i_2$$

For Region 3:

$$i_2 + i_5 = i_4$$

You may now ask, "Since there is no current from point *a* to point *b* (or vice versa) why is the connection (a short-circuit) between the points there?" If the connection between the two points is removed, two separate circuits result. The voltages and currents within each individual circuit remain the same as before. Having the connection present constrains points *a* and *b* to be the same node, and hence be at the same voltage. It also indicates that the two separate portions are physically connected (even though there is no current between them).

2.2 Kirchhoff's Voltage Law

Starting at any node in a circuit, we form a *loop* by traversing through elements (open-circuits included!) and returning to the starting node, never encountering any other node more than once.

Loop defined

For example, the paths *fabef* and *fdcef* are loops:

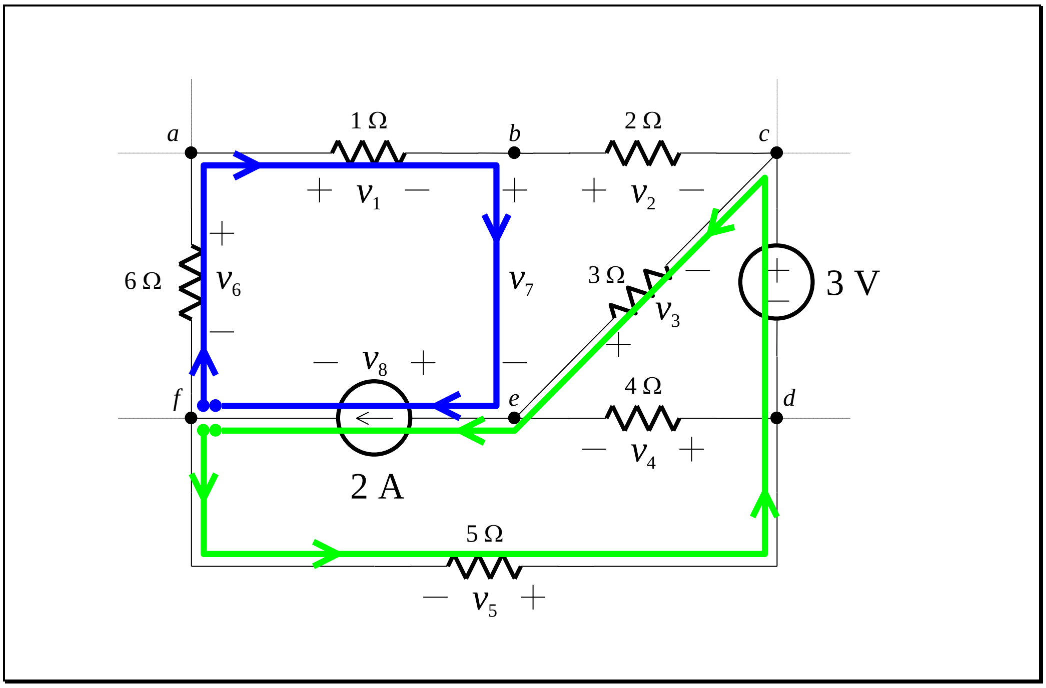


Figure 2.3

whereas the paths *becba* and *fde* are not:

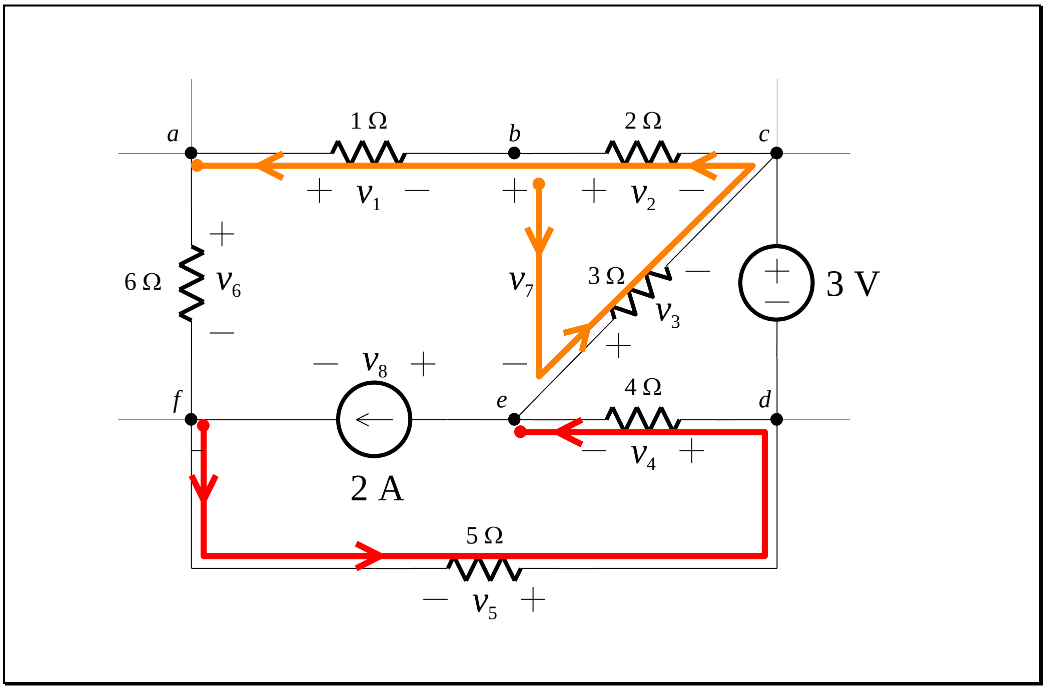


Figure 2.4

2.8

Kirchhoff's Voltage Law (KVL) is essentially the law of conservation of energy. If voltage drops across elements traversed from + to – are positive in sense, and voltage drops across elements that are traversed from – to + are negative in sense (or vice versa), then KVL can be stated as follows:

KVL defined

> KVL: Around any loop in a circuit, the voltages algebraically sum to zero. (2.3)

If there are n elements in the loop then, in symbols, KVL is:

$$\sum_{k=1}^{n} v_k = 0 \tag{2.4}$$

KVL can also be stated as: In traversing a loop, the sum of the voltage rises equals the sum of the voltage drops.

EXAMPLE 2.4 Kirchhoff's Voltage Law Around a Loop

In the circuit shown in Figure 2.3, we select a traversal from + to – to be positive in sense. Then KVL around the loop *abcefa* gives:

$$v_1 + v_2 - v_3 + v_8 - v_6 = 0$$

and around loop *bcdeb*, we have:

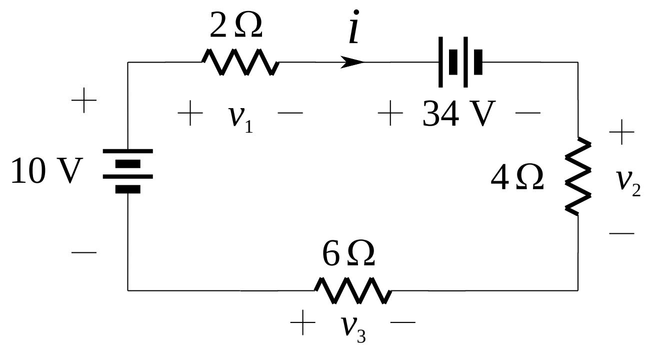
$$v_2 + 3 + v_4 - v_7 = 0$$

In this last loop, one of the elements traversed (the element between nodes *b* and *e*) is an open-circuit; however, KVL holds regardless of the nature of the elements in the circuit.

Although arbitrary, it will be seen later that we get simpler equations if we choose the algebraic "sense" when applying KVL so that we "add voltage drops". To do this, follow the simple rule: **write down the algebraic sign of the branch voltage that you first meet in traversing the loop**.

EXAMPLE 2.5 Kirchhoff's Voltage Law Around a Circuit

We want to find the current *i*, in the one-loop circuit shown below:



The polarities of v_1, v_2, v_3 and the direction of *i* were chosen arbitrarily (the polarities of the 10 V and 34 V sources are given). Applying KVL we get:

$$-10 + v_1 + 34 + v_2 - v_3 = 0$$

Thus:

$$v_1 + v_2 - v_3 = -24$$

From Ohm's Law:

$$v_1 = 2i \qquad v_2 = 4i \qquad v_3 = -6i$$

Substituting these into the previous equation yields:

$$\begin{aligned}(2i) + (4i) - (-6i) &= -24 \\ 2i + 4i + 6i &= -24 \\ 12i &= -24 \\ i &= -2\text{ A}\end{aligned}$$

Having solved for *i*, we now find that:

$$\begin{aligned}v_1 &= 2i = 2(-2) = -4\text{ V} \\ v_2 &= 4i = 4(-2) = -8\text{ V} \\ v_3 &= -6i = (-6)(-2) = 12\text{ V}\end{aligned}$$

2.3 Combining Resistors

Relatively complicated resistor combinations can be replaced by a single equivalent resistor whenever we are not specifically interested in the current, voltage or power associated with any of the individual resistors.

2.3.1 Series Resistors

Consider the series combination of N resistors shown in (a) below:

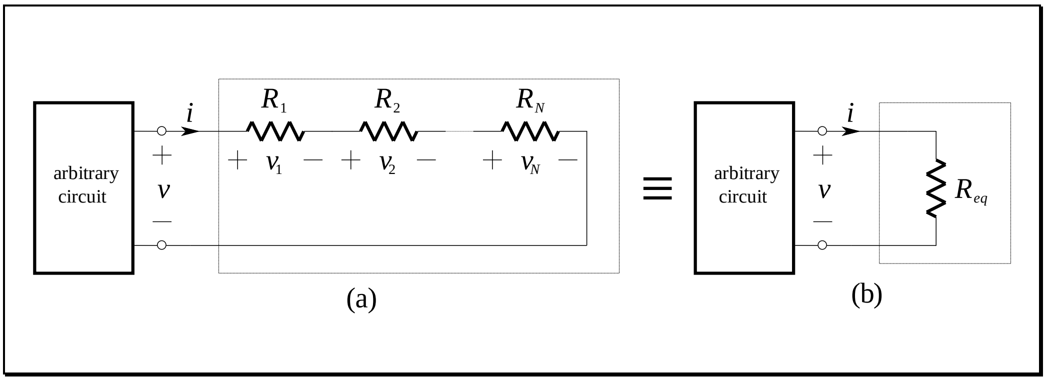


Figure 2.5

We apply KVL:

$$v = v_1 + v_2 + \cdots v_N \tag{2.5}$$

and Ohm's Law:

$$\begin{aligned} v &= R_1 i + R_2 i + \cdots R_N i \\ &= \left(R_1 + R_2 + \cdots R_N\right) i \end{aligned} \tag{2.6}$$

and then compare this result with the simple equation applying to the equivalent circuit shown in Figure 2.5b:

$$v = R_{eq} i \tag{2.7}$$

Thus, the value of the equivalent resistance for N series resistances is:

Combining series resistors

$$R_{eq} = R_1 + R_2 + \cdots + R_N \qquad \text{(series)} \tag{2.8}$$

2.3.2 Parallel Resistors

A similar simplification can be applied to parallel resistors. Consider the parallel combination of N conductances shown in (a) below:

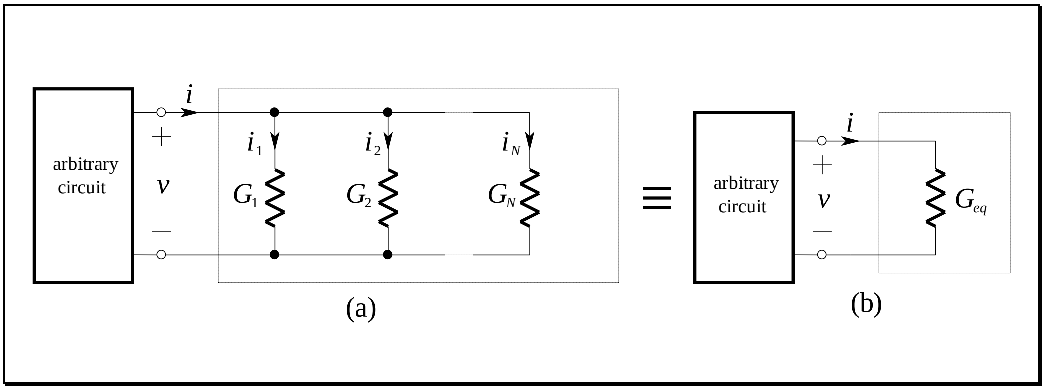


Figure 2.6

We apply KCL:

$$i = i_1 + i_2 + \cdots i_N \tag{2.9}$$

and Ohm's Law:

$$\begin{aligned} i &= G_1 v + G_2 v + \cdots G_N v \\ &= \left(G_1 + G_2 + \cdots G_N\right) v \end{aligned} \tag{2.10}$$

whereas the equivalent circuit shown in Figure 2.6b gives:

$$i = G_{eq} v \tag{2.11}$$

and thus the value of the equivalent conductance for N parallel conductances is:

$$G_{eq} = G_1 + G_2 + \cdots + G_N \qquad \text{(parallel)} \tag{2.12}$$

Combining parallel conductances

In terms of resistance instead of conductance:

Combining parallel resistors

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \quad \text{(parallel)} \tag{2.13}$$

The special case of only two parallel resistors is needed often:

Combining two resistors in parallel…

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \quad \text{(parallel)} \tag{2.14}$$

Note that since $G_{eq} = G_1 + G_2$ then we may deduce that:

$$G_{eq} > G_1 \qquad \text{and} \qquad G_{eq} > G_2 \tag{2.15}$$

Hence:

$$\frac{1}{R_{eq}} > \frac{1}{R_1} \qquad \text{and} \qquad \frac{1}{R_{eq}} > \frac{1}{R_2} \tag{2.16}$$

or:

…results in an equivalent resistance smaller than either resistor

$$R_{eq} < R_1 \qquad \text{and} \qquad R_{eq} < R_2 \tag{2.17}$$

Thus the equivalent resistance of two resistors in parallel is less than the value of either of the two resistors.

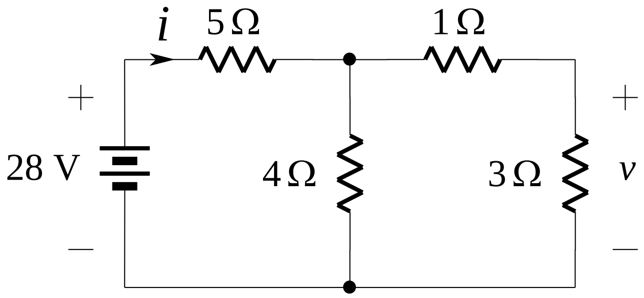
The special case of N resistors of equal value R in parallel is:

Combining the same valued resistors in parallel

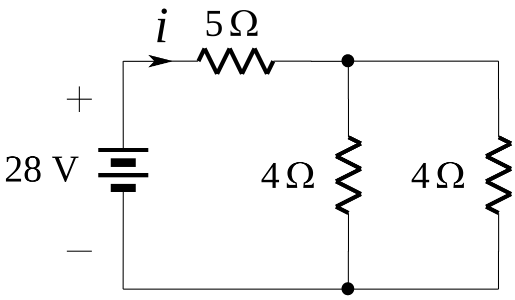
$$R_{eq} = \frac{R}{N} \quad \text{(parallel)} \tag{2.18}$$

EXAMPLE 2.6 Series and Parallel Resistors

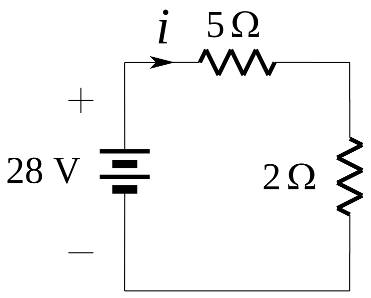
We want to find the current *i* in the circuit below:



In order to find *i*, we can replace series and parallel connections of resistors by their equivalent resistances. We begin by noting that the $1\,\Omega$ and $3\,\Omega$ resistors are in series. Combining them we obtain:



Note that it is not possible to display the original voltage *v* in this figure. Since the two $4\,\Omega$ resistors are connected in parallel, we can further simplify the circuit as shown below:



Here, the $5\,\Omega$ and $2\,\Omega$ resistors are in series, so we may combine them into one $7\,\Omega$ resistor. Then, from Ohm's Law, we have:

$$i = \frac{28}{7} = 4\text{ A}$$

2.4 Combining Independent Sources

An inspection of the KVL equations for a series circuit shows that the order in which elements are placed in a series circuit makes no difference. An inspection of the KCL equations for a parallel circuit shows that the order in which elements are placed in a parallel circuit makes no difference. We can use these facts to simplify voltage sources in series and current sources in parallel.

2.4.1 Combining Independent Voltage Sources in Series

It is not possible to combine independent voltage sources in parallel, since this would violate KVL. However, consider the series connection of two ideal voltage sources shown in (a) below:

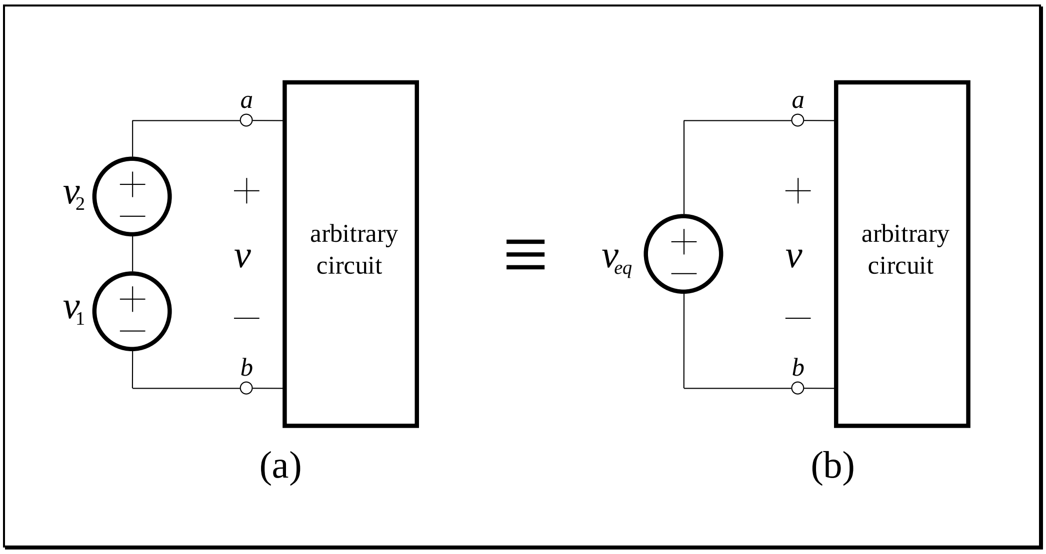


Figure 2.7

From KVL we know that $v = v_1 + v_2$, and by the definition of an ideal voltage source, this must be the voltage between nodes *a* and *b*, regardless of what is connected to them. Thus, the series connection of two ideal voltage sources is equivalent to a single independent voltage source given by:

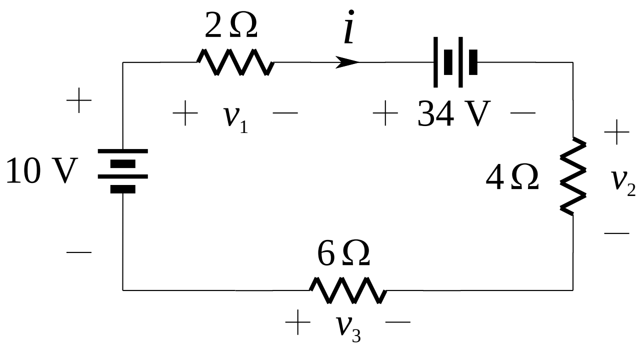
Combining independent voltage sources in series

$$v_{eq} = v_1 + v_2 \qquad \text{(series)} \tag{2.19}$$

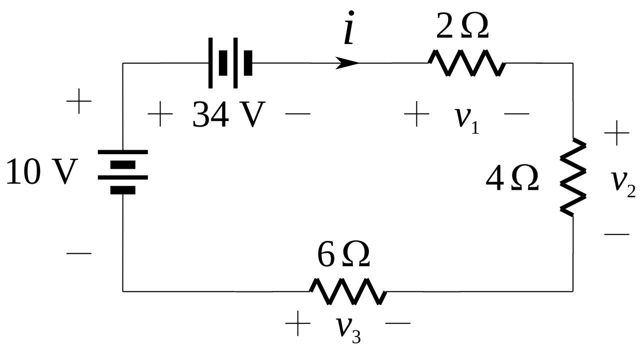
Clearly, the obvious generalization to *N* voltage sources in series holds.

EXAMPLE 2.7 Combining Independent Voltage Sources in Series

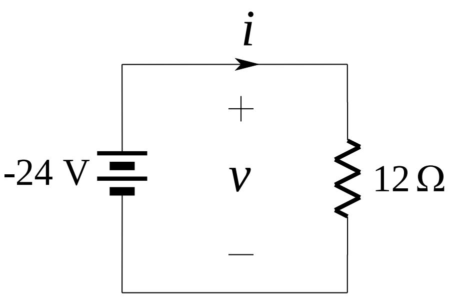
In a previous example we determined the current *i* in the one-loop circuit shown below:



By rearranging the order in this one loop circuit (of course this does not affect *i*), we obtain the circuit shown below:



We can now combine the series independent voltage sources and the series resistors into single equivalent elements:



By Ohm's Law:

$$i = \frac{-24}{12} = -2\text{ A}$$

2.4.2 Combining Independent Current Sources in Parallel

It is not possible to combine independent current sources in series, since this would violate KCL. However, consider the parallel connection of two ideal current sources shown in (a) below:

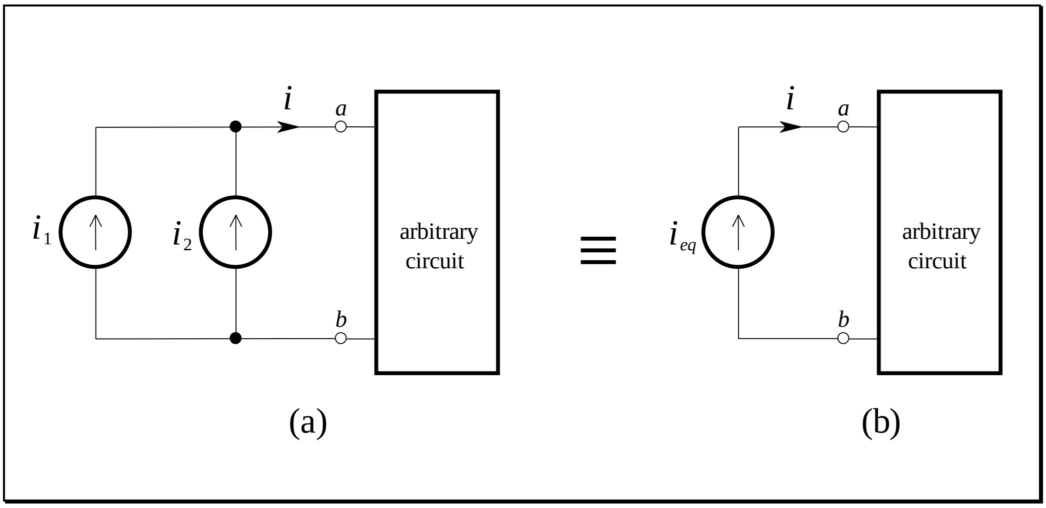


Figure 2.8

From KCL we find that $i = i_1 + i_2$, and by the definition of an ideal current source, this must always be the current into the arbitrary circuit. Thus, the parallel connection of two ideal current sources is equivalent to a single independent current source given by:

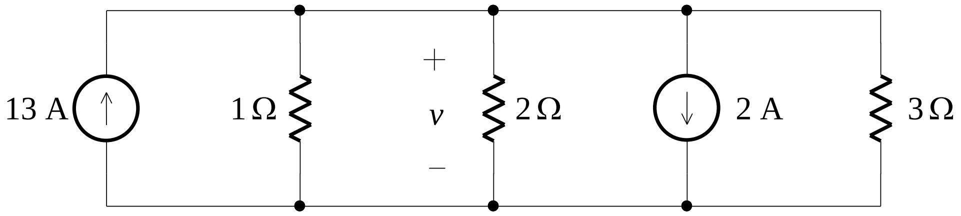
Combining independent current sources in parallel

$$i_{eq} = i_1 + i_2 \qquad \text{(parallel)} \tag{2.20}$$

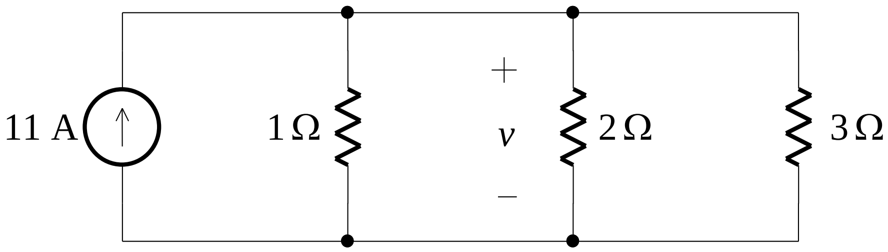
Clearly, the obvious generalization to N current sources in parallel holds.

EXAMPLE 2.8 Combining Independent Current Sources in Parallel

In a previous example, we determined the voltage v in the two-node circuit shown below:



Combining the parallel independent current sources into a single equivalent source, we obtain the circuit:



Since the equivalent resistance of the three resistors in parallel is given by:

$$\frac{1}{R_{eq}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{6+3+2}{6} = \frac{11}{6}$$

we obtain:

$$R_{eq} = \frac{6}{11}\,\Omega$$

Then, from Ohm's Law:

$$v = \frac{6}{11}(11) = 6\text{ V}$$

2.5 The Voltage Divider Rule

It can be quite useful to determine how a voltage appearing across two series resistors "divides" between them. Consider the circuit shown below:

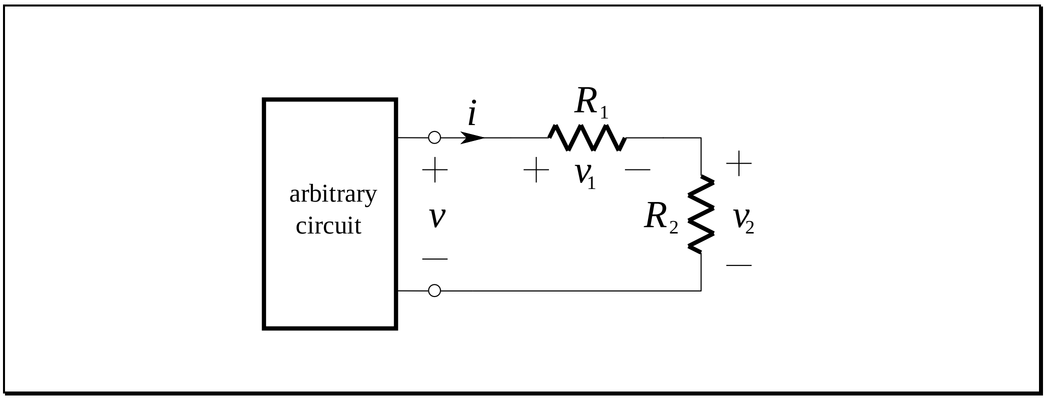


Figure 2.9

By Ohm's Law, the current in the resistors is:

$$i = \frac{v}{R_1 + R_2} \qquad (2.21)$$

By application of Ohm's Law again, the voltage across R_1 is:

$$v_1 = R_1 i \qquad (2.22)$$

and therefore:

Voltage divider rule defined

$$v_1 = \frac{R_1}{R_1 + R_2} v \qquad (2.23)$$

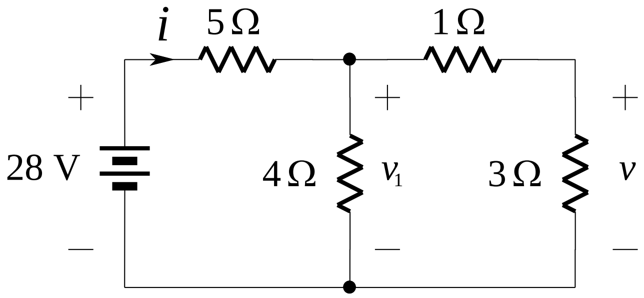
Similarly, the voltage across R_2 is:

$$v_2 = \frac{R_2}{R_1 + R_2} v \qquad (2.24)$$

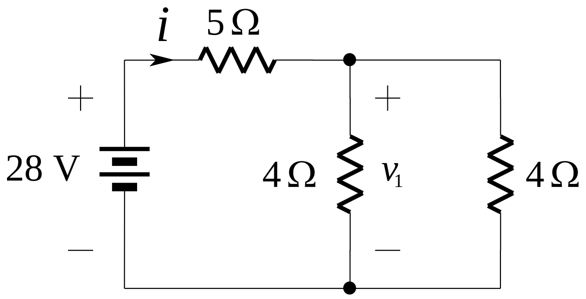
These equations describe how the voltage is divided between the resistors. Because of this, a pair of resistors in series is often called a *voltage divider*.

EXAMPLE 2.9 Voltage Divider Rule

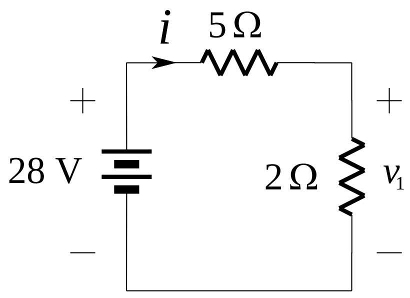
We want to find the voltage v in the circuit below:



Combining the series connection of the $1\,\Omega$ and $3\,\Omega$ resistors, we obtain the circuit below:



Now the pair of $4\,\Omega$ resistors in parallel can be combined as shown below:



By voltage division:

$$v_1 = \frac{2}{2+5} \times 28 = \frac{56}{7} = 8\,\text{V}$$

Returning to the original circuit and applying voltage division again yields:

$$v = \frac{3}{3+1} v_1 = \frac{3}{4} \times 8 = 6\,\text{V}$$

2.6 The Current Divider Rule

It can be quite useful to determine how a current entering two parallel resistors "divides" between them. Consider the circuit shown below:

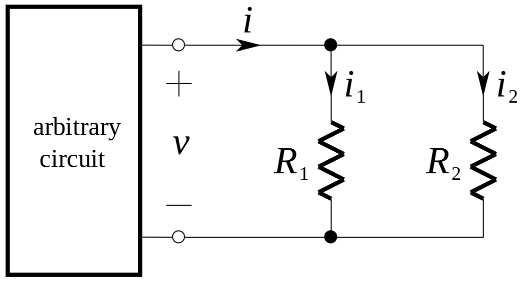


Figure 2.10

We replace the parallel connection of R_1 and R_2 by its equivalent resistance. Thus, Ohm's Law gives:

$$v = R_{eq} i = \frac{R_1 R_2}{R_1 + R_2} i \tag{2.25}$$

By application of Ohm's Law again, the current in R_1 is $i_1 = v/R_1$ and thus:

Current divider rule defined

$$i_1 = \frac{R_2}{R_1 + R_2} i \tag{2.26}$$

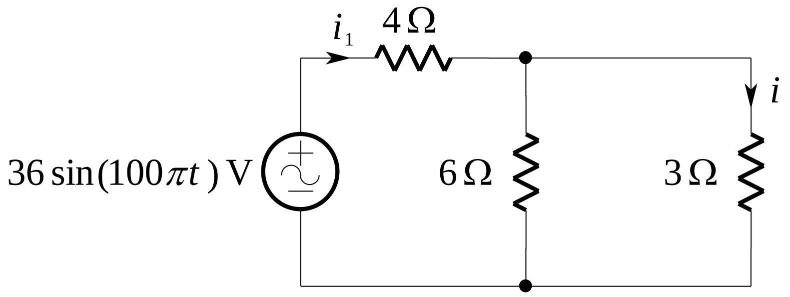
Similarly, the current in R_2 is:

$$i_2 = \frac{R_1}{R_1 + R_2} i \tag{2.27}$$

These equations describe how the current is divided between the resistors. Because of this, a pair of resistors in parallel is often called a *current divider*. Note that a larger amount of current will exist in the smaller resistor – thus current tends to take the path of least resistance!

EXAMPLE 2.10 Current Divider Rule

We want to find the current *i* in the circuit below:



The total current delivered by the source is:

$$\begin{aligned} i_1 &= \frac{36\sin(100\pi t)}{4+(6)(3)/(6+3)} \\ &= 6\sin(100\pi t)\,\text{A} \end{aligned}$$

Therefore the desired current is:

$$i = \frac{6}{6+3} i_1 = \frac{2}{3} \times 6\sin(100\pi t) = 4\sin(100\pi t)\,\text{A}$$

The current divider rule can also be derived using conductances. Referring to Figure 2.10, the voltage across the parallel resistors is:

$$v = R_{eq} i = \frac{i}{G_{eq}} = \frac{i}{G_1 + G_2} \tag{2.28}$$

The current in resistor R_1 is $i_1 = G_1 v$ and thus:

$$i_1 = \frac{G_1}{G_1 + G_2} i \tag{2.29}$$

A similar result obviously holds for current i_2. The advantage of this form of the current divider rule is that it is the *dual* of the voltage divider rule – we replace voltages with currents, and resistors with conductances.

2.7 Power

Power is the rate at which work is done or energy is expended. Taking the product of voltage (energy per unit charge) and current (charge per unit time) we get a quantity that measures energy per unit time. It's for this reason that we define p, the *instantaneous power absorbed* by an electrical circuit element, to be the product of voltage and current:

Instantaneous power defined

$$p = vi \tag{2.30}$$

The fundamental unit of power is the watt (W) and is equivalent to Js^{-1}. In using the formula for instantaneous power, we need to be careful in establishing the correct voltage polarity and current direction. Consider the circuit element:

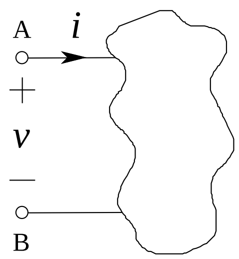


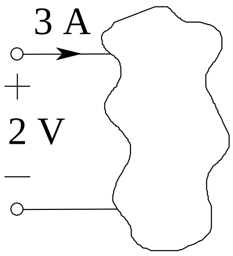
Figure 2.11

Passive sign convention defined – it gives power absorbed by a circuit element

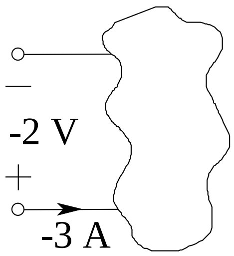
If one terminal of the element (A) is v volts positive with respect to the other terminal (B), and if a current i is entering the element through terminal A, then a power $p = vi$ is being *absorbed* or *delivered* to the element. When the current arrow is directed into the element at the plus-marked terminal, we satisfy the *passive sign convention*. If the numerical value of the power using this convention is negative, then we say that the element is *generating* or *delivering* power.

EXAMPLE 2.11 Power Absorbed

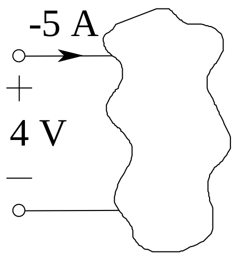
Note the power in the circuit elements below:



$p = (2)(3) = 6$ W absorbed



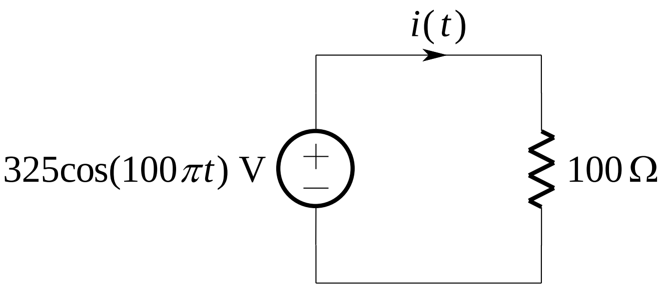
$p = (-2)(-3) = 6$ W absorbed



$p = (4)(-5) = -20$ W absorbed
(20 W generated)

EXAMPLE 2.12 Power Absorbed by a Resistor

Consider the circuit shown below:



By Ohm's Law:

$$i(t)=\frac{v(t)}{R}=\frac{325}{100}\cos(100\pi t)\ \text{A}$$

By definition, the power absorbed by the resistor is:

$$\begin{aligned}p_R(t)&=v(t)i(t)\\&=\frac{325^2}{100}\cos^2(100\pi t)\\&=1056\cos^2(100\pi t)\ \text{W}\end{aligned}$$

In particular, at time $t=0$ the power absorbed by the resistor is:

$$\begin{aligned}p_R(0)&=1056\cos^2(0)\\&=1056\ \text{W}\end{aligned}$$

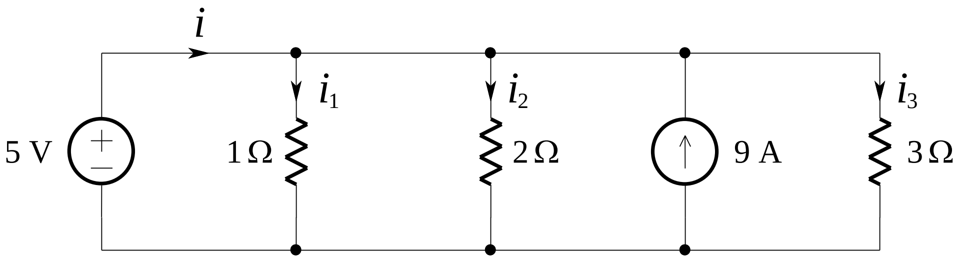
At time $t=5\ \text{ms}$, however, since:

$$\cos\left(100\pi\times5\times10^{-3}\right)=\cos\left(\frac{\pi}{2}\right)=0$$

then the resistor absorbs 0 watts.

EXAMPLE 2.13 Power Absorbed by Circuit Elements

Consider the circuit shown below:



We shall determine the power absorbed in each of the elements.

Note that the voltage across each of the elements is 5 V since all the elements are in parallel. Therefore, by Ohm's Law:

$$i_1 = \frac{5}{1} = 5\text{ A} \qquad i_2 = \frac{5}{2}\text{ A} \qquad i_3 = \frac{5}{3}\text{ A}$$

and the powers absorbed in the $1\,\Omega$, $2\,\Omega$ and $3\,\Omega$ resistors are:

$$p_1 = 5i_1 = 5(5) = 25\text{ W}$$

$$p_2 = 5i_2 = 5\left(\frac{5}{2}\right) = \frac{25}{2}\text{ W}$$

$$p_3 = 5i_3 = 5\left(\frac{5}{3}\right) = \frac{25}{3}\text{ W}$$

respectively, for a total of:

$$25 + \frac{25}{2} + \frac{25}{3} = \frac{150 + 75 + 50}{6} = \frac{275}{6}\text{ W}$$

absorbed by the resistors.

2.26

By KCL:

$$i + 9 = i_1 + i_2 + i_3 = \frac{5}{1} + \frac{5}{2} + \frac{5}{3}$$

or:

$$i = \frac{30 + 15 + 10}{6} - 9 = \frac{1}{6}\text{ A}$$

Thus the power delivered by the voltage source is:

$$p_v = 5i = \frac{5}{6}\text{ W}$$

Also, the power delivered by the current source is:

$$p_i = 5(9) = 45\text{ W}$$

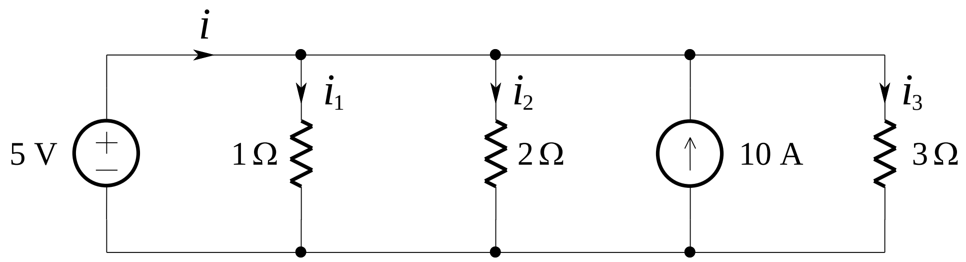
Hence the total power delivered by the sources is:

$$45 + \frac{5}{6} = \frac{270 + 5}{6} = \frac{275}{6}\text{ W}$$

We see that the total power delivered by the sources is equal to the total power absorbed by the resistors. Since power delivered by a circuit element is equal to the negative of the power absorbed, this is equivalent to saying that the total power absorbed by all circuit elements is zero. Thus, the principle of conservation of energy (and therefore power) is satisfied in this circuit (as it is in any circuit).

EXAMPLE 2.14 Power Conservation

Consider the circuit shown below, which is identical to the previous example except for the value of the current source:



In this case:

$$p_1 = 25\,\text{W} \qquad p_2 = \frac{25}{2}\,\text{W} \qquad p_3 = \frac{25}{3}\,\text{W}$$

as before. By KCL, however:

$$i + 10 = i_1 + i_2 + i_3$$

and thus:

$$i = \frac{55}{6} - 10 = -\frac{5}{6}\,\text{A}$$

Therefore, the powers delivered by the sources are:

$$p_v = 5\left(-\frac{5}{6}\right) = -\frac{25}{6}\,\text{W} \qquad p_i = 5(10) = 50\,\text{W}$$

Hence the total power absorbed is:

$$p_1 + p_2 + p_3 + p_v + p_i = \frac{275}{6} + \frac{25}{6} - 50 = 0\,\text{W}$$

and again energy (power) is conserved. However, in this case not only do the resistors absorb power, but so does the voltage source. It is the current source that supplies all the power absorbed in the rest of the circuit.

2.7.1 Power Absorbed in a Resistor

The power absorbed in every resistor is always a nonnegative number. Consider the resistor shown below:

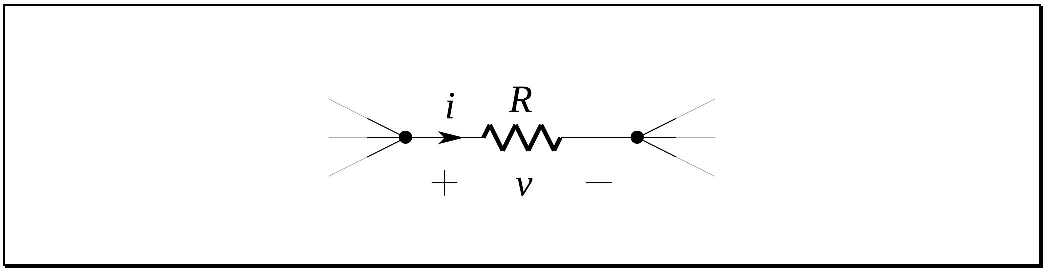


Figure 2.12

By definition, the power absorbed in the resistor is $p = vi$. But by Ohm's Law, $v = Ri$. Thus $p = (Ri)i$, or:

$$p = Ri^2 \tag{2.31}$$

Also, $i = v/R$, so that $p = v(v/R)$, or:

$$p = \frac{v^2}{R} \tag{2.32}$$

A real resistor always absorbs power

Both formulas for calculating power absorbed in a resistor R demonstrate that p is always a nonnegative number when R is positive. Therefore a resistor always absorbs power.

In a physical resistor, this power is dissipated as radiation (light and/or heat). In some types of resistors (such as an incandescent bulb, a toaster, or an electric heater), this property is desirable in that the net result may be light or warmth. In other types of resistors, such as those found in electronic circuits, the heat dissipated in a resistor may be a problem that cannot be ignored.

Real resistors have a power rating that must not be exceeded

The physical size of a resistor determines the amount of power it can safely dissipate. A power dissipation that exceeds the rating of a resistor can physically damage the resistor. In many electronic applications, resistors need dissipate only small amounts of power, allowing their use in integrated circuits.

2.8 Summary

- Kirchhoff's Current Law (KCL) states: "At any node of a circuit, the currents algebraically sum to zero":

$$\sum_{k=1}^{n} i_k = 0$$

- Kirchhoff's Voltage Law (KVL) states: "Around any loop in a circuit, the voltages algebraically sum to zero":

$$\sum_{k=1}^{n} v_k = 0$$

- Resistors in *series* can be combined into one equivalent resistor:

$$R_{eq} = R_1 + R_2 + \cdots + R_N$$

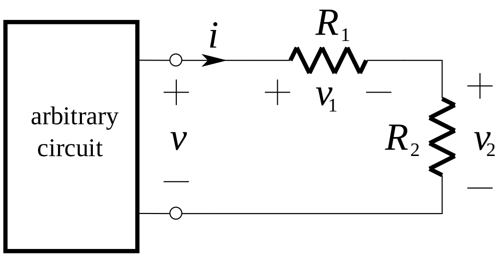
- Resistors in *parallel* can be combined into one equivalent resistor:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

- Independent voltage sources in *series* can be added. Independent current sources in *parallel* can be added.

2.30

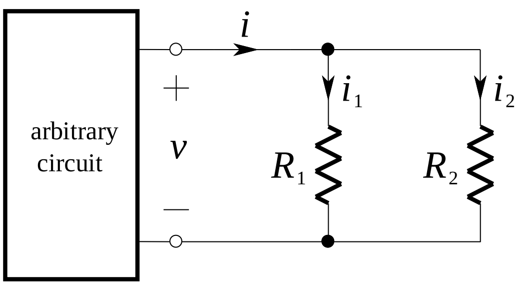
- Two resistors in series form a voltage divider:



The voltage divider rule is:

$$v_2 = \frac{R_2}{R_1 + R_2} v$$

- Two resistors in parallel form a current divider:



The current divider rule is:

$$i_2 = \frac{R_1}{R_1 + R_2} i$$

- The *instantaneous power absorbed* by an electrical circuit element is the product of voltage and current:

$$p = vi$$

The power absorbed in a resistor is:

$$p = Ri^2 = \frac{v^2}{R}$$

2.9 References

Hayt, W. & Kemmerly, J.: *Engineering Circuit Analysis*, 3rd *Ed*., McGraw-Hill, 1984.

Exercises

1.

The voltage v has its positive reference at terminal A of a certain circuit element. The power absorbed by the circuit element is $4(t-1)^2$ W for $t > 0$. If $v = (2t-2)$ V for $t > 0$, how much charge enters terminal A between $t = 0$ and $t = 2$ s?

2.

The resistance of 10 mm^2 copper wire is 1.725 Ω/km, and, with a certain type of insulation, it can safely carry 70 A without overheating. With a one kilometre length of wire operating at maximum current:

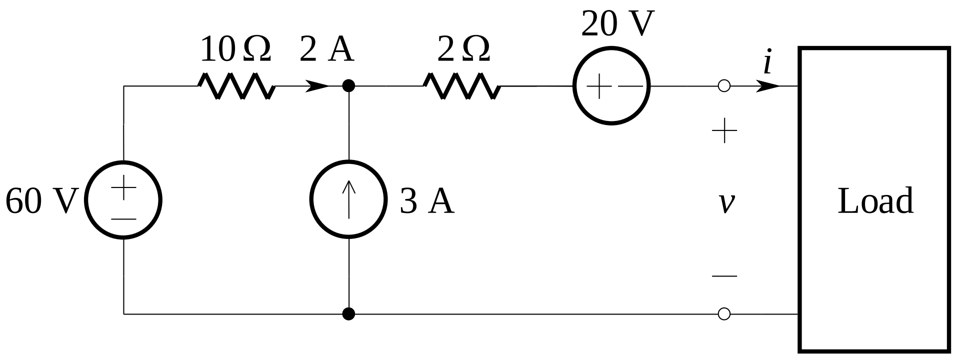
(a) What voltage exists between the ends of the wire?

(b) How much power is dissipated in the conductor?

(c) What is the power dissipation per square mm of surface area?

3.

For the circuit shown below:

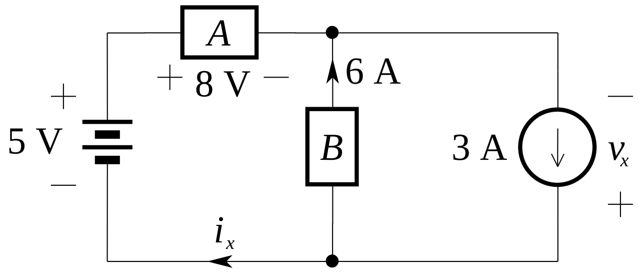


find:

(a) v (b) i (c) the power absorbed by the load.

4.

With reference to the network shown below:



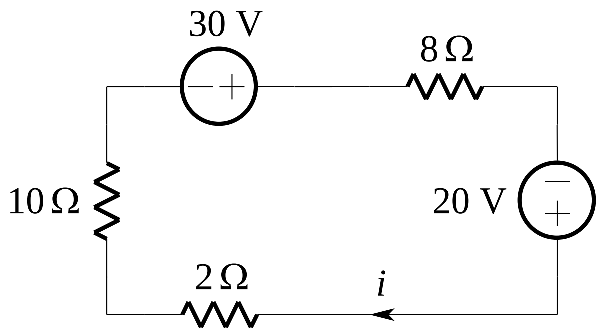
find:

(a) i_x (b) v_x (c) the power absorbed by the battery.

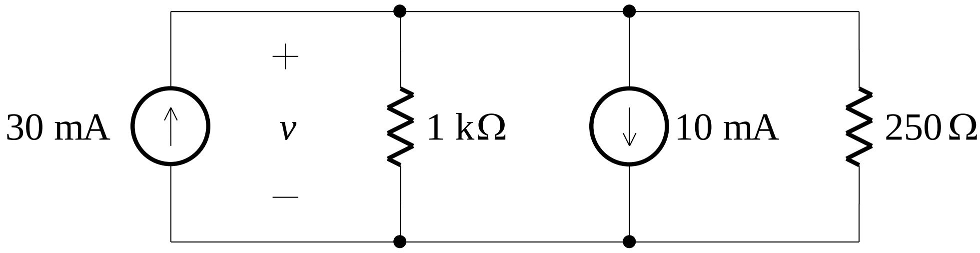
5.

By combining independent sources and resistances as appropriate, find:

(a) The current i in the circuit below.

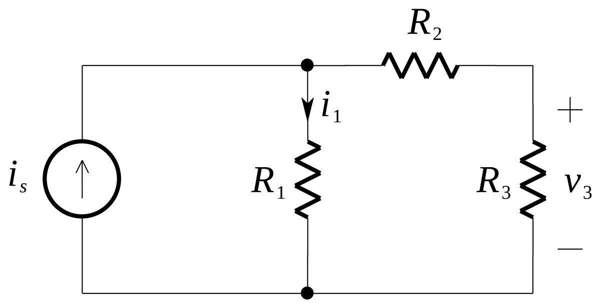


(b) The voltage v in the circuit below.



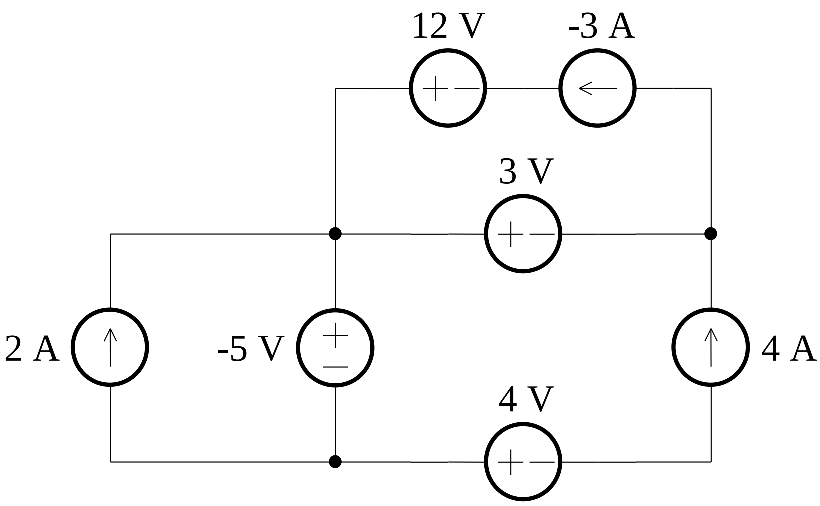
6.

Use the concepts of current division, voltage division, and resistance combination to write expressions (by inspection) for v_3 and i_1 in the circuit shown below:



7.

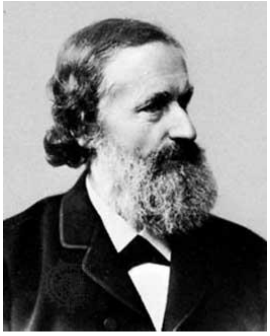
The circuit shown below exhibits several examples of independent current and voltage sources in series and in parallel.



(a) Find the power supplied by the -5 V source.

(b) To what value should the 4 A source be changed to reduce the power supplied by the -5 V source to zero?

Gustav Robert Kirchhoff (1824-1887)



Kirchhoff was born in Russia, and showed an early interest in mathematics. He studied at the University of Königsberg, and in 1845, while still a student, he pronounced Kirchhoff's Laws, which allow the calculation of current and voltage for *any* circuit. They are the Laws electrical engineers apply on a routine basis – they even apply to non-linear circuits such as those containing semiconductors, or distributed parameter circuits such as microwave striplines.

He graduated from university in 1847 and received a scholarship to study in Paris, but the revolutions of 1848 intervened. Instead, he moved to Berlin where he met and formed a close friendship with Robert Bunsen, the inorganic chemist and physicist who popularized use of the "Bunsen burner".

In 1857 Kirchhoff extended the work done by the German physicist Georg Simon Ohm, by describing charge flow in three dimensions. He also analysed circuits using topology. In further studies, he offered a general theory of how electricity is conducted. He based his calculations on experimental results which determine a constant for the speed of the propagation of electric charge. Kirchhoff noted that this constant is approximately the speed of light – but the greater implications of this fact escaped him. It remained for James Clerk Maxwell to propose that light belongs to the electromagnetic spectrum.

Kirchhoff's most significant work, from 1859 to 1862, involved his close collaboration with Bunsen. Bunsen was in his laboratory, analysing various salts that impart specific colours to a flame when burned. Bunsen was using coloured glasses to view the flame. When Kirchhoff visited the laboratory, he suggested that a better analysis might be achieved by passing the light from the flame through a prism. The value of spectroscopy became immediately clear. Each element and compound showed a spectrum as unique as any fingerprint, which could be viewed, measured, recorded and compared.

Spectral analysis, Kirchhoff and Bunsen wrote not long afterward, promises "the chemical exploration of a domain which up till now has been completely closed." They not only analysed the known elements, they discovered new ones. Analyzing salts from evaporated mineral water, Kirchhoff and Bunsen detected a blue spectral line – it belonged to an element they christened *caesium* (from the Latin *caesius*, sky blue). Studying lepidolite (a lithium-based mica) in 1861, Bunsen found an alkali metal he called rubidium (from the Latin *rubidius*, deepest red). Both of these elements are used today in atomic clocks. Using spectroscopy, ten more new elements were discovered before the end of the century, and the field had expanded enormously – between 1900 and 1912 a "handbook" of spectroscopy was published by Kayser in six volumes comprising five thousand pages!

"[Kirchhoff is] a perfect example of the true German investigator. To search after truth in its purest shape and to give utterance with almost an abstract self-forgetfulness, was the religion and purpose of his life." – Robert von Helmholtz, 1890.

Kirchhoff's work on spectrum analysis led on to a study of the composition of light from the Sun. He was the first to explain the dark lines (Fraunhofer lines) in the Sun's spectrum as caused by absorption of particular wavelengths as the light passes through a gas. Kirchhoff wrote "It is plausible that spectroscopy is also applicable to the solar atmosphere and the brighter fixed stars." We can now analyse the collective light of a hundred billion stars in a remote galaxy billions of light-years away – we can tell its composition, its age, and even how fast the galaxy is receding from us – simply by looking at its spectrum!

As a consequence of his work with Fraunhofer's lines, Kirchhoff developed a general theory of emission and radiation in terms of thermodynamics. It stated that a substance's capacity to emit light is equivalent to its ability to absorb it at the same temperature. One of the problems that this new theory created was the "blackbody" problem, which was to plague physics for forty years. This fundamental quandary arose because heating a black body – such as a metal bar – causes it to give off heat and light. The spectral radiation, which depends only on the temperature and not on the material, could not be predicted by classical physics. In 1900 Max Planck solved the problem by discovering quanta, which had enormous implications for twentieth-century science.

In 1875 he was appointed to the chair of mathematical physics at Berlin and he ceased his experimental work. An accident-related disability meant he had to spend much of his life on crutches or in a wheelchair. He remained at the University of Berlin until he retired in 1886, shortly before his death in 1887.