21 The Sinusoidal Steady-State Response

Contents

21.2

Introduction

In the analysis of resistive circuits of arbitrary complexity, we are able to employ many different circuit analysis techniques to determine the response – nodal analysis, mesh analysis, superposition, source transformations, Thévenin's and Norton's theorems.

Sometimes one method is sufficient, but more often we find it convenient to combine several methods to obtain the response in the most direct manner.

We now want to extend these techniques to the analysis of circuits in the sinusoidal steady-state. We have already seen that impedances combine in the same manner as do resistances. We have seen that KVL and KCL are obeyed by phasors, and we also have an Ohm-like law for the passive elements, $\mathbf{V} = \mathbf{ZI}$. We can therefore extend our resistive circuit analysis techniques to the frequency-domain to determine the phasor response, and therefore the sinusoidal steady-state response.

21.1 Analysis using Phasors

Phasors can only be used for sinusoidal steady-state analysis.

Phasor analysis is a *transform method* of analysis. In phasor analysis we transform a problem from the time-domain to the frequency-domain. To find a response in the frequency-domain, we solve equations using complex algebra. Once the response is found, we transform the solutions back to the time-domain. This is illustrated conceptually below:

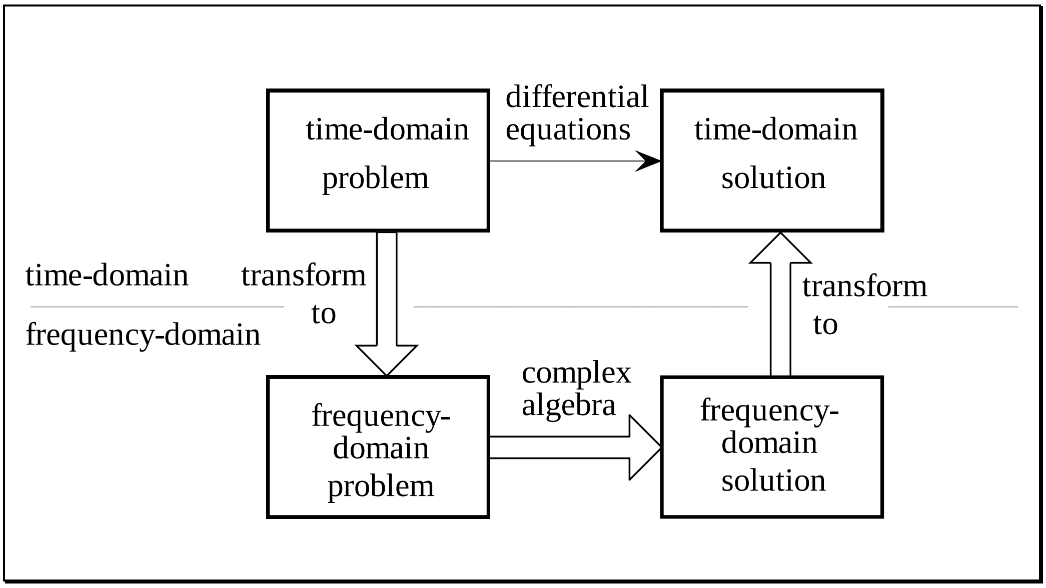


Figure 21.1

Transform methods are a common method of analysis in branches of engineering, and you will be introduced to more powerful transform methods in more advanced subjects.

21.2 Nodal Analysis

As an example of nodal analysis, consider the circuit shown below:

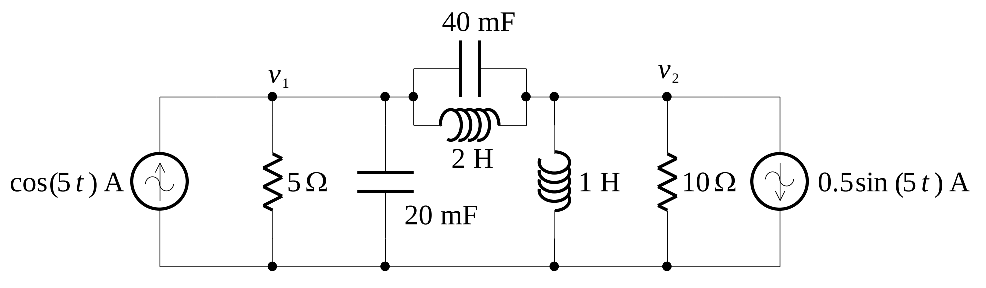


Figure 21.2

Noting from the sources that $\omega = 5\,\text{rads}^{-1}$, we draw the frequency-domain circuit and assign nodal voltages $\mathbf{V}_1$ and $\mathbf{V}_2$:

Nodal analysis in the frequency-domain

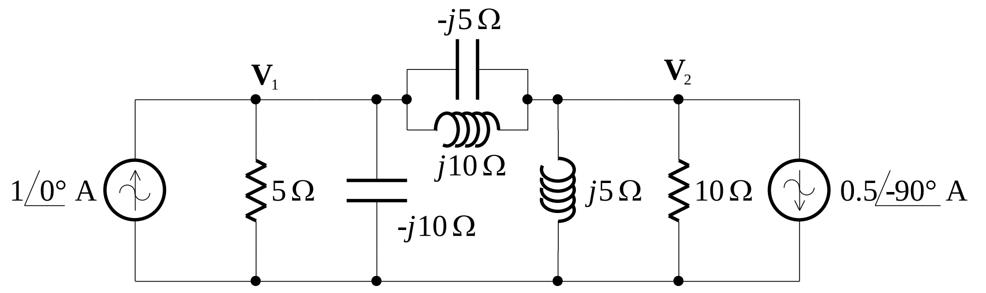


Figure 21.3

Each passive element is specified by its impedance, which has been determined by knowing the frequency of the sources (which are the same) and the element values. Two current sources are given as phasors, and phasor node voltages $\mathbf{V}_1$ and $\mathbf{V}_2$ are indicated.

At the left node, we apply KCL and $\mathbf{I} = \mathbf{V}/\mathbf{Z}$:

$$\frac{\mathbf{V}_1}{5} + \frac{\mathbf{V}_1}{-j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j10} = 1 + j0 \tag{21.1}$$

At the right node:

$$\frac{\mathbf{V}_2-\mathbf{V}_1}{-j5}+\frac{\mathbf{V}_2-\mathbf{V}_1}{j10}+\frac{\mathbf{V}_2}{j5}+\frac{\mathbf{V}_2}{10}=-(-j0.5) \tag{21.2}$$

Combining terms we have the two equations:

$$\begin{aligned}(0.2+j0.2)\mathbf{V}_1-j0.1\mathbf{V}_2&=1\\ -j0.1\mathbf{V}_1+(0.1-j0.1)\mathbf{V}_2&=j0.5\end{aligned} \tag{21.3}$$

Using Cramer's Rule to solve, we obtain:

$$\begin{aligned}\mathbf{V}_1&=\frac{\begin{vmatrix}1 & -j0.1\\ j0.5 & (0.1-j0.1)\end{vmatrix}}{\begin{vmatrix}(0.2+j0.2) & -j0.1\\ -j0.1 & (0.1-j0.1)\end{vmatrix}}=\frac{0.1-j0.1-0.05}{0.02-j0.02+j0.02+0.02+0.01}\\ &=\frac{0.05-j0.1}{0.05}=1-j2\text{ V}\\ \mathbf{V}_2&=\frac{\begin{vmatrix}(0.2+j0.2) & 1\\ -j0.1 & j0.5\end{vmatrix}}{0.05}=\frac{-0.1+j0.1+j0.1}{0.05}=-2+j4\text{ V}\end{aligned} \tag{21.4}$$

The time-domain solutions are best obtained by representing the phasors in polar form:

$$\begin{aligned}\mathbf{V}_1&=\sqrt{5}\angle-63.4^\circ\text{ V}\\ \mathbf{V}_2&=2\sqrt{5}\angle116.6^\circ\text{ V}\end{aligned} \tag{21.5}$$

and passing to the time-domain:

$$\begin{aligned}v_1(t)&=\sqrt{5}\cos(5t-63.4^\circ)\text{ V}\\ v_2(t)&=2\sqrt{5}\cos(5t+116.6^\circ)\text{ V}\end{aligned} \tag{21.6}$$

Note how simple phasor analysis is compared to the work involved if we stayed in the time-domain solving differential equations!

21.3 Mesh Analysis

As an example of mesh analysis, consider the circuit shown below:

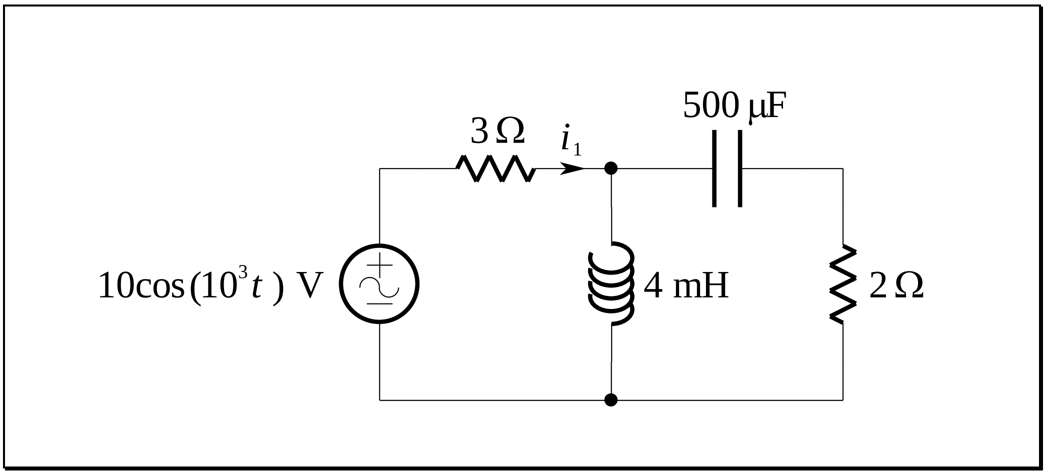


Figure 21.4

Noting from the source that $\omega = 10^3 \text{ rads}^{-1}$, we draw the frequency-domain circuit and assign mesh currents $\mathbf{I}_1$ and $\mathbf{I}_2$:

Mesh analysis in the frequency-domain

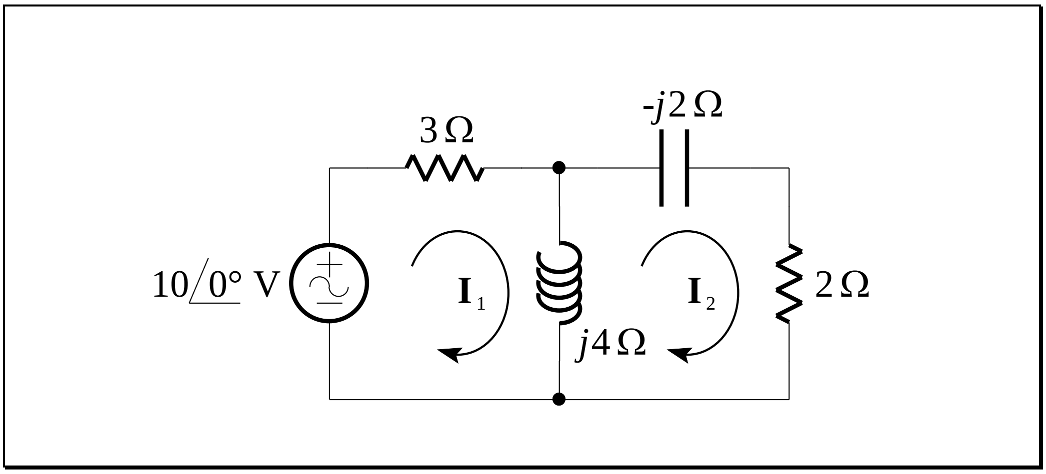


Figure 21.5

Around mesh 1:

$$3\mathbf{I}_1 + j4(\mathbf{I}_1 - \mathbf{I}_2) = 10\angle 0° \tag{21.7}$$

while mesh 2 leads to:

$$j4(\mathbf{I}_2 - \mathbf{I}_1) - j2\mathbf{I}_2 + 2\mathbf{I}_2 = 0 \tag{21.8}$$

Combining terms we have the two equations:

$$\begin{aligned} (3+j4)\mathbf{I}_1 - j4\mathbf{I}_2 &= 10 \\ -j4\mathbf{I}_1 + (2+j2)\mathbf{I}_2 &= 0 \end{aligned} \tag{21.9}$$

Solving:

$$\begin{aligned} \mathbf{I}_1 &= \frac{10}{7} = \frac{10}{7}\angle 0^\circ \text{ A} \\ \mathbf{I}_2 &= \frac{10}{7}(1+j) = \frac{10\sqrt{2}}{7}\angle 45^\circ \text{ A} \end{aligned} \tag{21.10}$$

or:

$$\begin{aligned} i_1(t) &= \frac{10}{7}\cos\left(10^3 t\right) \text{ A} \\ i_2(t) &= \frac{10\sqrt{2}}{7}\cos\left(10^3 t + 45^\circ\right) \text{ A} \end{aligned} \tag{21.11}$$

The solution above could be checked by working entirely in the time-domain, but it would be quite an undertaking!

21.4 Superposition

Linear circuits are those that consist of idealised linear passive circuit elements (R, L and C) and ideal independent voltage and current sources. Such circuits are amenable to the superposition principle.

We can analyse linear circuits with phasors and the principle of superposition. (You may remember that linearity and superposition were invoked when we combined real and imaginary sources to obtain a complex source).

Let's look again at the circuit of Figure 21.3, redrawn below with each pair of parallel impedances replaced by a single equivalent impedance (for example, 5 and $-j10$ in parallel yield $4-j2$):

Superposition in the frequency-domain

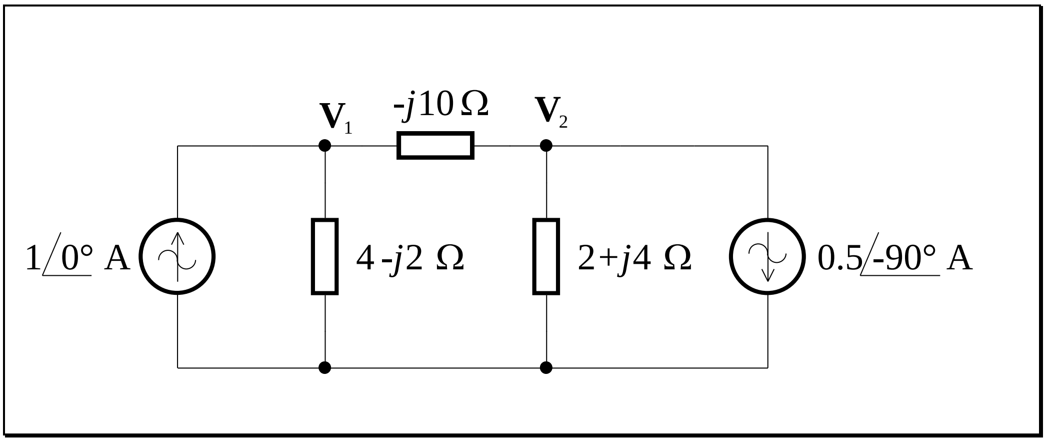


Figure 21.6

To find $\mathbf{V}_1$ we first activate only the left source and find the partial response:

$$\mathbf{V}_{1L} = 1\angle 0° \frac{(4-j2)(-j10+2+j4)}{4-j2-j10+2+j4} = \frac{-4-j28}{6-j8} = 2-j2 \tag{21.12}$$

With only the right source active, current division helps us to obtain:

$$\mathbf{V}_{1R} = (-0.5\angle -90°)\left(\frac{2+j4}{4-j2-j10+2+j4}\right)(4-j2) = \frac{-6+j8}{6-j8} = -1 \tag{21.13}$$

Summing, we get:

$$\mathbf{V}_1 = 2-j2-1 = 1-j2 \text{ V} \tag{21.14}$$

which agrees with our previous result.

21.5 Thévenin's Theorem

We will use the same circuit to see whether Thévenin's Theorem can help us:

Thévenin's Theorem in the frequency-domain

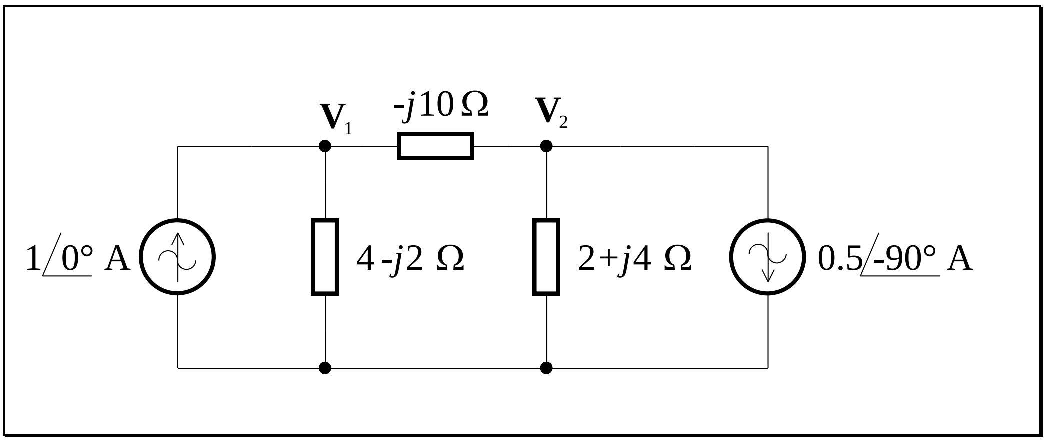


Figure 21.7

Suppose we determine the Thévenin equivalent faced by the $-j10\,\Omega$ impedance. The open circuit voltage (+ reference to the left) is:

$$\begin{aligned}\mathbf{V}_{oc} &= (1\angle 0°)(4-j2)+(0.5\angle -90°)(2+j4) \\ &= 4-j2+2-j1=6-j3\end{aligned} \tag{21.15}$$

The impedance of the inactive circuit, as viewed from the load terminals, is simply the sum of the two remaining impedances (because the current sources are set to zero – open circuits). Hence:

$$\mathbf{Z}_{th} = 6+j2 \tag{21.16}$$

21.10

Thus, when we reconnect the circuit, the current directed from node 1 toward node 2 through the $-j10\,\Omega$ load is:

$$\mathbf{I}_{12}=\frac{6-j3}{6+j2-j10}=0.6+j0.3 \tag{21.17}$$

Subtracting this from the left source current, the downward current through the $4-j2\,\Omega$ branch is found:

$$\mathbf{I}_1=1-(0.6+j0.3)=0.4-j0.3 \tag{21.18}$$

and , thus:

$$\mathbf{V}_1=\mathbf{Z}_1\mathbf{I}_1=(4-j2)(0.4-j0.3)=1-j2\text{ V} \tag{21.19}$$

21.6 Norton's Theorem

Again using the same circuit, if our chief interest is in $\mathbf{V}_1$ we could use Norton's Theorem on the three right elements:

Norton's Theorem in the frequency-domain

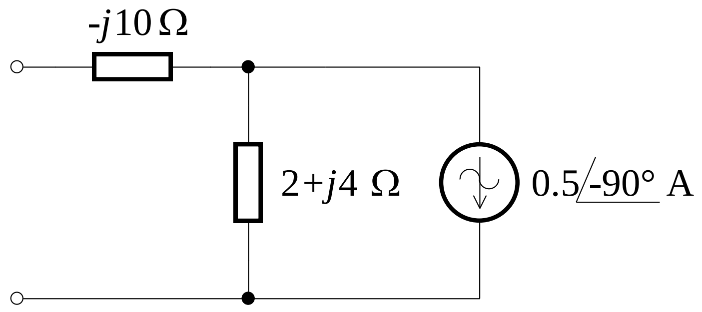


Figure 21.8

The short circuit current is obtained using current division:

$$\mathbf{I}_{sc} = \frac{2+j4}{2+j4-j10}\left(-0.5\angle-90°\right)$$
$$= \frac{-2+j}{2-j6} = -\frac{1+j}{4} \text{ A} \qquad (21.20)$$

and the Norton impedance (equal to the Thévenin impedance) is simply:

$$\mathbf{Z}_{th} = 2-j6 \qquad (21.21)$$

We thus need to analyse the circuit:

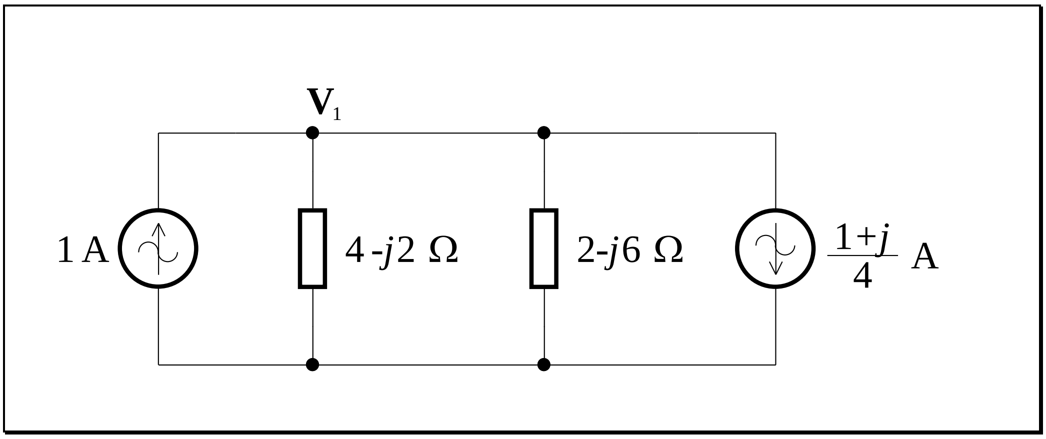


Figure 21.9

The voltage $\mathbf{V}_1$ is therefore:

$$\mathbf{V}_1 = \frac{(4-j2)(2-j6)}{(4-j2+2-j6)}(1-(0.25+j0.25))$$
$$= \frac{-4-j28}{6-j8}(0.75-j0.25) = (2-j2)(0.75-j0.25) = 1-j2 \text{ V} \quad (21.22)$$

It should now be clear that all methods available for linear circuit analysis can be applied to the frequency-domain. The slight additional complexity that is apparent now arises from the necessity of using complex numbers and not from any more involved theoretical considerations.

21.7 Phasor Diagrams

The phasor diagram is a sketch in the complex plane of the phasor voltages and currents throughout a specific circuit. It provides a graphical method for solving problems which may be used to check more exact analytical methods.

A phasor diagram is a graphical sketch of phasors in the complex plane

Since phasor voltages and currents are complex numbers, they may be identified as points in a complex plane. For example, the phasor voltage $\mathbf{V}_1 = 6 + j8 = 10\angle 53.1°$ is identified on the complex voltage plane shown below:

A simple phasor diagram

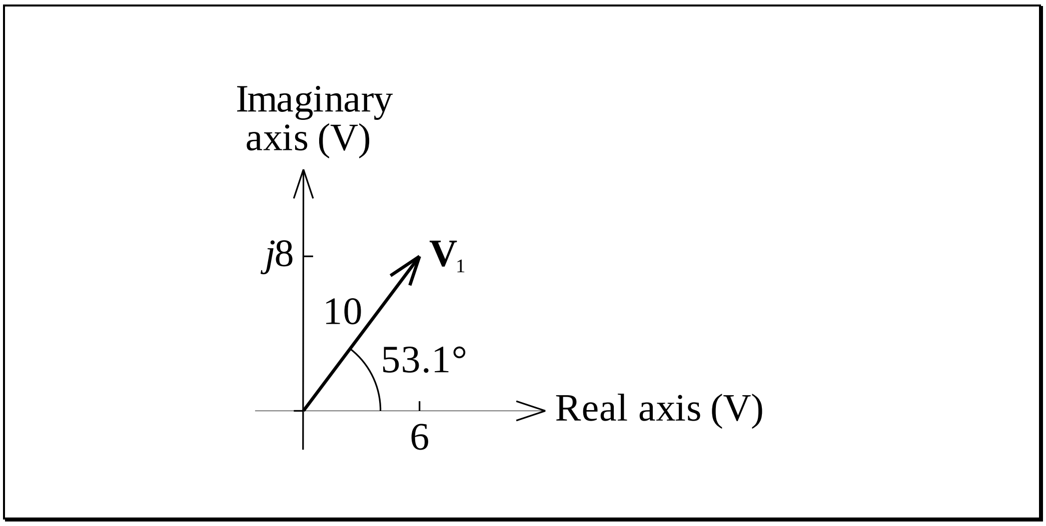


Figure 21.10

The axes are the real voltage axis and the imaginary voltage axis. The voltage $\mathbf{V}_1$ is located by an arrow drawn from the origin. Since addition and subtraction are particularly easy to perform and display on a complex plane, it is apparent that phasors may be easily added and subtracted in a phasor diagram. Multiplication and addition result in a change in magnitude and the addition and subtraction of angles.

Figure 21.11 shows the sum of $\mathbf{V}_1$ and a second phasor voltage $\mathbf{V}_2 = 3 - j4 = 5\angle -53.1°$:

Phasor diagram showing addition

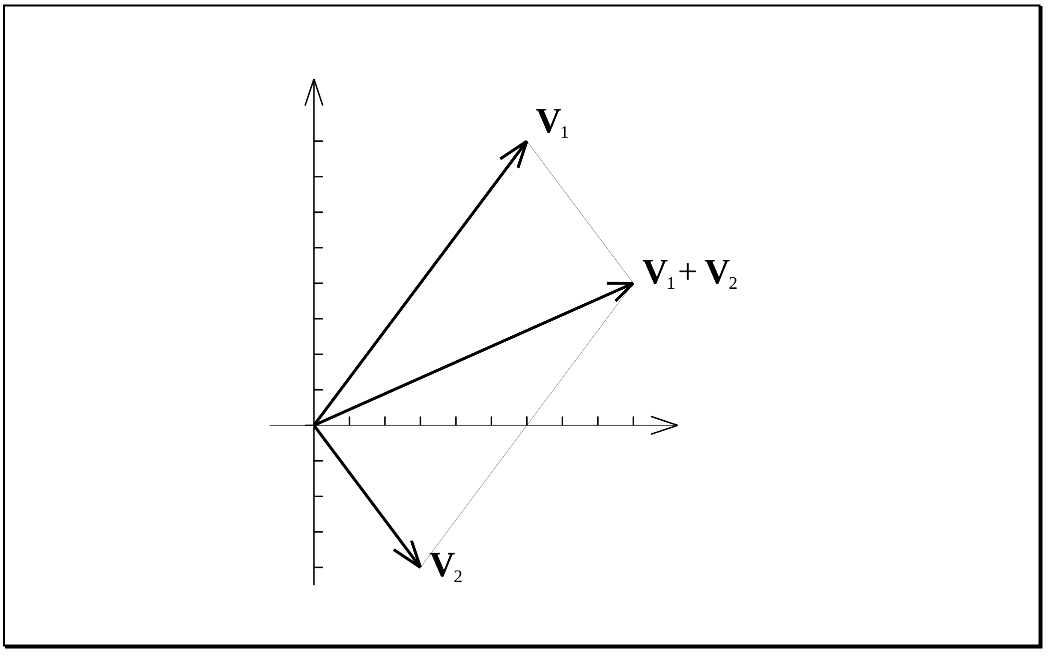


Figure 21.11

Figure 21.12 shows the current $\mathbf{I}_1$, which is the product of $\mathbf{V}_1$ and the admittance $\mathbf{Y} = 1 + j1$:

Phasor diagram showing multiplication

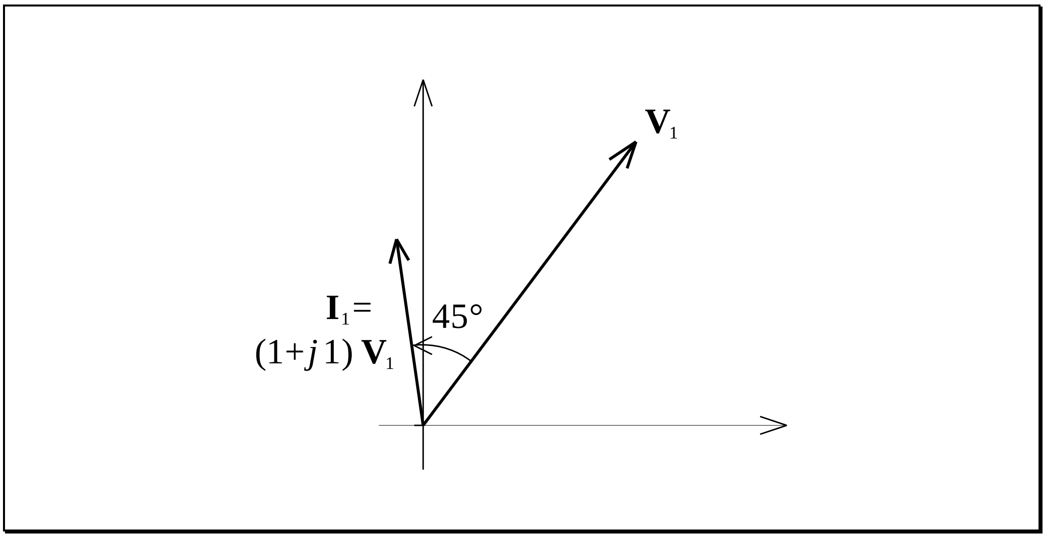


Figure 21.12

This last phasor diagram shows both current and voltage phasors on the same complex plane – it is understood that each will have its own amplitude scale, but a common angle scale.

The phasor diagram can also show the connection between the frequency-domain and the time-domain. For example, let us show the phasor $\mathbf{V} = V_m \angle \theta$ on the phasor diagram, as in (a) below:

Phasor diagram showing transformation to the time-domain by rotating the phasor and taking the real part

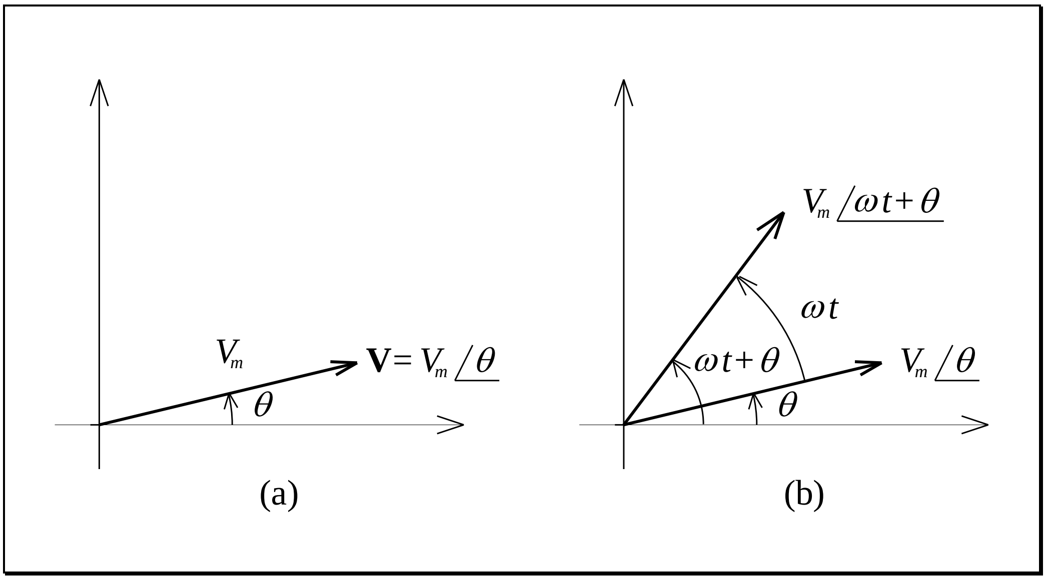


Figure 21.13

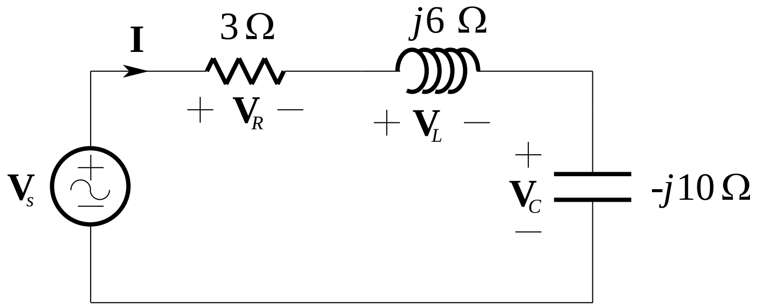
In order to transform $\mathbf{V}$ to the time-domain, we first need to multiply by $e^{j\omega t}$. We now have the complex voltage $\mathbf{V} = V_m e^{j\theta} e^{j\omega t} = V_m \angle \omega t + \theta$. This voltage may be interpreted as a phasor which possesses a phase angle that increases linearly with time. On a phasor diagram it therefore represents a rotating line segment, the instantaneous position being ωt rad ahead (counterclockwise) of $V_m \angle \theta$. Both $V_m \angle \theta$ and $V_m \angle \omega t + \theta$ are shown on the phasor diagram in (b).

The transformation to the time-domain is completed by taking the real part of $V_m \angle \omega t + \theta$, which is the projection of the phasor onto the real axis. It is helpful to think of the arrow representing the phasor $\mathbf{V}$ on the phasor diagram as the snapshot, taken at $t = 0$, of a rotating arrow whose projection onto the real axis is the instantaneous voltage $v(t)$.

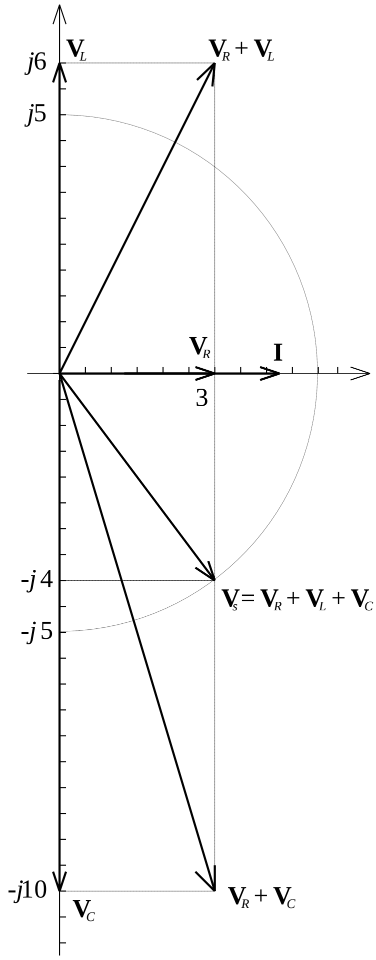
See the "Phasors" PC program

EXAMPLE 21.1 Phasor Diagram of a Series *RLC* Circuit

The series *RLC* circuit shown below has several different voltages associated with it, but only a single current:

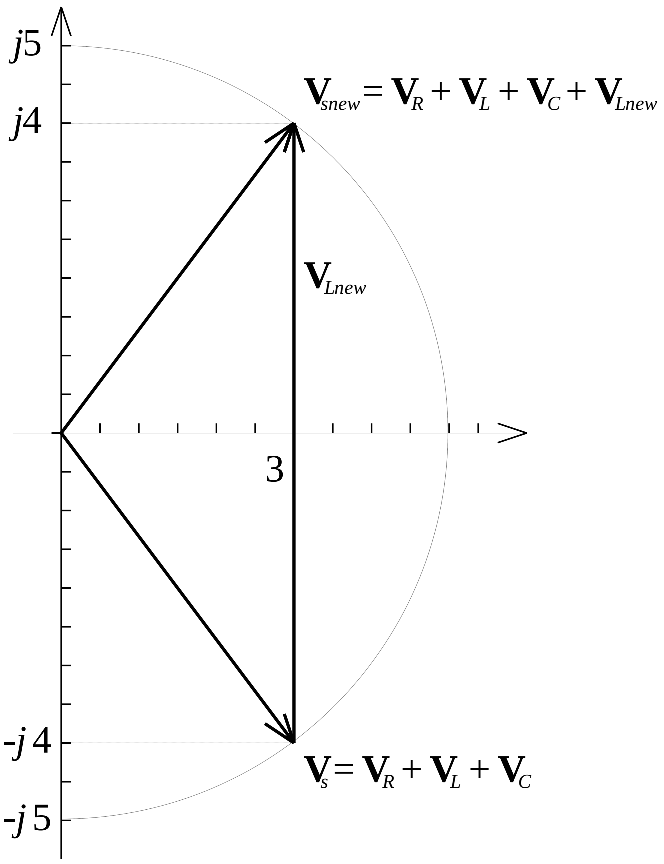


The phasor diagram is constructed most easily by employing the single current as the *reference* phasor – all other phasors with have their angles measured with respect to the reference. Let us arbitrarily set $\mathbf{I} = I_m \angle 0°$ and place it along the real axis of the phasor diagram, as shown below:



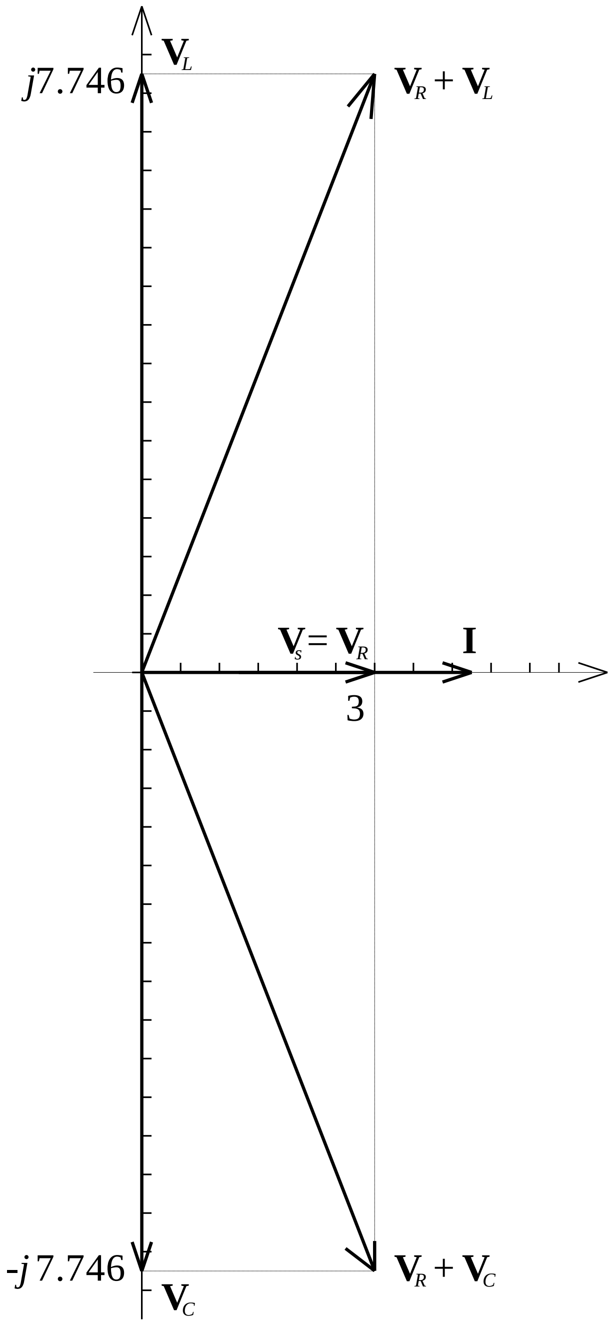
The resistor, inductor and capacitor voltages may next be calculated and placed on the diagram, where the $90°$ phase relationships stand out clearly. The sum of these three voltages is the source voltage for this circuit. The total voltage across the resistance and inductance or resistance and capacitance or inductance and capacitance is easily obtained from the phasor diagram.

We can design using the phasor diagram quite easily instead of embarking on complex algebraic manipulation. For example, suppose we would like to determine a single extra passive element that can be added in series with the circuit so that the magnitude of the current does not change. This additional circuit element will contribute to an additional voltage drop, but we still must have KVL satisfied so that the total voltage drop magnitude equals the source voltage magnitude. Therefore, the addition of the voltage drop due to the new element must keep the source voltage on a circle of radius $|\mathbf{V}_s|$. From the phasor diagram, we can see that we can only add an inductor with an impedance $\mathbf{Z}_{Lnew} = j8\,\Omega$, so that the additional voltage drop still brings us onto the circle of radius $|\mathbf{V}_s|$:



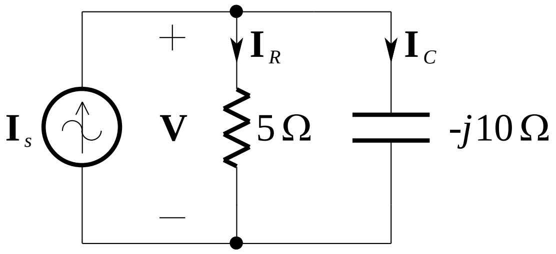
21.18

As another example of the use of the phasor diagram, consider the original circuit again. We know that an increase in frequency will cause the voltage across the inductor to increase, whilst simultaneously decreasing the voltage across the capacitor (although not linearly). In fact, if we increase the frequency by 29.1%, the inductor voltage and capacitor voltage will exactly cancel one another, and we have a condition known as resonance. In this case the supply voltage and current are precisely in phase, and the circuit appears resistive:

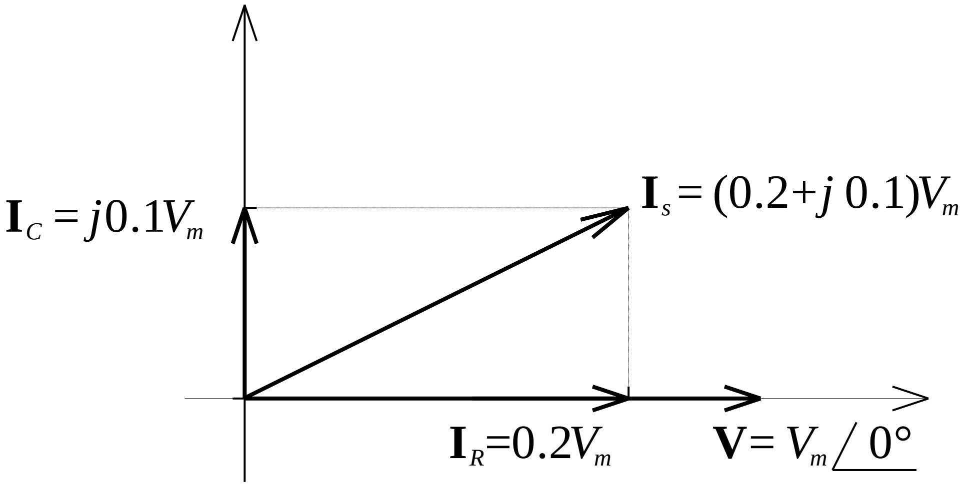


EXAMPLE 21.2 Phasor Diagram of a Parallel *RC* Circuit

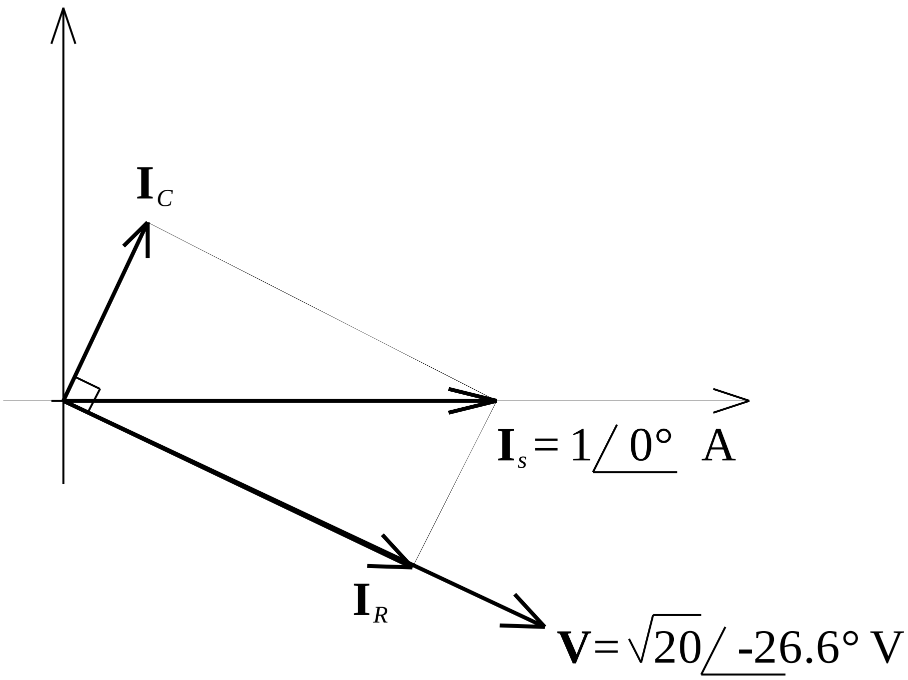
The figure below shows a simple parallel circuit in which it is logical to use the single voltage between the two nodes as a reference phasor:



Let us arbitrarily set $\mathbf{V} = V_m \angle 0°$ and place it along the real axis of the phasor diagram. The resistor current is in phase with this voltage, $\mathbf{I}_R = 0.2V_m$ A, and the capacitor current leads the reference voltage by $90°$, $\mathbf{I}_C = j0.1V_m$ A. After these two currents are added to the phasor diagram, shown below, they may be summed to obtain the source current. The result is $\mathbf{I}_s = (0.2 + j0.1)V_m$ A.



If the source current were specified initially as, for example, $\mathbf{I}_s = 1\angle 0°\ \text{A}$, and the node voltage is not initially known, it is still convenient to begin construction of the phasor diagram by assuming, say $\mathbf{V} = 1\angle 0°$. The source current, as a result of the assumed node voltage, is now $\mathbf{I}_s = 0.2 + j0.1\ \text{A}$. The true source current is $1\angle 0°\ \text{A}$, however, and thus the true node voltage is greater by the factor $1/(0.2 + j0.1)$; the true node voltage is therefore $4 - j2\ \text{V}$. The assumed voltage leads to a phasor diagram which differs from the true phasor diagram by a change of scale (the assumed diagram is smaller by a factor of $1/\sqrt{20}$) and an angular rotation (the assumed diagram is rotated clockwise through $26.6°$). The true phasor diagram in this case is shown below:



Phasor diagrams are usually very simple to construct, and most sinusoidal steady-state analyses will be more meaningful if such a diagram is included.

21.8 Summary

- A linear circuit can be converted to the frequency-domain where we use the concept of phasors and impedances, and in particular the branch phasor relationship $\mathbf{V} = \mathbf{ZI}$. The circuit is then amenable to normal circuit analysis techniques: nodal analysis, mesh analysis, superposition, source transformations, Thévenin's theorem, Norton's theorem, etc. Time-domain responses are obtained by transforming phasor responses back to the time-domain.

- A phasor diagram is a sketch in the complex plane of the phasor voltages and currents throughout a circuit and is a useful graphical tool to illustrate, analyse and design the sinusoidal steady-state response of the circuit.

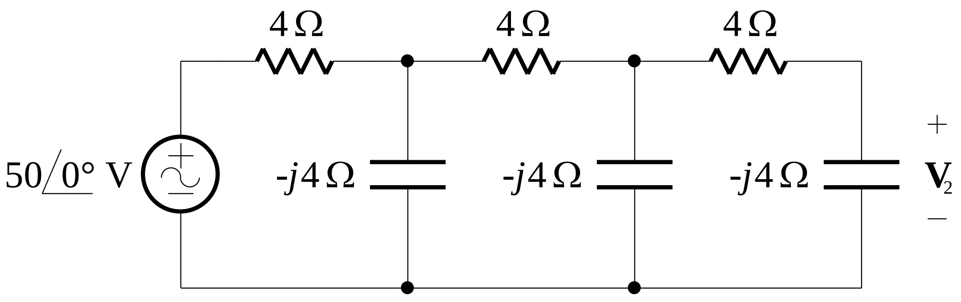
21.9 References

Hayt, W. & Kemmerly, J.: *Engineering Circuit Analysis*, 3rd *Ed*., McGraw-Hill, 1984.

Exercises

1.

Consider the circuit shown below:

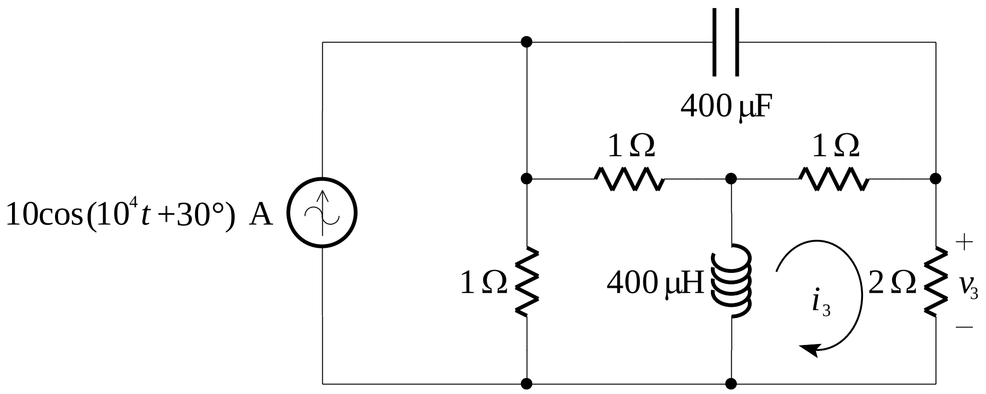


(a) Find $\mathbf{V}_2$.

(b) To what identical value should each of the $4\ \Omega$ resistors be changed so that $\mathbf{V}_2$ is $180°$ out of phase with the source voltage?

2.

Consider the circuit shown below:

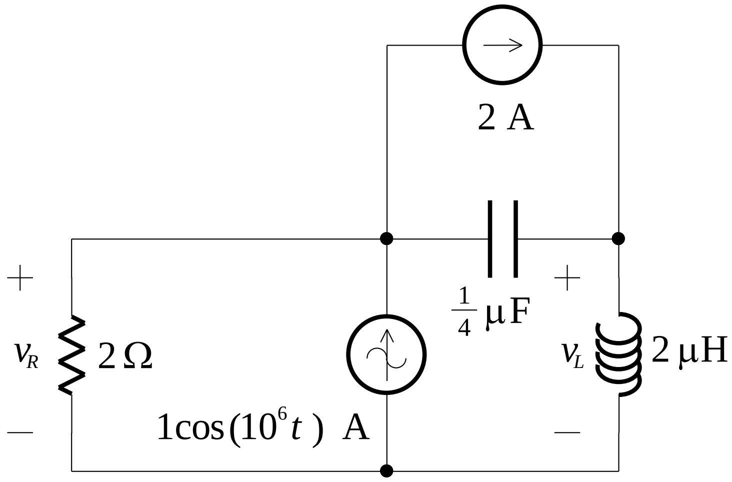


(a) Find $v_3(t)$ in the steady-state by using nodal analysis.

(b) Find $i_3(t)$ in the steady-state by using mesh analysis

3.

Consider the circuit shown below:

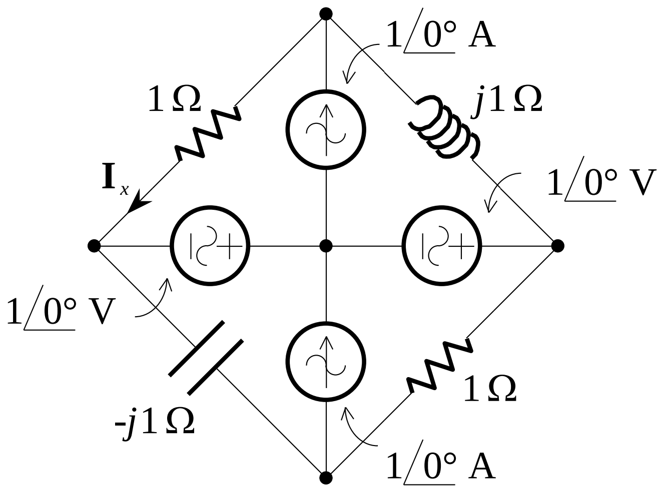


(a) Find $v_R(t)$.

(b) Find $v_L(t)$.

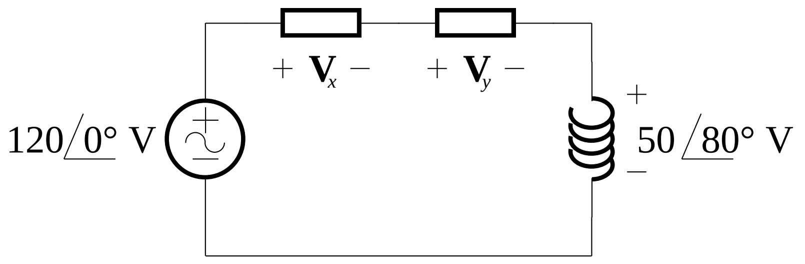
4.

Use superposition to find $\mathbf{I}_x$ in the circuit below:



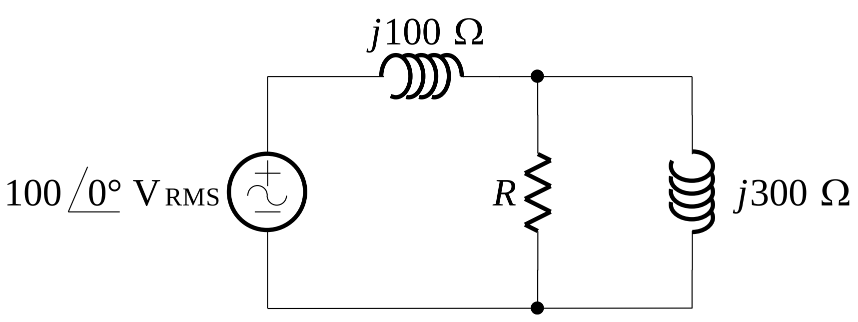
5.

For the circuit below, find the phase angles of $\mathbf{V}_x$ and $\mathbf{V}_y$ graphically if $|\mathbf{V}_x| = 90\text{ V}$ and $|\mathbf{V}_y| = 150\text{ V}$.



6.

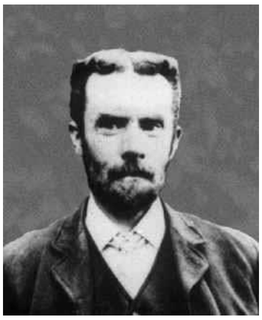
Consider the following circuit:



(a) What value of R will cause the RMS voltages across the inductors to be equal?

(b) What is the value of that RMS voltage?

Oliver Heaviside (1850-1925)



The mid-Victorian age was a time when the divide between the rich and the poor was immense (and almost insurmountable), a time of unimaginable disease and lack of sanitation, a time of steam engines belching forth a steady rain of coal dust, a time of horses clattering along cobblestoned streets, a time when social services were the fantasy of utopian dreamers. It was into this smelly, noisy, unhealthy and class-conscious world that Oliver Heaviside was born the son of a poor man on 18 May, 1850.

A lucky marriage made Charles Wheatstone (of Wheatstone Bridge fame) Heaviside's uncle. This enabled Heaviside to be reasonably well educated, and at the age of sixteen he obtained his first (and last) job as a telegraph operator with the Danish-Norwegian-English Telegraph Company. It was during this job that he developed an interest in the physical operation of the telegraph cable. At that time, telegraph cable theory was in a state left by Professor William Thomson (later Lord Kelvin) – a diffusion theory modelling the passage of electricity through a cable with the same mathematics that describes heat flow.

I remember my first look at the great treatise of Maxwell's…I saw that it was great, greater and greatest, with prodigious possibilities in its power. – Oliver Heaviside

By the early 1870's, Heaviside was contributing technical papers to various publications – he had taught himself calculus, differential equations, solid geometry and partial differential equations. But the greatest impact on Heaviside was Maxwell's treatise on electricity and magnetism – Heaviside was swept up by its power.

In 1874 Heaviside resigned from his job as telegraph operator and went back to live with his parents. He was to live off his parents, and other relatives, for the rest of his life. He dedicated his life to writing technical papers on telegraphy and electrical theory – much of his work forms the basis of modern circuit theory and field theory.

In 1876 he published a paper entitled *On the extra current* which made it clear that Heaviside (a 26-year-old unemployed nobody) was a brilliant talent. He had extended the mathematical understanding of telegraphy far beyond

Thomson's submarine cable theory. It showed that inductance was needed to permit finite-velocity wave propagation, and would be the key to solving the problems of long distance telephony. Unfortunately, although Heaviside's paper was correct, it was also unreadable by all except a few – this was a trait of Heaviside that would last all his life, and led to his eventual isolation from the "academic world". In 1878, he wrote a paper *On electromagnets, etc.* which introduced the expressions for the AC impedances of resistors, capacitors and inductors. In 1879, his paper *On the theory of faults* showed that by "faulting" a long telegraph line with an inductance, it would actually improve the signalling rate of the line – thus was born the idea of "inductive loading", which allowed transcontinental telegraphy and long-distance telephony to be developed in the USA.

Now all has been blended into one theory, the main equations of which can be written on a page of a pocket notebook. That we have got so far is due in the first place to Maxwell, and next to him to Heaviside and Hertz. – H.A. Lorentz

When Maxwell died in 1879 he left his electromagnetic theory as twenty equations in twenty variables! It was Heaviside (and independently, Hertz) who recast the equations in modern form, using a symmetrical vector calculus notation (also championed by Josiah Willard Gibbs (1839-1903)). From these equations, he was able to solve an enormous amount of problems involving field theory, as well as contributing to the *ideas* behind field theory, such as energy being carried by fields, and not electric charges.

Rigorous mathematics is narrow, physical mathematics bold and broad. – Oliver Heaviside

A major portion of Heaviside's work was devoted to "operational calculus".[1] This caused a controversy with the mathematicians of the day because although it seemed to solve physical problems, it's mathematical rigor was not at all clear. His knowledge of the physics of problems guided him correctly in many instances to the development of suitable mathematical processes. In 1887 Heaviside introduced the concept of a *resistance operator*, which in modern terms would be called *impedance*, and Heaviside introduced the symbol Z for it. He let p be equal to time-differentiation, and thus the resistance operator for an inductor would be written as pL. He would then treat p just like an algebraic

[1] The Ukrainian Mikhail Egorovich Vashchenko-Zakharchenko published *The Symbolic Calculus and its Application to the Integration of Linear Differential Equations* in 1862. Heaviside independently invented (and applied) his *own* version of the operational calculus.

quantity, and solve for voltage and current in terms of a power series in *p*. In other words, Heaviside's operators allowed the reduction of the *differential* equations of a physical system to equivalent *algebraic* equations.

Heaviside was fond of using the unit-step as an input to electrical circuits, especially since it was a very practical matter to send such pulses down a telegraph line. The unit-step was even called the Heaviside step, and given the symbol $H(t)$, but Heaviside simply used the notation **1**. He was tantalizingly close to discovering the impulse by stating "… $p \cdot \mathbf{1}$ means a function of t which is wholly concentrated at the moment $t = 0$, of total amount 1. It is an impulsive function, so to speak…[it] involves only ordinary ideas of differentiation and integration pushed to their limit."

Paul Dirac derived the modern notion of the impulse, when he used it in 1927, at age 25, in a paper on quantum mechanics. He did his undergraduate work in electrical engineering and was both familiar with all of Heaviside's work and a great admirer of his.

Heaviside also played a role in the debate raging at the end of the 19th century about the age of the Earth, with obvious implications for Darwin's theory of evolution. In 1862 Thomson wrote his famous paper *On the secular cooling of the Earth*, in which he imagined the Earth to be a uniformly heated ball of molten rock, modelled as a semi-infinite mass. Based on experimentally derived thermal conductivity of rock, sand and sandstone, he then mathematically allowed the globe to cool according to the physical law of thermodynamics embedded in Fourier's famous partial differential equation for heat flow. The resulting age of the Earth (100 million years) fell short of that needed by Darwin's theory, and also went against geologic and palaeontologic evidence. John Perry (a professor of mechanical engineering) redid Thomson's analysis using discontinuous diffusivity, and arrived at approximate results that could (based on the conductivity and specific heat of marble and quartz) put the age of the Earth into the billions of years. But Heaviside, using his operational calculus, was able to solve the diffusion equation for a finite spherical Earth. We now know that such a simple model is based on faulty premises – radioactive decay within the Earth maintains the thermal gradient without a continual cooling of the planet. But the power of Heaviside's methods to solve remarkably complex problems became readily apparent.

The practice of eliminating the physics by reducing a problem to a purely mathematical exercise should be avoided as much as possible. The physics should be carried on right through, to give life and reality to the problem, and to obtain the great assistance which the physics gives to the mathematics. – Oliver Heaviside, *Collected Works*, Vol II, p.4

Throughout his "career", Heaviside released 3 volumes of work entitled *Electromagnetic Theory*, which was really just a collection of his papers.

Heaviside shunned all honours, brushing aside his honorary doctorate from the University of Göttingen and even refusing to accept the medal associated with his election as a Fellow of the Royal Society, in 1891.

In 1902, Heaviside wrote an article for the *Encyclopedia Britannica* entitled *The theory of electric telegraphy*. Apart from developing the wave propagation theory of telegraphy, he extended his essay to include "wireless" telegraphy, and explained how the remarkable success of Marconi transmitting from Ireland to Newfoundland might be due to the presence of a permanently conducting upper layer in the atmosphere. This supposed layer was referred to as the "Heaviside layer", which was directly detected by Edward Appleton and M.A.F. Barnett in the mid-1920s. Today we merely call it the "ionosphere".

Heaviside spent much of his life being bitter at those who didn't recognise his genius – he had disdain for those that could not accept his mathematics without formal proof, and he felt betrayed and cheated by the scientific community who often ignored his results or used them later without recognising his prior work. It was with much bitterness that he eventually retired and lived out the rest of his life in Torquay on a government pension. He withdrew from public and private life, and was taunted by "insolently rude imbeciles". Objects were thrown at his windows and doors and numerous practical tricks were played on him.

Heaviside should be remembered for his vectors, his field theory analyses, his brilliant discovery of the distortionless circuit, his pioneering applied mathematics, and for his wit and humor. – P.J. Nahin

Today, the historical obscurity of Heaviside's work is evident in the fact that his vector analysis and vector formulation of Maxwell's theory have become "basic knowledge". His operational calculus was made obsolete with the 1937 publication of a book by the German mathematician Gustav Doetsch – it showed how, with the Laplace transform, Heaviside's operators could be replaced with a mathematically rigorous and systematic method.

The last five years of Heaviside's life, with both hearing and sight failing, were years of great privation and mystery. He died on 3rd February, 1925.

References

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