Lecture 4 –Transformer Connections

Parallel connection. The autotransformer. Regulation. Three- to two-phase conversion.

Transformers in Parallel

To be suitable for parallel operation the transformers must have compatible polarity, ratio and phase displacement.

For example, transformers of identical rated voltage connected Yd1 and Dy1 are compatible for parallel operation, but transformers Yd1 and Yd11 are not, as their phase displacements differ by 60°.

If all of the above conditions of compatibility are satisfied, the parallel connected transformers may still fail to share the load in a fair manner. The sharing of load depends on:

(i) Transformer impedances

(ii) Small differences in ratio

Let:

$$a_n = \text{nominal voltage ratio (primary/secondary)} \tag{4.1}$$

$$a_I = a_n - \frac{\Delta a}{2} = \text{actual ratio of transformer I} \tag{4.2}$$

$$a_{II} = a_n + \frac{\Delta a}{2} = \text{actual ratio of transformer II} \tag{4.3}$$

$$\Delta a << a_n \tag{4.4}$$

An equivalent circuit of two transformers in parallel is shown below:

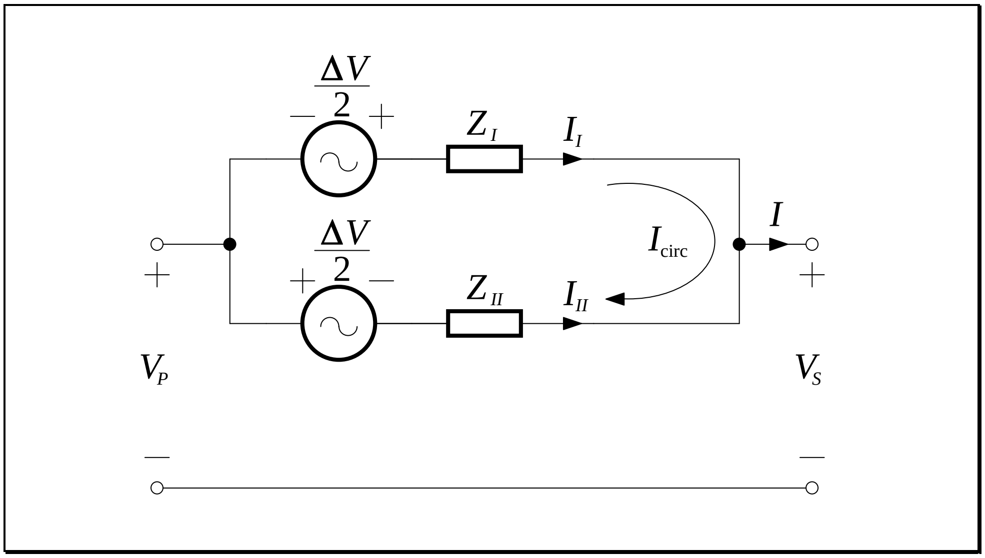


Figure 4.1 – Two transformers in parallel

On no load ($I = 0$) we have a circulating current between the transformers (using per-unit values):

$$I_{\text{circ}} = \frac{\Delta V}{Z_I + Z_{II}} \approx \frac{\Delta a}{a_n} \frac{V_P}{Z_I + Z_{II}} \approx \frac{\Delta a}{a_n} \frac{V_S}{Z_I + Z_{II}} \tag{4.5}$$

where:

$$\frac{\Delta a}{a_n} = \frac{a_{II} - a_I}{a_n} = \text{per unit difference in ratio} \tag{4.6}$$

When $Z_I = Z_{II}$ then we have a balanced bridge circuit, and hence the circulating current is independent of the load current. Even if the transformer impedances are not equal, because the load impedance is much larger than the transformer impedances, the coupling between the load current and the circulating current is weak and can be ignored.

Therefore we can first calculate the branch currents by assuming $\Delta V = 0$, and then add the circulating current to obtain the total current in each transformer:

$$I_I = I\frac{Z_{II}}{Z_I + Z_{II}} + I_{\text{circ}} \qquad I_{II} = I\frac{Z_I}{Z_I + Z_{II}} - I_{\text{circ}} \tag{4.7}$$

Example

Two 50 MVA 11 / 66 kV transformers (T1 and T2) are operated in parallel, supplying a load of 80 MW and 60 Mvar at 66 kV. The reactances ar 6% and 8% for T1 and T2 respectively, resistances are considered negligible. Both transformers have on-load tap changers providing ratio adjustments in 32 discrete steps of 0.625% over the total range of 90% to 110% of the nominal ratio.

Calculate:

a) Per-unit magnitudes of transformer currents when both transformers are operated at nominal ratio.

b) Tap adjustments to optimize load sharing.

Solution:

Use $S_{base} = 50\,\text{MVA}$, and load voltage $V_S = 1 + j0\,\text{p.u.}$ (reference).

a) <u>Both transformers at nominal ratio</u>

Load power $S_S = \dfrac{80 + j60}{50} = 1.6 + j1.2\,\text{p.u.}$

Load current $I_S = \left(\dfrac{S_S}{V_S}\right)^* = 1.6 - j1.2 = 2.0\angle -36.9°\,\text{p.u.}$

$$I_I = (1.6 - j1.2)\frac{j8}{j6 + j8} = 0.914 - j0.686 = 1.143\angle -36.9°\,\text{p.u.} \text{ (overload)}$$

4.4

$$I_{II} = (1.6 - j1.2)\frac{j6}{j6 + j8} = 0.686 - j0.514 = 0.857\angle -36.9^\circ \text{ p.u.}$$

b) <u>Ratio adjustment</u>

$$I_{\text{circ}} = \frac{\Delta a}{a_n}\frac{1}{j0.06 + j0.08} = -j7.143\frac{\Delta a}{a_n}$$

$$I_I = 0.914 - j0.686 - j7.143\frac{\Delta a}{a_n}$$

$$I_{II} = 0.686 - j0.514 + j7.143\frac{\Delta a}{a_n}$$

Note that we can alter only the reactive components, the real components are fixed.

Optimum load sharing occurs when $|I_I| = |I_{II}|$.

Putting $x = 7.143\frac{\Delta a}{a_n}$, we have:

$$|I_I|^2 = 0.914^2 + (0.686 + x)^2 = |I_{II}|^2 = 0.686^2 + (0.514 + x)^2$$

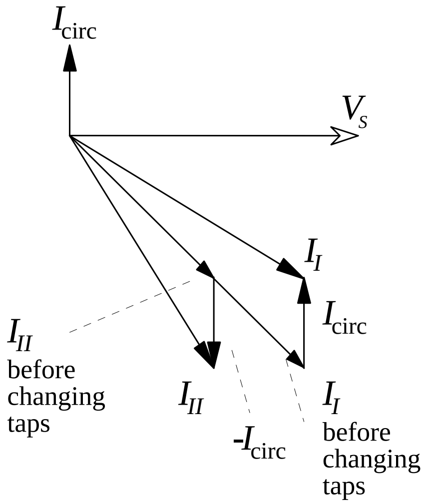
Solving we obtain $x = -0.238$, hence $\frac{\Delta a}{a_n} = \frac{-0.238}{7.143} = -0.0333 \text{ p.u.} = -3.33\%$.

$\frac{3.33\%}{0.625\%} = 5.3$ increment difference, with $a_I > a_{II}$.

Usually the taps are provided in the HV winding. Therefore increase the 66 kV tap setting of Transformer 2 by 3 increments, and decrease Transformer 1 by 2 increments.

Then $\frac{\Delta a}{a_n} = -5 \times 0.625\% = -0.03125 \text{ p.u.}$

The phasor diagram is shown below:



$$I_{\text{circ}} = \frac{-0.03125}{j0.06 + j0.08} = j0.223\,\text{p.u.}$$

$$\begin{aligned} I_I &= 0.914 - j0.686 + j0.223 \\ &= 0.914 - j0.463 \\ &= 1.025\angle -26.9^\circ \end{aligned}$$

$$\begin{aligned} I_{II} &= 0.686 - j0.514 - j0.223 \\ &= 0.686 - j0.737 \\ &= 1.007\angle -47.0^\circ \end{aligned}$$

The Autotransformer

Consider the *step-up autotransformer* with N_P primary turns and N_S secondary turns, with $N_P < N_S$. There is a saving of materials by making the primary serve as part of the secondary.

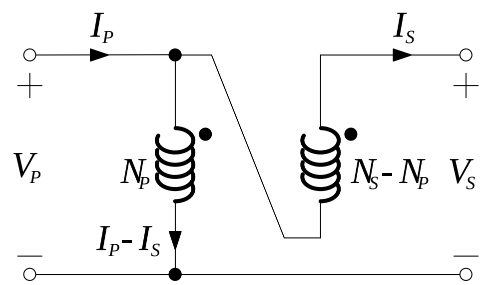


Figure 4.2 – Step-up autotransformer

In actual construction there are two windings: a *common winding* with N_P turns, and a *series winding* with $N_S - N_P$ turns. The reasons for this arrangement are:

- Different conductor sizes
- Low leakage reactance
- Mechanical forces on short circuit

Using the ideal transformer model, the mmf balance is:

$$N_P\left(I_P - I_S\right) = \left(N_S - N_P\right)I_S \tag{4.8}$$

or:

$$N_P I_P = N_S I_S \tag{4.9}$$

Also:

$$\frac{V_P}{V_S} = \frac{N_P}{N_S} = a \tag{4.10}$$

Therefore:

$$\frac{I_P}{I_S} = \frac{1}{a} \tag{4.11}$$

Let the currents and voltages be the rated values for the autotransformer. Then the rated power is:

$$|S_n| = |V_P||I_P| = |V_S||I_S| \tag{4.12}$$

If the coils were rewired so that the transformer has two separate windings with turns N_P and $(N_S - N_P)$, then the rated power would be:

$$\begin{aligned} |S_e| &= |V_S - V_P||I_S| \\ &= (1-a)|V_S||I_S| \\ &= (1-a)|S_n| \quad \text{VA} \quad \text{for } a < 1 \end{aligned} \tag{4.13}$$

$|S_e|$ is the *equivalent two-winding rating* of the autotransformer. Because $|S_e| < |S_n|$ the autotransformer can be made physically smaller than a conventional two-winding transformer of the same power rating.

Now let the equivalent two-winding transformer have a leakage impedance Z_S ohms referred to the secondary winding, which is the series winding of the transformer.

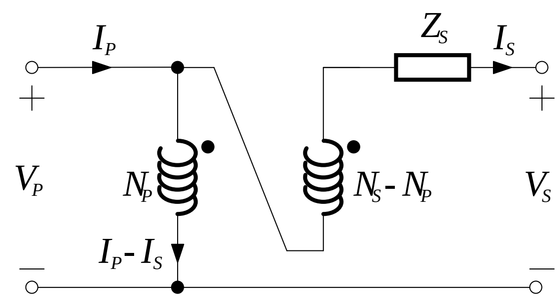


Figure 4.3 – Step-up autotransformer with leakage impedance

The per-unit leakage impedance of the autotransformer, based on its rating $|S_n|$ is now:

$$Z_{auto} = Z_S \frac{|S_n|}{|V_S|^2} \quad \text{p.u.} \tag{4.14}$$

The per-unit leakage impedance of the equivalent two-winding transformer, based on its rating $|S_e|$ is:

$$\begin{aligned} Z_{2w} &= Z_S \frac{|S_e|}{|V_S - V_P|^2} \\ &= Z_S \frac{|S_n|(1-a)}{|V_S|^2(1-a)^2} \\ &= Z_{auto} \frac{1}{(1-\text{a})} \quad \text{p.u.} \end{aligned} \tag{4.15}$$

Therefore:

$$Z_{auto} = (1 - \mathrm{a}) Z_{2w} \quad \text{p.u.} \quad \text{for } a < 1 \tag{4.16}$$

Note that the same factor $(1 - a)$ appears in Eqs. (4.16) and (4.16).

To obtain a *step-down autotransformer* $(a > 1)$, swap the primary and secondary terminals. Then:

$$\begin{aligned} |S_e| &= |V_P - V_S||I_P| \\ &= \left(1 - \frac{1}{a}\right)|V_P||I_P| \\ &= \left(1 - \frac{1}{a}\right)|S_n| \quad \text{VA} \quad \text{for } a > 1 \end{aligned} \tag{4.17}$$

Similarly:

$$Z_{auto} = \left(1 - \frac{1}{\mathrm{a}}\right) Z_{2w} \quad \text{p.u.} \quad \text{for } a > 1 \tag{4.18}$$

Regulation

Per-unit regulation of a transformer is defined as:

$$\varepsilon = \frac{\left|V_{S(NL)}\right| - \left|V_S\right|}{\left|V_{S(NL)}\right|} \quad \text{p.u.} \tag{4.19}$$

where:

$V_{S(NL)}$ = secondary voltage on no load

V_S = secondary voltage on specified load (4.20)

When the magnetising current and capacitive effects are negligible, as is the case in a power transformer, and we use per-unit values for all quantities, then Eq. (4.19) gives:

$$\varepsilon = \frac{\left|V_P\right| - \left|V_S\right|}{\left|V_P\right|} \quad \text{p.u.} \tag{4.21}$$

An equivalent circuit and phasor diagram in this case is shown below:

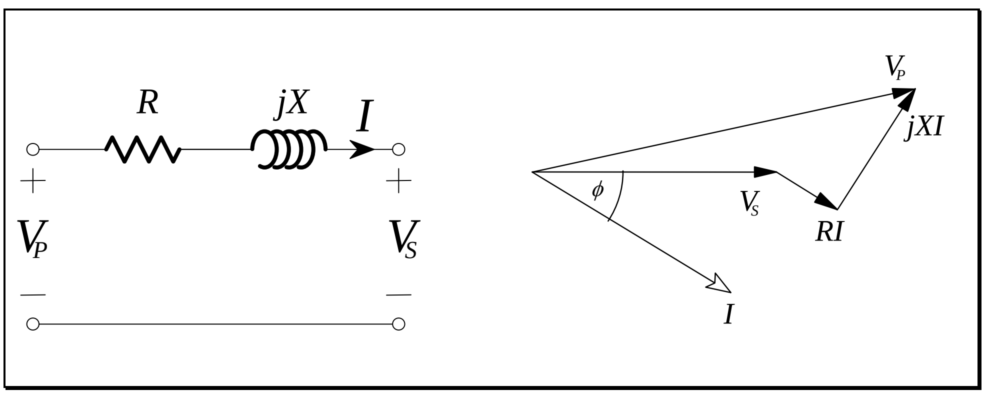


Figure 4.4 – Transformer regulation

Usually V_P is the rated voltage, and the magnitude of the impedance voltage is small, i.e. $|(R+jX)I| << |V_P|$ and $V_P = 1$ p.u. Then V_P and V_S are nearly in phase, and the following approximation can be used:

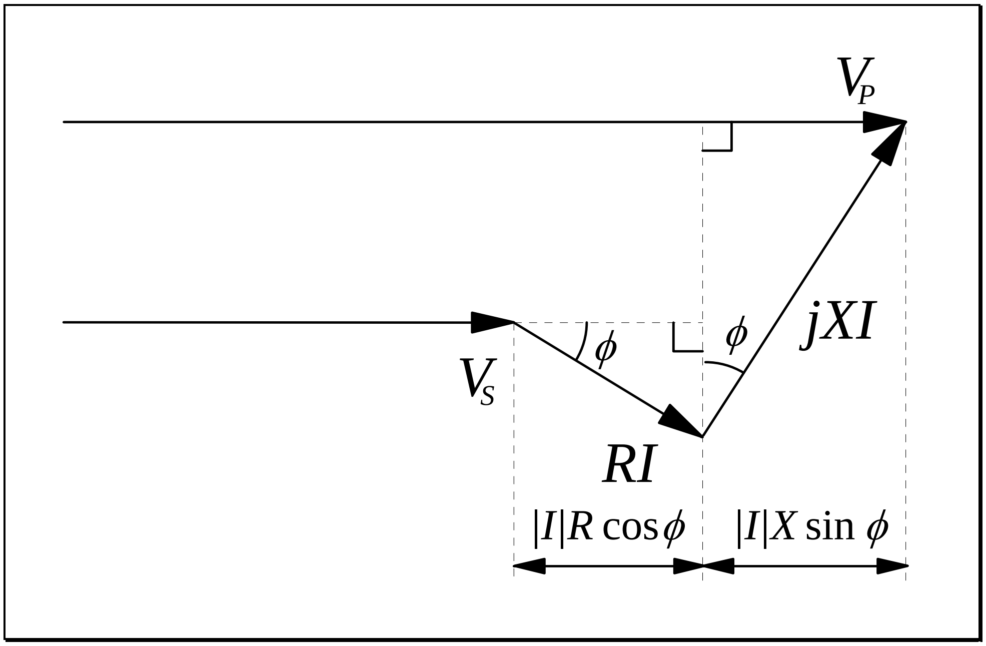


Figure 4.5 – Approximate transformer regulation

Therefore:

$$\varepsilon \approx |I|R\cos\phi + |I|X\sin\phi \tag{4.22}$$

Three-Phase to Two-Phase Conversion

Two-phase systems were the first polyphase systems. Two-phase generators were used in the first hydroelectric power station at Niagara Falls in 1895. The advantage of two-phase electrical power was that it allowed for simple, self-starting electric motors. Three-phase systems have replaced two-phase systems for commercial distribution of electrical energy, but two-phase circuits are still found in certain control systems.

Two single-phase transformers are connected as shown. This is known as the Scott connection.

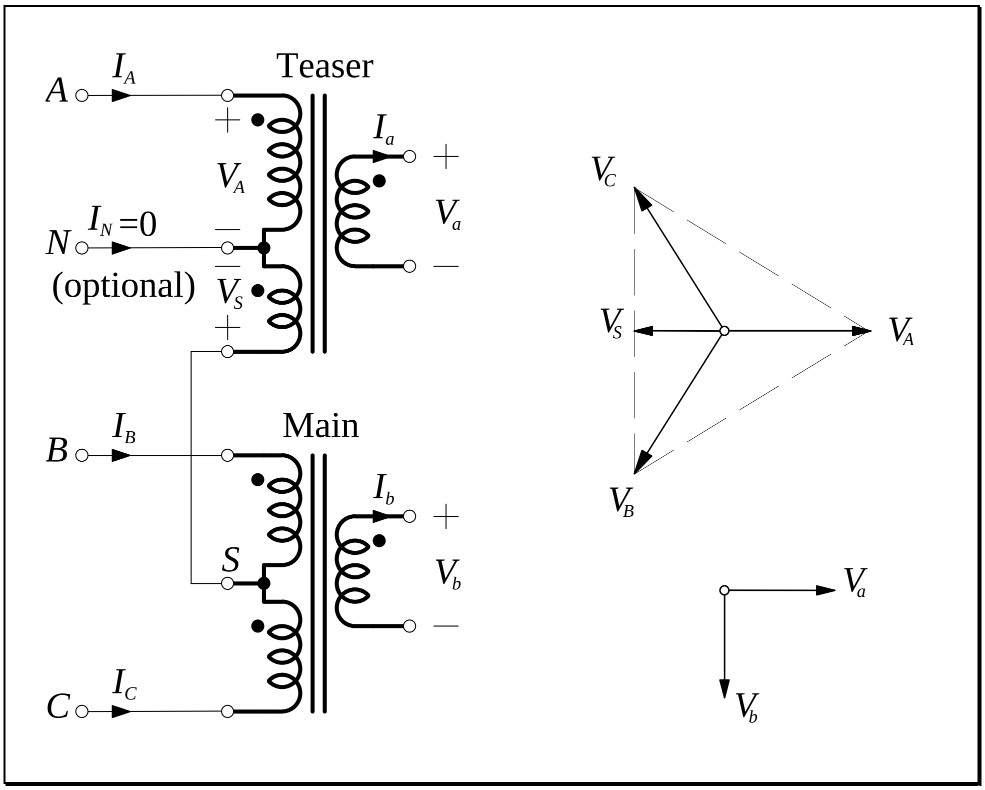


Figure 4.6 – Scott connection

The Scott connection can be found in a few special applications, such as 2 Scott connections back-to-back for low-power three-phase transformers.

Power flow can be in either direction (3-phase to 2-phase or in reverse), but we will assume the three-phase side to be the primary.

The two single-phase transformers have equal power rating:

"Main transformer" – turns ratio $\frac{N_P}{N_S} = a$. Centre tap, labelled "*S*", on primary winding.

"Teaser transformer" – turns ratio $= \frac{\sqrt{3}}{2} a$. Optional tap at $\frac{1}{3}$ primary turns.

Assume a symmetrical three-phase voltage source is connected to the primary, and use ideal transformer modelling. For the moment, assume the neutral (tapping point *N*) to be disconnected.

The secondary voltages are:

$$V_a = \frac{V_{AS}}{\frac{\sqrt{3}}{2} a} = \frac{\sqrt{3}}{a} V_A \tag{4.23}$$

$$V_b = \frac{V_{BC}}{a} = -j\frac{\sqrt{3}}{a} V_A \tag{4.24}$$

giving:

$$V_b = -jV_a \tag{4.25}$$

Now assume that the secondary is connected to a balanced two-phase load (or two equal single-phase loads) with some arbitrary power factor. Then $I_b = -jI_a$.

Since the primary voltages are balanced, the phasor diagram shows the tapping point *N* to be at zero voltage, therefore no neutral current can exist when the optional neutral connection is made. $I_N = 0$ always, and $I_B + I_C = -I_A$. Also, $I_B - I_C = \frac{2}{a} I_b = -j\frac{2}{a} I_a = -j\sqrt{3} I_A$.

Now we have two simultaneous equations:

$$\begin{aligned} I_B + I_C &= -I_A \\ I_B - I_C &= -j\sqrt{3} I_A \end{aligned} \tag{4.26}$$

where:

$$I_A = \frac{2}{a\sqrt{3}} I_a \tag{4.27}$$

Solving Eq. (4.26) gives $I_B = h^2 I_A$ and $I_C = hI_A$. This proves that for balanced two-phase secondary currents we have balanced three-phase primary currents. From Eq.

(4.27) and Eq. (4.23) we also conclude that the primary power factor is the same as the secondary power factor.

Summary

- To be suitable for parallel operation the transformers must have compatible polarity, ratio and phase displacement.

- Parallel connected transformers share the load in manner which depends on the transformer impedances and differences in the turns ratio.

- Autotransformers provide a higher rating (compared to a standard transformer) for the same cost. They therefore operate more efficiently since the losses are the same as for the ordinary connection. The loss of electrical isolation between the high- and low-voltage sides is usually the reason they are not used extensively.

- Regulation of a transformer is a measure of the voltage variation between full-load and no-load conditions.

- The Scott connection can be used to convert between a three-phase system and a two-phase system.

References

Carmo, J.: *Power Circuit Theory Notes*, UTS, 1994.

Truupold, E.: *Power Circuit Theory Notes*, UTS, 1993.

4.16

Exercises

1.

Two transformers of power ratings $|S_A|$ and $|S_B|$ have identical voltage ratings and winding connections. Their per-unit impedances Z_A and Z_B are based on $|S_A|$ and $|S_B|$ respectively. Prove that the transformers, operated in parallel, share the load according to their capability if $Z_A = Z_B$.

2.

A 500 kVA transformer having an impedance of $(1.2 + j4.2)\%$ is paralleled with a similar 300 kVA transformer having an impedance of $(1.5 + j5.0)\%$. Both impedances are referred to their respective kVA ratings. The combined load is 600 kVA at 0.75 p.f. lag. Find the complex power delivered by each transformer.

3.

The transformer in Q6 has a tapping switch to change the number of HV turns in increments of 1.25 % of the principal tapping. Two of such transformers, labelled 1 and 2, are operated in parallel (including the tertiary windings). Transformer 1 is on the principal tapping, while transformer 2 is on the next higher tap. Calculate the resulting circulating currents.

4.

Scott connected transformers are used to supply 100 V to two independently switched single-phase loads. The three-phase supply voltage is 415 V, and each single-phase load is 5 kVA, unity power factor. Determine the three-phase input currents (magnitude and phase angle) for the following conditions:

(a) Both loads connected.

(b) Load on the main transformer.

(c) Load on the teaser transformer only.