4 Mesh Analysis

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Introduction

Mesh analysis is a method that can be applied to any *planar* circuit (i.e. to circuits that are able to be laid out on a 2D surface without crossing elements). Both of these methods are widely used in hand design and computer simulation. A third technique, known as *loop analysis*, generalises mesh analysis and can be applied to any circuit – it is effectively the "dual" of nodal analysis.

We will find that the judicious selection of an analysis technique can lead to a drastic reduction in the number of equations to solve, and we should therefore try to develop an ability to select the most convenient analysis method for a particular circuit.

4.1 Mesh Analysis

Before we embark on mesh analysis, we need to define the concept of: a planar circuit; a path through a circuit; a loop and a mesh. We can then outline the analysis strategy using these terms.

4.1.1 Planar Circuits

A *planar* circuit is one where it is possible to draw the circuit on a plane surface in such a way that no branch passes over or under any other branch.

Planar circuits defined

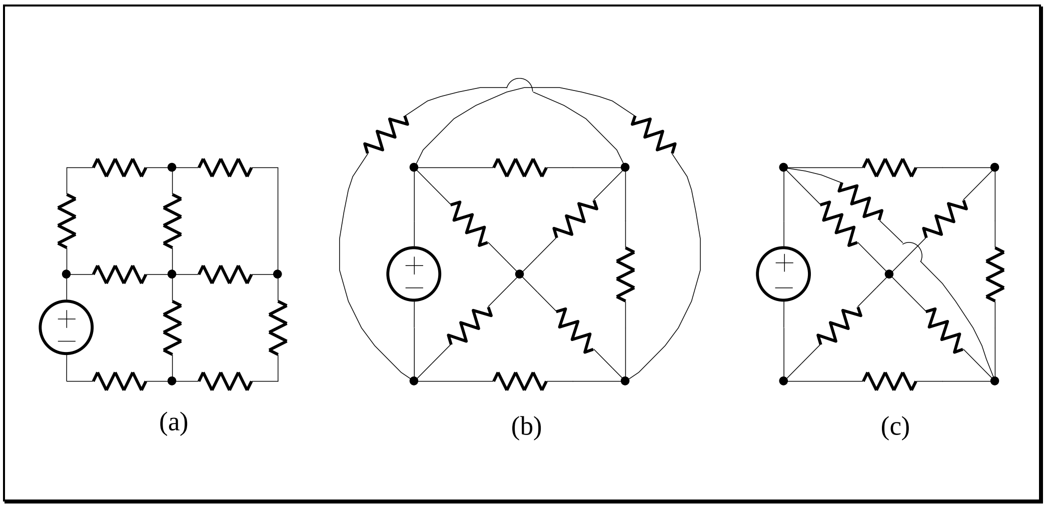


Figure 4.1

In the figure above, circuit (a) is planar, circuit (b) is nonplanar and circuit (c) is planar, but drawn so that it appears nonplanar.

4.1.2 Paths, Loops and Meshes

A *path* is made through a circuit when we start on one node and traverse elements and nodes without encountering any nodes previously visited. A *loop* is any *closed path* – i.e. the last node visited is the same as the starting node. A *mesh* is a loop which does not contain any other loops within it. A mesh is a property of a planar circuit and is not defined for a nonplanar circuit.

Examples of loops and meshes

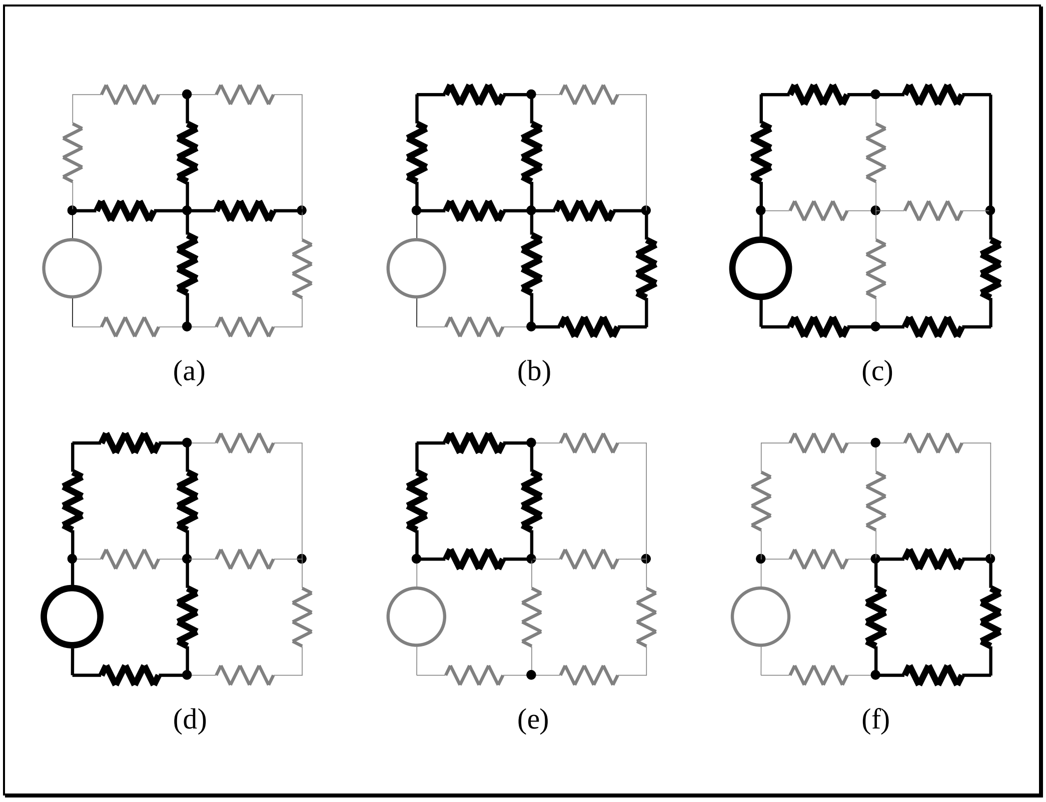


Figure 4.2

In the figure above, the set of branches in (a) identified by the heavy lines is neither a path nor a loop. In (b) the set of branches is not a path since it can be traversed only by passing through the central node twice. In (c) the closed path is a loop but not a mesh. In (d) the closed path is also a loop but not a mesh. In (e) and (f) each of the closed paths is both a loop and a mesh. This circuit contains four meshes.

4.1.3 Mesh Current

We define a *mesh current*[1] as a "mathematical" (or imaginary) current in which charge flows only around the perimeter of a mesh. A mesh current is indicated by a curved arrow that almost closes on itself and is drawn inside the appropriate mesh.

Mesh currents defined

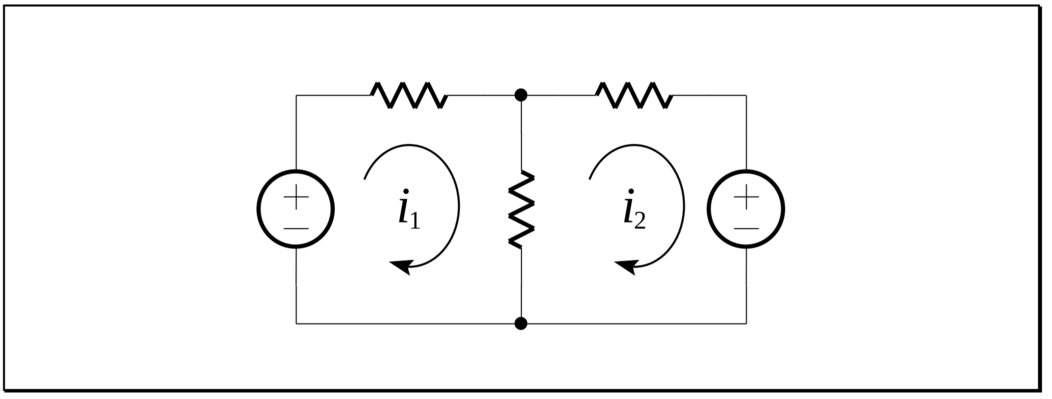


Figure 4.3

Although the direction of mesh currents is arbitrary, we draw the mesh currents in a clockwise direction so that a symmetry in the equations results when performing mesh analysis. One of the great advantages of mesh currents is that KCL is automatically satisfied, and no branch can appear in more than two meshes.

We no longer have a current or current arrow shown on each branch in the circuit. The current through any branch may be determined by superimposing each mesh current that exists in it. For example, the branch current heading *down* in the middle resistor in the circuit above is given by $(i_1 - i_2)$.

Branch currents can be expressed in terms of mesh currents

[1] It was the famous Scottish mathematical physicist James Clark Maxwell who invented the concept of a mesh current, and the associated methodology of formulating the "mesh equations". The analysis of planar circuits using mesh currents was thus reduced to solving a set of linear equations, in the same manner as nodal analysis.

4.1.4 Mesh Analysis Methodology

In general terms, mesh analysis for a planar circuit with *M* meshes proceeds as follows:

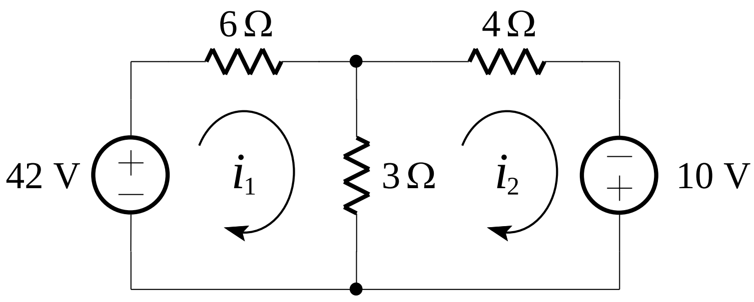
The general principle of mesh analysis

1. Assign a clockwise mesh current in each of the *M* meshes.
2. Write KVL around each mesh, in terms of the mesh currents.
3. Solve the resulting set of simultaneous equations.

As will be seen, the method outlined above becomes a little complicated if the circuit contains current sources and / or controlled sources, but the principle remains the same.

EXAMPLE 4.1 Mesh Analysis with Independent Sources

A two-mesh circuit is shown below.



We apply KVL to each mesh. For the left-hand mesh:

$$42 - 6i_1 - 3(i_1 - i_2) = 0$$
$$9i_1 - 3i_2 = 42$$

For the right-hand mesh:

$$-3(i_2 - i_1) - 4i_2 + 10 = 0$$
$$-3i_1 + 7i_2 = 10$$

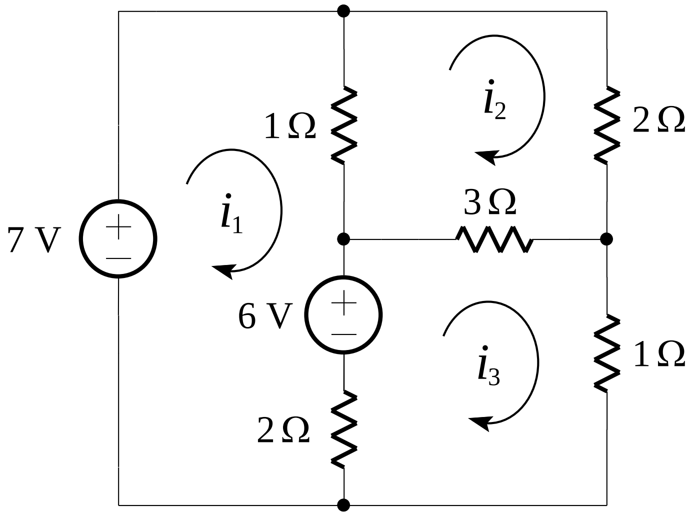
The solution is obtained by solving simultaneously: $i_1 = 6\text{ A}$ and $i_2 = 4\text{ A}$.

4.1.5 Circuits with Resistors and Independent Voltage Sources Only

When the circuit contains only resistors and voltages sources, the KVL equations have a certain symmetrical form and we can define a *resistance matrix* with the circuit. We will find again that the matrix equation can be formulated by inspection of the circuit.

EXAMPLE 4.2 Mesh Analysis with Independent Sources Only

Consider the five-node, three-mesh circuit shown below.



The three required mesh currents are assigned as indicated, and we methodically apply KVL about each mesh:

$$\begin{aligned}
7-1(i_1-i_2)-6-2(i_1-i_3)&=0\\
-1(i_2-i_1)-2i_2-3(i_2-i_3)&=0\\
-2(i_3-i_1)+6-3(i_3-i_2)-1i_3&=0
\end{aligned}$$

Simplifying and writing as a matrix equation:

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}=\begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$

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The resistance matrix defined

For circuits that contain only resistors and independent voltage sources, we define the *resistance matrix* of the circuit as:

$$\mathbf{R} = \begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix}$$

Once again we note the symmetry about the major diagonal. This occurs *only* for circuits with resistors and independent voltage sources when we order the equations correctly (rows correspond to meshes).

We also define the current and voltage source vectors as:

$$\mathbf{i} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$

Our KVL equations can therefore be written succinctly in matrix notation as:

Mesh analysis expressed in matrix notation

$$\mathbf{Ri} = \mathbf{v}$$

Applying Cramer's rule to the formulation for i_1 gives:

$$i_1 = \frac{\begin{vmatrix} 1 & -1 & -2 \\ 0 & 6 & -3 \\ 6 & -3 & 6 \end{vmatrix}}{\begin{vmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{vmatrix}} = \frac{27 - 0 + 90}{81 - 12 - 30} = \frac{117}{39} = 3\,\text{A}$$

The other mesh currents are: $i_2 = 2\,\text{A}$ and $i_3 = 3\,\text{A}$.

4.1.6 Circuits with Current Sources

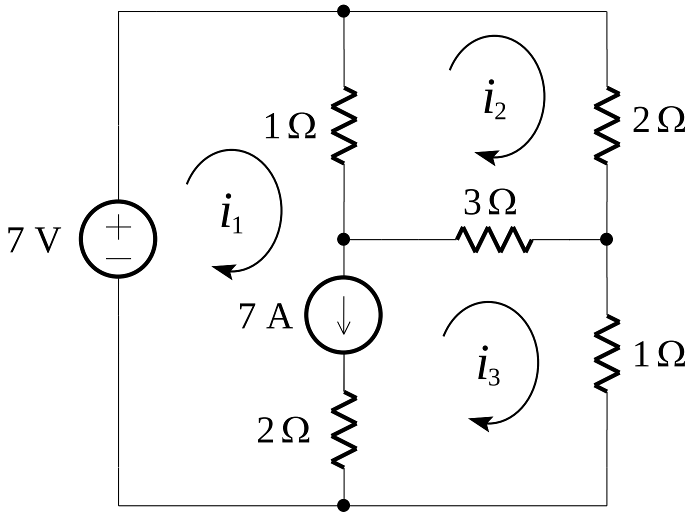
When a mesh has a current source in it, we must modify the procedure for forming the circuit equations. There are two possible methods. In the first method, we can relate the source current to the assigned mesh currents, assign an arbitrary voltage across it (thereby increasing the number of variables by one) and write KVL equations using this voltage. Alternately, a better method is to take a lead from nodal analysis and formulate the *dual* of a supernode - a *supermesh*.

To create a supermesh, we open-circuit or remove current sources, thereby reducing the total number of meshes. We apply KVL only to those meshes in the modified circuit.

The concept of a supermesh

EXAMPLE 4.3 Mesh Analysis with Current Sources

Consider the circuit shown below in which a 7 A independent current source is in the common boundary of two meshes.



For the independent current source, we relate the source current to the mesh currents:

$$i_1 - i_3 = 7$$

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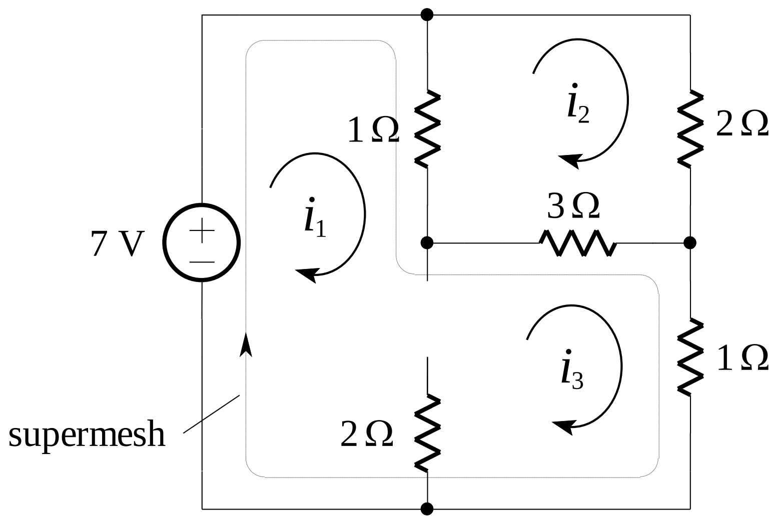
We then mentally open-circuit the current source, and form a supermesh whose interior is that of meshes 1 and 3:



Applying KVL about the supermesh:

$$7 - 1(i_1 - i_2) - 3(i_3 - i_2) - 1i_3 = 0$$

Around mesh 2 we have:

$$-1(i_2 - i_1) - 2i_2 - 3(i_2 - i_3) = 0$$

Rewriting these last three equations in matrix form, we have:

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & -4 & 4 \\ -1 & 6 & -3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 7 \\ 0 \end{bmatrix}$$

Notice that we have lost all symmetry in the matrix equation $\mathbf{Ri} = \mathbf{v}$, and we can no longer call $\mathbf{R}$ the resistance matrix. Applying Cramer's rule for i_1:

$$i_1 = \frac{\begin{vmatrix} 7 & 0 & -1 \\ 7 & -4 & 4 \\ 0 & 6 & -3 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & -1 \\ 1 & -4 & 4 \\ -1 & 6 & -3 \end{vmatrix}} = \frac{-84 - 42}{-12 - 2} = \frac{-126}{-14} = 9\text{ A}$$

The other mesh currents are: $i_2 = 2.5\text{ A}$ and $i_3 = 2\text{ A}$.

4.1.7 Summary of Mesh Analysis

We perform mesh analysis for any resistive circuit with M meshes by the following method:

The general procedure to follow when undertaking mesh analysis

1. Make certain that the circuit is a planar circuit. If it is nonplanar, then mesh analysis is not applicable.

2. Make a neat, simple, circuit diagram. Indicate all element and source values. Each source should have its reference symbol.

3. Assuming that the circuit has M meshes, assign a clockwise mesh current in each mesh, i_1, i_2, …, i_M.

4. If the circuit contains current sources, temporarily modify the given circuit by replacing each current source by an open-circuit to form *supermeshes*, thus reducing the number of meshes by one for each current source that is present. The assigned mesh currents should not be changed. Relate each source current to the mesh currents.

5. Apply KVL around each of the meshes or supermeshes. If the circuit has only resistors and independent voltage sources, then the equations may be formed by inspection.

6. Solve the resulting set of simultaneous equations.

4.2 Summary

- Mesh analysis can only be applied to planar circuits. Apart from relating source currents to mesh currents, the equations of mesh analysis are formed from application of Kirchhoff's Voltage Law.

- In mesh analysis, a supermesh is formed by open-circuiting a current source and treating the perimeter of the original two meshes as a single mesh.

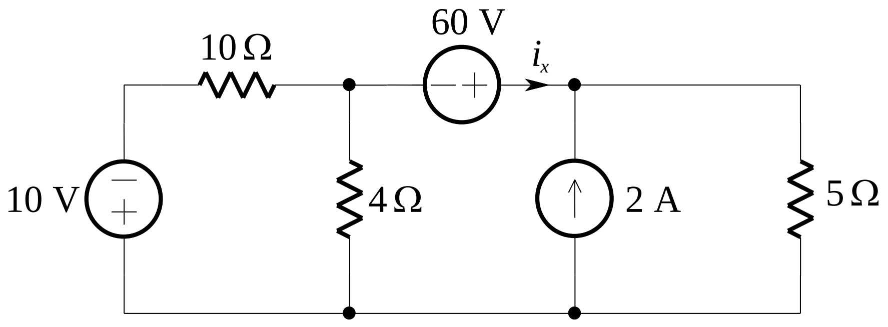
4.3 References

Hayt, W. & Kemmerly, J.: *Engineering Circuit Analysis*, 3rd *Ed*., McGraw-Hill, 1984.

Exercises

1.

Write mesh equations and then determine i_x in the circuit shown below:



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